Multi-country matrix multipliers

Coordinated expansion and trade (im)balances in the *Eurozone*

Massimo Cingolani [1] Nadia Garbellini [2] Ariel L. Wirkierman [3]

[1] European Investment Bank (opinions expressed are personal)
[2] Università degli Studi di Bergamo
[3] IMS, Goldsmiths, University of London

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Topic and research questions

Topic: Fiscal policy coordination in the Eurozone

Triggering idea: foreign trade multiplier (Harrod, 1933)

- Domestic autonomous injections have

  positive income multiplier effects: response of domestic output to an increase in autonomous demand

  negative trade balance consequences: import leakages may defeat a domestic expansion

Research questions:

- How to coordinate a fiscal expansion in the Eurozone exploiting *intra*-regional feedbacks and spillovers effects? i.e. in which *proportion* should each country expand?
- What are the gains from a coordinated action with respect to an isolated policy?
- How should the intra-EZ BoT consequences of a coordinated expansion be dealt with?

Accounting for income and expenditure

GDP (Y) = Domestic absorption (D) + [Exports (E) - Imports (M)]

$$Y = D + (E - M)$$

$$\underbrace{Z}_{\text{output}} = \underbrace{X}_{\text{interm}} + Y = X + D + (E - M)$$

$$Z = X + Y = \underbrace{X_d + D_d}_{\text{dom. prod.}} + E \qquad (X + D - M = X_d + D_d)$$

$$Z = X + Y = \underbrace{(X_d + a_d Z)}_{\text{induced}} + \underbrace{(\overline{D}_d + E)}_{\text{autonomous}} \qquad (D_d = \overline{D}_d + a_d Z)$$

$$Z = X + Y = H + (\overline{D}_d + E) \qquad (H = X_d + a_d Z)$$

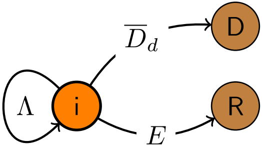
$$\underbrace{Z}_{\text{output}} = \underbrace{a_x Z + Y}_{\text{income}} = \underbrace{\Lambda Z + (\overline{D}_d + E)}_{\text{expenditure}}$$

Income matrix multipliers: theory

Scalar Multiplier (Kennedy and Thirlwall, 1979)

Expenditure: $Z = \Lambda Z + \overline{D}_d + E$

Income: $Z = a_x Z + Y$

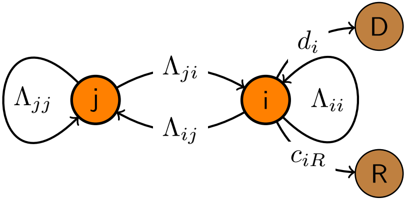


Multiplier: $\Delta Y = \left[\dfrac{1 - a_x}{1 - \Lambda}\right] \Delta(\overline{D}_d + E)$

Matrix Multiplier (Goodwin, 1983)

$z_i = \Lambda_{ii} z_i + \sum_{j \neq i} \Lambda_{ij} z_j + d_i + c_{iR}$

$z_i = a_{x_i} z_i + y_i$



$$\Delta \mathbf{y} = \underbrace{(\mathbf{I} - \widehat{\mathbf{a}}_x)(\mathbf{I} - \boldsymbol{\Lambda})^{-1}}_{\boldsymbol{\Gamma} = [\gamma_{ij}]} \Delta(\mathbf{d} + \mathbf{c}_R)$$

$$\boldsymbol{\Gamma} = [\gamma_{ij}] = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix}$$

γ_{ij}: income change of country i induced per dollar change in autonomous expenditure of country j

Balance of Trade (BoT) effects: theory

Scalar Multiplier

$$\underbrace{B}_{BoT} = E - M = E - \underbrace{\alpha_R Z}_{M}$$

$$\Delta Z = f(\Delta \overline{D}_d)$$

$$\Delta B = -\left[\frac{\alpha_R}{1-\Lambda}\right]\overline{D}_d$$

$$\mathbf{\Phi} = [\varphi_{ij}] = \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1n} \\ \vdots & \ddots & \vdots \\ \varphi_{n1} & \cdots & \varphi_{nn} \end{bmatrix}$$

Matrix Multiplier

$$b_i = \underbrace{(\underbrace{\sum_{j\neq i} \Lambda_{ij} z_j}_{\text{exports}} - \underbrace{\sum_{j\neq i} \Lambda_{ji} z_i}_{\text{imports}})}_{\text{endogenous trade}} + \underbrace{(c_{iR} - \alpha_{Ri} z_i)}_{\text{residual trade}}$$

$$\Delta z_i = f(\Delta d_1, \ldots, \Delta d_i, \ldots, \Delta d_n)$$

$$\Delta \mathbf{b} = \underbrace{(\mathbf{\Lambda} - \widehat{\boldsymbol{\alpha}})(\mathbf{I} - \mathbf{\Lambda})^{-1}}_{\mathbf{\Phi}=[\varphi_{ij}]} \Delta \mathbf{d}$$

φ_{ij}: trade balance change in country i induced per dollar change in domestic autonomous expenditures of country j

Intra/Inter-regional partition

- Divide *endogenous* countries into two regions:

  Eurozone (Region 1) vs. All other countries (Region 2)

- Matrix of production and trade coefficients ($\mathbf{\Lambda}$) and its partitioned inverse:

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} \\ \mathbf{\Lambda}_{21} & \mathbf{\Lambda}_{22} \end{bmatrix}; \qquad (\mathbf{I} - \mathbf{\Lambda})^{-1} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$$

  $\mathbf{\Lambda}_{11}$: *Intra*-regional $\mathbf{\Lambda}_{12}$, $\mathbf{\Lambda}_{21}$: *Inter*-regional

- On the basis of this partition:

  Assume: given change in domestic autonomous injections in each country of region 1, i.e. the Eurozone ($\Delta\mathbf{d}_1 \neq \mathbf{0}$), while all other autonomous components held constant ($\Delta\mathbf{d}_2 = \mathbf{0}, \Delta\mathbf{c}_{1R} = \mathbf{0}, \Delta\mathbf{c}_{2R} = \mathbf{0}$);

  Effects for **region 1**:

$$\Delta\mathbf{y}_1 = (\mathbf{I} - \widehat{\mathbf{a}}_{x1})\mathbf{B}_{11}\Delta\mathbf{d}_1 \qquad \text{(Income)}$$

$$\Delta\mathbf{b}_{1,1} = [(\mathbf{\Lambda}_{11} - \widehat{\boldsymbol{\alpha}}_{11})\mathbf{B}_{11}]\Delta\mathbf{d}_1 \qquad \text{(\textit{intra}-regional BoT)}$$

$$\Delta\mathbf{b}_{1,2R} = (\mathbf{\Lambda}_{12}\mathbf{B}_{21} - \widehat{\boldsymbol{\alpha}}_{2R,1}\mathbf{B}_{11})\Delta\mathbf{d}_1 \qquad \text{(\textit{extra}-regional BoT)}$$

Coordinated expansion and the *definition* of net gains

- **Thought experiment**: Quantify the effects of a coordinated expansion of autonomous injections in the Eurozone ($\Delta \mathbf{d}_1$), *supported by* a system of compensating accounts that collects and redistributes *intra*-regional BoT surpluses/deficits ($\Delta \mathbf{b}_{1,1}$)
- **Net gains** (in current €): GDP increase - exogenous injection + BoT variations with respect to *extra*-EZ countries

$$\boldsymbol{\pi} := \Delta \mathbf{y}_1 - \Delta \mathbf{d}_1 + \Delta \mathbf{b}_{1,2R} \quad \text{(Net Gains)}$$

- **'The' problem**: What is the country composition of $\Delta \mathbf{d}_1$ such that *all* countries:

(i) have positive net gains ($\boldsymbol{\pi} > \mathbf{0}$) and

(ii) can expand their autonomous injections ($\Delta \mathbf{d}_1 > \mathbf{0}$)?

- Non-trivial: *wrong* proportions may lead to fiscal expansions defeated by BoT consequences *or* requirements of fiscal contraction for some countries to avoid negative BoT effects

Solving the problem: a coordination eigen-system

- What is the country composition of $\Delta\mathbf{d}_1$ such that $\boldsymbol{\pi} > \mathbf{0}$ and $\Delta\mathbf{d}_1 > \mathbf{0}$ for all countries?

  Define: $\mathbf{M} := (\mathbf{I} - \widehat{\mathbf{a}}_{x1} - \widehat{\boldsymbol{\alpha}}_{2R,1})\mathbf{B}_{11} + \boldsymbol{\Lambda}_{12}\mathbf{B}_{21}$, then

$$\boldsymbol{\pi} = (\mathbf{M} - \mathbf{I})\Delta\mathbf{d}_1 \geq \mathbf{0} \iff \mathbf{M}\Delta\mathbf{d}_1 \geq \Delta\mathbf{d}_1$$

  In particular, by adding $\psi\Delta\mathbf{d}_1$ to the RHS of the inequality — and turning it into an equality — we aim to find $\Delta\mathbf{d}_1$ such that:

$$\mathbf{M}\Delta\mathbf{d}_1 = \Delta\mathbf{d}_1 + \psi\Delta\mathbf{d}_1 \iff (1+\psi)\Delta\mathbf{d}_1 = \mathbf{M}\Delta\mathbf{d}_1 \quad (\psi > 0)$$

  For $\mathbf{M} \geq \mathbf{0}$ and irreducible:
  (Meyer, 2000, p. 673, Perron-Frobenius Theorem)

  $\Delta\mathbf{d}_1^*$ is the eigenvector associated to the *leading* eigenvalue of $\mathbf{M}$;

  and is the *only* set of coordinated autonomous injections (up to a scalar) that satisfies the desired conditions.

Coordinated Eurozone expansion: what we compute

With $\Delta \mathbf{d}_1^*$ at hand, we compute:

(i) Proportional distribution of coordinated fiscal expansion:

$$\frac{\Delta \mathbf{d}_1^*}{\mathbf{e}^T \Delta \mathbf{d}_1^*}$$

(ii) Comparison between a coordinated and an equivalent isolated expansion:

$$\Delta \mathbf{d}_1^* \Rightarrow \underbrace{\begin{bmatrix} \mathbf{y}_1^* \\ \Delta \mathbf{b}_{1,1}^* \\ \Delta \mathbf{b}_{1,2R}^* \end{bmatrix}}_{\text{coordinated}} \quad \text{vs.} \quad \Delta \tilde{\mathbf{d}}_1 \Rightarrow \underbrace{\begin{bmatrix} \tilde{\mathbf{y}}_1 (= \Delta \mathbf{y}_1^*) \\ \Delta \tilde{\mathbf{b}}_{1,1} \\ \Delta \tilde{\mathbf{b}}_{1,2R} \end{bmatrix}}_{\text{isolated}}$$

(iii) Contribution to consolidated intra-regional surplus/deficit (as a proportion of total *intra*-EZ BoT changes):

$$\frac{\Delta \mathbf{b}_{1,1}^*}{(1/2)\mathbf{e}^T |\Delta \mathbf{b}_{1,1}^*|}$$

Dataset and Accounting Framework

- Purposely built multi-country scheme of trade and production for **62 endogenous countries** plus a residual RoW region (2004-2011)
- Commodity categories: intermediate inputs, fixed capital goods, final consumption products
- Capacity-generating investment induced by effective demand for final uses, i.e. accelerator principle (Pasinetti, 1981, p. 176)
- Main data sources:
  - (i) OECD BTDIxE Database (Zhu et al., 2011), UNSD Trade in Services (`comtrade.un.org`)
  - (ii) UNSD SNA Main Aggregates and Detailed Accounts (`unstats.un.org/unsd/snaama/`)
  - (iii) IMF International Financial Statistics
  - (iv) National Statistical Offices of individual countries
- Reconciliation of different data sources via bi-proportional matrix updating methods

(i) Distribution of a coordinated fiscal expansion

	2004	2005	2006	2007	2008	2009	2010	2011
AUT	3.05	3.08	3.12	3.01	3.00	2.95	2.89	2.93
BEL	2.35	2.29	2.33	2.31	2.19	2.20	2.08	2.07
CYP	0.07	0.08	0.08	0.08	0.08	0.13	0.22	0.19
DEU	23.84	22.82	21.94	22.15	21.37	19.57	19.25	19.01
ESP	11.59	12.53	13.21	13.46	14.20	15.69	14.37	15.29
EST	0.05	0.05	0.06	0.07	0.07	0.06	0.06	0.05
FIN	1.05	0.99	0.98	1.00	0.98	0.91	0.88	0.85
FRA	22.30	22.95	23.96	24.48	24.73	22.37	21.68	23.22
GRC	2.03	2.59	2.85	2.47	3.19	**4.57**	**9.60**	**7.11**
IRL	0.82	0.77	0.90	0.93	0.91	0.83	0.74	0.72
ITA	26.04	24.98	23.93	23.46	22.68	23.75	21.97	22.00
LUX	0.20	0.19	0.18	0.16	0.15	0.15	0.17	0.17
MLT	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
NLD	3.18	3.13	2.97	2.71	2.86	2.70	2.30	2.42
PRT	2.80	2.89	2.81	2.98	2.82	3.33	3.05	3.23
SVK	0.26	0.29	0.30	0.35	0.41	0.41	0.38	0.39
SVN	0.35	0.34	0.35	0.35	0.35	0.35	0.32	0.31
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

- Highest injections from France and Italy, then Germany and Spain
- Injections from **Greece** increase since 2009 and especially 2010

(ii) Coordinated vs. Isolated action, 2011 (€bln.)

	Panel (A): Coordinated Fiscal Stimulus					Panel (B): Equivalent Isolated Action				
	$\Delta \mathbf{y}^*$	$-\Delta \mathbf{d}^*$	$\Delta \mathbf{b}^*_{1,1}$	$\Delta \mathbf{b}^*_{1,2R}$	$\boldsymbol{\pi}^*$	$\Delta \tilde{\mathbf{y}}$	$-\Delta \tilde{\mathbf{d}}$	$\Delta \tilde{\mathbf{b}}_{1,1}$	$\Delta \tilde{\mathbf{b}}_{1,2R}$	$\tilde{\boldsymbol{\pi}}$
AUT	7.08	-3.21	-0.50	-0.82	3.05	7.08	-5.60	-2.46	-1.24	-2.22
DEU	49.33	**-20.84**	2.64	-8.68	**19.81**	49.33	**-33.85**	-8.01	-11.05	**-3.57**
ESP	38.13	**-16.76**	-1.17	-5.45	**15.93**	38.13	**-21.53**	-5.49	-5.91	**5.21**
FRA	56.72	**-25.44**	-2.58	-7.08	**24.19**	56.72	**-31.72**	-8.08	-7.85	**9.06**
GRC	18.20	**-7.80**	-2.19	-2.99	**7.41**	18.20	**-8.30**	-2.57	-3.12	**4.22**
IRL	2.06	-0.79	0.63	-0.51	0.75	2.06	-2.63	-0.83	-0.89	-2.29
ITA	55.31	**-24.11**	-2.29	-8.27	**22.92**	55.31	**-29.57**	-6.89	-9.13	**9.71**
NLD	8.45	-2.65	4.16	-3.28	2.52	8.45	-9.48	-1.84	-4.12	-6.99
PRT	7.90	**-3.54**	-1.00	-1.00	**3.36**	7.90	**-4.60**	-2.00	-1.07	**0.23**
Total	**255.03**	**-109.59**	0.00	-41.23	104.20	255.03	-159.70	-42.57	-48.77	3.98
	Net Gains: $\boldsymbol{\pi}^* = \Delta \mathbf{y}^* - \Delta \mathbf{d}^* + \Delta \mathbf{b}^*_{1,2R}$					Net Gains: $\tilde{\boldsymbol{\pi}} = \Delta \tilde{\mathbf{y}} - \Delta \tilde{\mathbf{d}} + \Delta \tilde{\mathbf{b}}_{1,1} + \Delta \tilde{\mathbf{b}}_{1,2R}$				

- Injections for €109 bln. induce €255 bln increase in GDP;
- To achieve the same result, Germany should spend €34 bln. as against €21 bln. (obtaining a negative rather than positive net gain);
- **Italy** and **France** would also benefit from a coordinated rather than isolated policy, but the losses would be smaller than for Germany;
- The same holds for **Spain**, **Portugal** and **Greece**.

(iii) Contribution to *intra*-regional trade balances

Intra-EZ BoTs do not participate into net gains: a system of **intra-regional** transfers can compensate imbalances

Contribution to consolidated intra-regional surplus/deficit
(as a proportion of total intra-EZ BoT changes)

	2004	2005	2006	2007	2008	2009	2010	2011
AUT	-11.64	-5.88	-5.40	-4.36	-4.44	-4.72	-4.08	-5.12
BEL	31.56	14.88	14.14	13.70	13.56	12.06	12.12	**12.56**
CYP	0.64	0.40	0.46	0.38	0.10	0.54	1.04	0.62
DEU	40.58	22.56	24.26	28.20	26.74	30.16	29.54	**27.16**
ESP	-27.64	-14.34	-18.52	-19.44	-18.62	-16.02	-11.36	**-12.02**
EST	0.20	0.14	0.04	-0.04	0.04	0.20	0.20	0.28
FIN	5.26	2.52	2.64	2.32	2.76	1.94	1.84	1.42
FRA	-30.08	-21.42	-25.98	-29.04	-31.52	-20.65	-19.08	**-26.5**
GRC	-13.22	-8.42	-8.90	-8.14	-12.60	-17.42	-35.98	**-22.5**
IRL	23.42	11.80	9.50	8.30	6.58	7.50	6.04	6.44
ITA	-90.58	-37.88	-30.56	-28.04	-22.24	-27.95	-19.00	**-23.52**
LUX	8.02	4.42	5.08	5.72	5.78	5.74	5.28	5.14
MLT	0.40	0.22	0.22	0.20	0.28	0.28	0.38	0.38
NLD	87.40	41.56	41.76	38.58	41.94	39.18	40.74	**42.74**
PRT	-25.26	-11.62	-10.12	-10.54	-10.32	-13.22	-10.50	-10.34
SVK	2.52	1.52	1.90	2.60	2.22	2.32	2.64	2.96
SVN	-1.58	-0.46	-0.52	-0.40	-0.26	0.06	0.18	0.30
Total	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

- ▷ **Germany** almost compensates **France**
- ▷ **Netherlands** almost compensates **Italy** and **Greece**
- ▷ **Belgium** almost compensates **Spain**

Final remarks

- Required Coordination Mechanisms:

(i) political will to clear intra-regional trade balances (surpluses/deficits)

(ii) an agreed structure of domestic autonomous injections

- Eigenvector proportions:

(i) solve a *coordination* problem: distribution of the joint effort among countries with positive overall net gains for each trade partner;

(ii) define eigen-centrality: most central economies trigger the rest of the region and — at the same time — benefit from such a coordinated expansion;

(iii) represent an endogenous desirable structure of expenditure subject to trade balance constraints, taking into account *trade in intermediates*

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