EDUCATION AND ‘HUMAN CAPITALISTS’ IN A CLASSICAL-MARXIAN MODEL OF GROWTH AND DISTRIBUTION*

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Abstract: A simple classical-Marxian model of growth and distribution is developed in which education transforms low-skilled workers into high-skilled ones and in which high-skilled workers save and hold capital, therefore receiving both high-skilled wages and profit income. We analyze the implications for class divisions, growth and distribution, of the transformation of the modern capitalist economy from one in which the main class division is between capitalists who own capital and workers who only receive wage income into one in which education and human capital play a major role. We show than an expansion in education can have a positive effect on growth but by altering the distribution of income rather than by fostering technological change, and that it yields some changes in income distribution and the class structure of the capitalist economy, but need not alter its fundamental features.

JEL classification codes: E2, E11, O41, I24

Keywords: education, human capital, workers’ savings, growth, distribution

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1. Introduction

Advanced capitalist economies are often described as "knowledge economies" and economic growth and development are increasingly seen as resulting from "human capital" accumulation, education and knowledge creation and diffusion. The knowledge economy has been described as "production of services based on knowledge-intensive activities that contribute to an accelerated pace of technological and scientific advance", its key components including "a greater reliance on intellectual capabilities than on physical inputs or natural resources" (Powell and Snellman 2004: 201). The analysis of economic growth and development has long focused on the accumulation of physical and financial capital, but has also stressed the importance of technological change and education. Although the first use of the term "human capital" and the popularization of the concept are relatively recent, dating to the early 1960s, the role of education and knowledge in economic growth has been examined for a long time (see Dutt and Veneziani 2011-12), as has the concept and measurement of human capital (Kiker 1966). Schultz (1960) referred to the term human capital, which is integral to the person and cannot be bought and sold, but which improves the capabilities of people as they work and increases income and production as the consequence of investment in human being due to formal education. He and others subsequently expanded the concept to include expenditures on health and internal migration, as well as on-the-job training (Schultz 1961, Becker 1962).

Now the importance of education, knowledge and human capital in the growth process are routinely emphasized (Lucas 1988, Savvides and Stengos 2009). Becker (2002: 3) estimates that around 70 per cent of all capital in the United States consists of capital invested in people, in the form of schooling, on-the-job training, health, information and R&D. He argues that while economies such as the US are called capitalist, it is more accurate to call them 'human capital' or 'knowledge capital economies', although physical and financial capital continue to have a role.

Lindsey (2013: 23) states that Marx's term for modern social systems dominated by the owners of physical capital, capitalism, is more appropriately called 'human capitalism'.

Suppose that there has indeed been a change in the socio-economic system due to the increasing importance of education and human capital. What are the implications of this shift? What does it imply for economic growth and the distribution of income, the relation between growth and inequality, its ethical implications, and the possibilities for improvement through public action? We address these questions by means of an extended classical-Marxian model of growth and distribution.

In the basic classical-Marxian framework, physical capital and homogenous labor are the productive inputs and there are two classes, capitalists who own and accumulate capital, and workers who own no capital and work for a wage. The real wage is determined by the state of class struggle, and consumed entirely by workers. Capitalists receive profits and save a portion of it, investing to accumulate more capital, which allows them to hire more workers and produce more (given capital-output and labor-output ratios). The growth rate of output is equal to the growth rate of capital, which depends on saving and investment and which in turn is affected by the distribution of income. Although accumulation tends to increase labor demand and improve workers' bargaining position, labor-displacing technological change and the expansion of labor supply in the capitalist sector – due to new entrants into the labor force and to the closing of self-employed and family businesses – act as countervailing forces.

As a result, the distribution of income between capitalists and workers is likely to remain unequal. Moreover, there is little or no within-generation or intergenerational mobility between classes: workers hardly ever become capitalists, and vice versa, because of imperfect credit markets and high fixed costs of starting a capitalist enterprise. Even when workers save, this is usually only in order to sustain consumption later in life. Even if real wages increase, this is often associated with increases in consumption norms, which prevents workers from saving and accumulating assets, squeezes profits, and reduces capital accumulation.

Thus, in a society divided into two classes, the prospect of reducing inequality through "market" forces (especially while maintaining a high growth rate) seems limited. In such a context it is difficult to provide an ethical justification of inequality, which is explained largely by class structure and wealth ownership rather than merit, effort, and individual choices. It may thus be supposed that the division of society into two distinct classes with clearly opposed interests can create a high degree of within-class solidarity, and the possibility of radical political and institutional changes. Nevertheless, the concentration of income and hence power in the hands of capitalists may prevent changes in distribution without changing the structure of the economy, *and* make it difficult to restructure society, because of their opposition to any asset redistribution.

The introduction of education and human capital can be, and has been, argued to fundamentally change the structure of the economy.[1] Adding human capital as an input into production implies that economic growth depends on the accumulation of human capital. It also alters the class structure of the economy because, for instance, high-skilled and low-skilled workers, may be thought of as being two different classes, with distinct interests. Income distribution depends not just on who owns physical and financial capital, but also on who obtains more education and possesses higher skills. If skills spread, and workers receive higher wages, increases in human capital can be equalizing by increasing the workers' share in total income, so that greater equality can accompany higher growth.

Obtaining education is likely to be less difficult than saving enough from low wages to become a capitalist, especially if the government helps people to obtain education. Inequality is likely to be limited because, unlike financial capital, human capital cannot be accumulated without limit. There is likely to be more mobility between classes, especially from low- to high-skilled categories, as workers become human capitalists. The increase in wages made possible by acquiring more education can allow high-skilled workers to save more and acquire financial and even productive assets. The expansion of education and human capital accumulation promote distributional equity because education has externality and public goods properties, and can raise the productivity of all, unlike physical and financial capital (although capital can increase the productivity of those workers who work with it).[2]

In terms of ethical considerations, if income distribution is determined more by education and training, the system can be argued to take on meritocratic features, as inequality can be explained more by choice and merit than by birth, and because "capital" becomes inseparable from people and, in fact, a part of them. The division of workers into different types based on skills can make the emergence of working class solidarity more difficult, especially if higher-income workers also become capitalists. Thus, political change may become less feasible, but it is also less pressing if society becomes more egalitarian and social mobility increases.

To examine these issues we build on the formal framework developed in Dutt and Veneziani (2010). To be specific, we extend the classical-Marxian model to include human capital accumulation due to education, and assume that there are *three* classes: capitalists, low-skilled workers and high-skilled workers (henceforth, L-workers and H-workers, respectively).[3] Both kinds of workers have basic education, but only H-workers obtain higher education and thereby acquire high skills. We assume that capital, low-skilled and high-skilled labor are inputs into production, and the two kinds of labor are substitutes only to some degree; that growth occurs with unemployed workers and the low-skilled real wage is determined by class struggle; that education converts L-workers into H-workers who receive a (market clearing) skill premium; that investment is determined by saving (so that we are abstracting from the role of effective demand); and that labor productivity growth is driven by human capital accumulation.

In Dutt and Veneziani (2010), we assumed that capitalists save and accumulate physical capital while both types of workers consume all of their income. This assumption is not suitable to examine the changing landscape of class cleavages and the relevance of various class distinctions in advanced capitalist economies. Therefore in this paper we introduce a major modification to our earlier model, by allowing H-workers to save and hold capital, like capitalists. Thus as in Pasinetti's (1962) seminal model, we can analyze changes in the dynamics of capital ownership. However, while Pasinetti examined the distribution of capital between capitalists and workers, we focus on capitalists and H-workers, while L-workers do not save.

Our analysis differs from a large literature on human capital, some of which simply *replaces* physical and financial capital with human capital, rather than considering all types of capital, and much of which takes a (sometimes implicit) neoclassical full employment approach that assumes labor supply to grow at an exogenously-given rate, rather than the classical-Marxian model with unemployed labor or an endogenous labor supply.

Empirical studies suggest that even without major shifts in the functional distribution of income between wages and profits, income inequality has increased in many countries, including the United States, as a result of a rise in *wage* inequality (see, for instance, Lawrence and Slaughter 1993, Autor et al 2008). Neoclassical growth models explain this in terms of the differential accumulation of education in the presence of neighborhood schooling, credit market imperfections, and intergenerational externalities in the productivity of education, focusing on the gap between high-skilled and low-skilled wages (Galor and Zeira 1993, Bénabou 1996, Durlauf 1996). Neoclassical and new (neoclassical) growth models (see Uzawa 1965, Lucas 1988, among others) examine both physical capital and human capital accumulation due to education, but make full employment assumptions that seem difficult to justify, both historically and in the light of the current crisis.

Trade-theoretic models emphasize increases in wage inequality in high-income countries due to increasing North-South trade, owing to trade liberalization and lower transport costs, which has increased imports of low-skilled labor intensive goods from low-income countries, and increased high-skilled labor intensive exports, thereby increasing the demand for, and the relative wage of high-skilled workers (Wood 1994). Including both physical and human capital makes possible the analysis of the relative roles of capital ownership and human capital in the growth process and in income distribution, and the relative power of different types of capitalists and workers.

In summary, compared with the standard, neoclassical approach, we believe that a classical-Marxian model incorporates some key empirical regularities of advanced economies (such as persistent unemployment, relatively limited substitutability between productive factors, class-based differential savings rates, and so on) and provides a simple and flexible theoretical framework to analyze the evolution of class cleavages in advanced capitalist economies, and their effect on growth and distribution.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the dynamics of the economy. Section 4 examines the growth and distributional effects of changes in policy variables and behavioral parameters. Section 5 concludes.

2. Structure of the model

Consider a closed economy that produces one good with three factors of production: capital, high-skilled labor and low-skilled labor. There are three classes in the economy: capitalists who do not work and derive their income only from the ownership of capital; H-workers who possess higher education, supply high-skilled labor to firms or serve as educators, receiving a high-skilled wage, and also own capital; and L-workers who possess basic education and supply low-skilled labor, own no capital, and receive a low-skilled wage.

Production uses fixed coefficients input-output relations with capital and a mixture of high-skilled, H_P, and low-skilled, L, labor as inputs. The symbol H_P refers to H-workers employed by capitalist firms as distinct from H-workers employed as educators, which we denote by H_E. The productivities of high-skilled and low-skilled labor are given at a point in time, t, by A_H and A_L, respectively, and the maximum output that can be produced by a unit of capital is a_K. The production function of the standard firm is

$$Y = min[a_K K, f(A_L L, A_H H_P)], \tag{1}$$

where Y is the output of the good, K is the capital stock, and f is an index of the two types of (effective) labor. This production function is in line with standard heterodox assumptions in rejecting the substitutability between labor and capital, but in principle it allows for some substitutability between the two types of labor. By distinguishing qualitatively between H-workers and L-workers, we are departing from much of the neoclassical growth-theoretic approach which assumes that education simply makes L-workers more productive by increasing their quantity in effective units. In this respect, our approach is closer to the neoclassical trade-theoretic literature (Wood 1994) and to those classical-Marxian notions which see workers of different skills as belonging to different classes. We assume that H-workers are more productive at all *t*, and that their productivity advantage remains constant over time. Formally, there is a scalar $\mu \geq 1$, such that $A_H = \mu A_L$, all *t*.[4]

Let $\sigma = \frac{w_H}{w_L}$ denote the ratio of high-skilled and low-skilled wages. Concerning firm behavior, we stipulate that at all *t* the demand for high-skilled labor is given by

$$H_P^D = \frac{b(\sigma)K}{A_L} \tag{2}$$

where $b(\sigma) = \frac{\alpha_0}{\sigma}$ and α_0 is a positive parameter. Similarly, the demand for low-skilled labor is

$$L^D = \frac{c(\sigma)K}{A_L} \tag{3}$$

where $c(\sigma) = \alpha_1 + \alpha_2\sigma$, and α_1 and α_2 are positive parameters. Clearly, *b*(.) is decreasing and *c*(.) is increasing in σ, so that firms substitute H-workers by L-workers when the relative wage of H-workers increases. Equations (2) and (3) can be interpreted either as the reduced form of a general model with identical profit-maximizing, perfectly competitive firms, as in Dutt and Veneziani (2010); or, from a macro perspective, and more in line with a classical-Marxian approach, as a concise way of capturing the average behavior of capitalist firms.

The key role of the government is to provide education (we abstract from other fiscal activities in order to focus on the main topic of this paper). Education requires only high-skilled labor and is organized by the government, which employs a fraction, ε, of the total stock of H-workers at *t*, H^S. Government expenditure on education is $w_H \varepsilon H^S$ and we assume that it is financed entirely by taxes on profit income.

Labor markets are modeled as follows. L-workers are in unlimited supply, and along standard neo-Marxian lines, their real wage is determined by the relative bargaining power of L-workers and firms, or what has been called the “state of class struggle”. We parameterize this state in terms of the real wage of L-workers relative to their efficiency factor, so that given the state of class struggle, an increase in A_L results in a proportionate increase in w_L. Formally, there exists a positive parameter, λ, such that $w_L = \lambda A_L$.

The market for H-workers is flexprice, and the skill premium adjusts to clear the market, given w_L. Our assumption is similar to the approach famously advocated by Mill, according to whom the wage differential between H-workers and L-workers is determined by the supply of H-workers (for a discussion, see Dutt and Veneziani 2010). In our model, w_L serves as a reference point, and given the skill premium, high and low-skilled wages increase proportionately. Formally, given H^s, at any t, σ solves the following equation

$$(1\text{-}\varepsilon)H^s = b(\sigma)K/A_L. \tag{4}$$

Given the assumptions on the labor market, in what follows we use the symbols H and L, to denote the quantities of H-workers and L-workers employed (including, in the case of H-workers, in the education sector).

Productivity increases derive from learning-by-doing processes and innovation activity by H-workers – both those employed by capitalist firms and those working in the education sector. To be specific, we assume that the growth rate of labor productivity of H-workers depends positively on the amount of high-skilled labor in efficiency units as a ratio of the stock of capital as a scaling factor representing the size of the productive economy. We adopt a simple linear functional form, and – denoting growth rates by overhats – assume that there exist positive scalars τ_0 and τ_1 such that

$$\hat{A}_H = \tau = \tau_0 + \tau_1 \frac{A_H H}{K}. \tag{5}$$

Because we abstract from firm heterogeneity, A_H can be thought of as representing the average productivity of H-workers. Given that the productivity differential between the two types of workers remains constant over time, $\hat{A}_L = \hat{A}_H$, all t, and we can write

$$\hat{A}_L = \tau = \tau_0 + \tau_1 \mu \frac{A_L H}{K}. \tag{5a}$$

In other words, we conceptualize innovations as non-rival products of learning-by-doing and innovative activity with a spillover to L-workers. As noted earlier, this approach stresses the qualitative difference between H-workers and L-workers: L-workers are employed in routine production activities, while H-workers are innovators and contribute to labor productivity growth.[5] We therefore assume that education converts workers who could only do repetitive activities into discoverers and innovators, although they continue to be engaged in some routine activities, activities which are qualitatively different from those of L-workers.

Low-skilled labor is converted into high-skilled labor through education. The dynamics of the stock of high-skilled labor H is given by the following equation, where $\theta > 0$ and $\sigma_m \geq 1$,

$$\frac{dH}{dt} = \theta(\sigma - \sigma_m)\varepsilon H. \tag{6}$$

whenever $\sigma \geq \sigma_m$, whereas for all $\sigma < \sigma_m$, $dH/dt = 0$. According to equation (6), the change in the stock of H-workers depends, first, on the demand for education which, in turn, depends positively on the skill premium and the 'return' to education. No one seeks education if the skill premium is at or below some level which is a measure of the cost and trouble of obtaining higher skills, as famously argued by Smith (1776, Book I, Chapter 10). Second, it depends on the stock of H-workers, and in particular on the availability of mentors and educators. Third, it depends on a parameter, θ, which captures the openness of the education system, either through government policy or through other institutional features. Easier access to low-cost public education and greater access to student loans and grants, the extent to which people from low-income families can obtain better basic education to prepare them for higher education, and a more open private education system, which is less elitist on the basis of class and income, all increase θ.

Our approach differs from much of neoclassical theory, which focuses on individual educational choices, and in which the stock of human capital depends on individual preferences (reflected, for instance, in the rate of time preference) and on the private returns to schooling. Because in our model the wage differential affects the rate of education, it is not inconsistent with the choice approach. However, it stresses other factors, such as the degree of access to education, and the wage differential may reflect increases in the opportunity to obtain education because of subsidies provided by businesses that react to the relative cost of educated workers. Our approach is therefore less specific than the neoclassical one, but we consider this lack of specificity to be a virtue because it opens up space for other determinants of the spread of education, which are usually crowded out in the neoclassical approach.

Concerning consumption and saving behavior, we assume that L-workers consume their entire income, H-workers save a fraction, s_H of their income, and capitalists save a fraction, s_C, of their profits, where $s_H \leq s_C$. This can be justified on the ground that capitalists are richer, or that saving out of profits includes saving by firms, which makes the saving rate out of profits higher even if capitalists and H-workers save at the same rate.

Total profit net of taxes is given by

$$rK = Y - w_L L - w_H(1-\varepsilon)H - w_H \varepsilon H = Y - w_L L - w_H H, \tag{7}$$

where $w_H \varepsilon H$ denotes the taxes on profits to cover the government's education expenditure, and *r* is the rate of profit net of taxes.

The capital stocks owned by capitalists and H-workers are, respectively, K_C and K_H, where $K_C + K_H = K$. We assume that capitalists and H-workers receive the same return on their capital, r. Aggregate consumption expenditure is therefore given by

$$C = (1 - s_C)rK_C + (1 - s_H)(w_H H + rK_H) + w_L L. \tag{8}$$

Finally, we assume that saving, S, and investment, I, are always equal, since firms invest whatever saving is available. This version of Say's law is a standard assumption of the classical-Marxian modeling approach (although aggregate demand considerations were discussed by Malthus, and especially, Marx).[6] Together with equation (8) it implies

$$S = I = s_C rK_C + s_H(w_H H + rK_H), \tag{9}$$

Hence there is no effective demand problem, and, given the existence of unemployed L-workers:

$$Y = a_K K. \tag{10}$$

3. The dynamics of the economy

We examine the dynamics of the economy by considering in turn the short and long run. In the short run, the levels of K_C, K_H (and hence, K), H and A_L are fixed, and equations (3), (4), (7), (9), and (10) solve for Y, L, σ, r and I . In short-run equilibrium, the profit rate is given by

$$r = a_K - \frac{w_L c(\sigma)}{A_L} - \sigma \frac{w_L}{A_L}\frac{b(\sigma)}{1-\varepsilon} = \Pi - \lambda \alpha_2 \sigma, \tag{11}$$

where $\Pi = a_K - \lambda\left(\frac{\alpha_0}{1-\varepsilon} + \alpha_1\right)$, and we assume that $\Pi > 0$. Without this condition the profit rate would never be positive for a non-negative skill premium.

The short-run equilibrium value of σ can be obtained as shown in Figure 1. Defining the state variable $h = \frac{A_L H}{K}$, this is given by

$$\sigma = \frac{\alpha_0}{h(1-\varepsilon)}. \tag{12}$$

As h increases σ falls, approaching zero in the limit.

In the long run, K_C, K_H, K, H and A_L can all change. Assuming capital depreciation away for simplicity, the change in capital stock is given by

$$dK/dt = I, \tag{13}$$

and changes in H and A_L are governed by equations (6) and (5a). The change in the stock of capital owned by capitalists is given by

$$dK_C/dt = s_C\, r\, K_C. \tag{14}$$

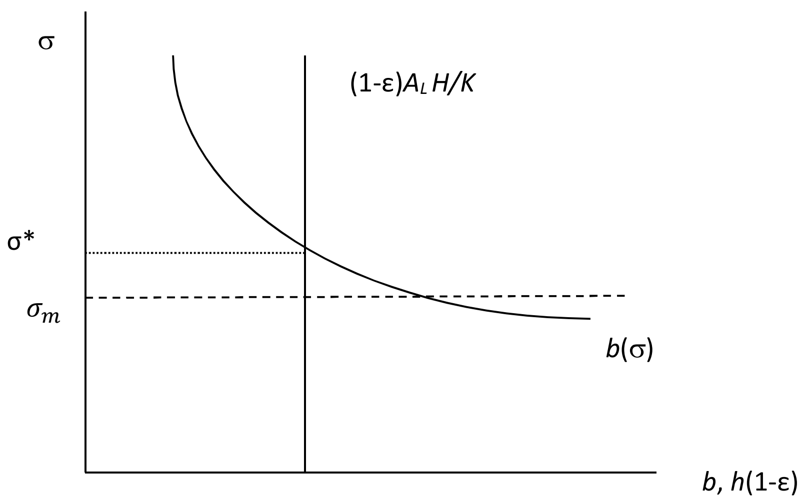

Figure 1. Determination of the skill premium in the short run

We examine the long run dynamics of the economy by focusing on h and on the share of total capital owned by capitalists, $k = K_C/K$. The growth rate of k is given by $\hat{k} = \hat{K}_c - \hat{K}$, and substituting from equations (9) and (11)-(14) we obtain

$$\hat{k} = (s_C - s_H)(1-k)\left[\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right] - s_H\frac{\lambda\alpha_0}{1-\varepsilon}. \qquad (15)$$

Because the growth rate of h is given by $\hat{h} = \hat{A}_L + \hat{H} - \hat{K}$, we can substitute from equations (5a), (6), and (9) and (11)-(12), to obtain,

$$\hat{h} = \Omega + \tau_1\mu h + \frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h} - [(s_C - s_H)k + s_H]\left[\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right], \qquad (16)$$

where $\Omega = \tau_0 - \theta\varepsilon\sigma_m - s_H\frac{\lambda\alpha_0}{1-\varepsilon}$.

The economy is defined as being in long-run equilibrium when $\hat{k} = \hat{h} = 0$. We can examine its long-run dynamics using a phase diagram in <k,h> space.

Setting $\hat{k} = 0$ in equation (15) we obtain

$$k = 1 - \frac{s_H\lambda\alpha_0}{(s_C - s_H)\left[(1-\varepsilon)\Pi - \frac{\lambda\alpha_0\alpha_2}{h}\right]}. \qquad (17)$$

The functional relation between h and k is increasing and concave, and takes the form of a hyperbola. The k-asymptote is given by

$$k_a = 1 - \frac{s_H\lambda\alpha_0}{(s_C - s_H)(1-\varepsilon)\Pi},$$

so that $k_a \leq 1$, while $k_a > 0$ if and only if $(s_C - s_H)(1-\varepsilon)\Pi - s_H\lambda\alpha_0 > 0$, which sets an upper bound to s_H, given the other parameters. The h-asymptote is given by

$$h_a = \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)\Pi} > 0.$$

We confine our attention to values of *h* and *k* for which the profit rate is positive: by equations (11)-(12) this requires that $h > h_a$ and the relevant portion of the $\hat{k} = 0$ curve is shown in Figure 2. The positive slope can be explained as follows. If we start from a position on the isocline and increase h, $\hat{k}$ increases. This happens because the increase in *h* reduces the skill premium, increasing the profit rate and the growth rates of the stock of capital owned by capitalists *and* of total capital. But the former increases more because the growth rate of total capital is tempered by the reduction in the growth rate of the capital stock owned by H-workers whose wage income decreases. To reduce $\hat{k}$ back to zero *k* must increase, which leaves unchanged the growth rate of the capitalists' capital stock (which depends on the capitalists' saving rate and on the profit rate which are both independent of *k*), but increases total capital accumulation since it shifts profit income from H-workers to capitalists who have a higher saving rate. This argument also shows that above (below) the $\hat{k} = 0$ line, *k* must be falling (rising), which explains the direction of the vertical arrows in Figure 2.

Setting $\hat{h} = 0$ in equation (16) we obtain

$$k = \frac{1}{(s_C - s_H)}\left[\frac{\Omega + \tau_1 \mu h + \frac{\theta\varepsilon\ _0}{(1-\varepsilon)h}}{\left[\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right]} - s_H\right], \tag{18}$$

which gives us a relation between *h* and *k* at which *h* is stationary provided that $\frac{\alpha_0}{(1-\varepsilon)h} \geq \sigma_m$. If $\frac{\alpha_0}{(1-\varepsilon)h} < \sigma_m$, equation (18) becomes

$$k = \frac{1}{(s_C - s_H)}\left[\frac{\Omega' + \tau_1 \mu h}{\left[\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right]} - s_H\right], \tag{19}$$

where $\Omega' = \tau_0 - s_H \frac{\lambda\alpha_0}{1-\varepsilon}$. The $\hat{h} = 0$ isocline is the upper envelope of the curves defined by equations (18) and (19) and is shown in Figure 2.

The shape of the $\hat{h} = 0$ curve is derived formally in the Appendix but can be explained intuitively as follows. An increase in *k* redistributes profit income from H-workers to capitalists, thus increasing the overall rate of saving and investment, lowering $\hat{h}$ below zero. To restore the value of $\hat{h}$ to zero *h* may have either to increase or to decrease. Both cases are possible. An increase in *h* affects $\hat{h}$ by affecting τ, $\hat{H}$ and $\hat{K}$. The increase in *h* increases τ; call it the *productivity effect*. It reduces $\hat{H}$ by increasing the relative supply of H-workers, reducing the skill premium and the incentive to acquire education; call it the *education effect*. It increases $\hat{K}$ by reducing high-skilled wages and, given the limited responsiveness in the demand for H-workers, increasing the profit rate; call it the *capital accumulation effect*. At relatively low levels of *h*, the latter effects are stronger than the productivity effect, and thus *h* must decrease to increase $\hat{h}$ back up to zero. At relatively high levels of *h* the productivity effect becomes strong compared to the capital accumulation effect and education effect (which even disappears as σ falls to σ_m), and *h* must increase to bring $\hat{h}$ back up to zero.

Figure 2 shows the case in which the $\hat{h} = 0$ and the $\hat{k} = 0$ curves intersect twice, so that there are two long-run equilibria. More formally, the Jacobian, $\mathbf{J} = (J_{ij})$, of the dynamic system shown by equations (15) and (16) is given by[7]

$$\mathbf{J} = \begin{bmatrix} -(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) & (s_C - s_H)(1-k)\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} \\ -(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) & \tau_1\mu - \frac{\varepsilon\theta\alpha_0}{(1-\varepsilon)h^2} - [(s_C - s_H)k + s_H]\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} \end{bmatrix}.$$

The conditions for local stability involve the trace of **J**, *Tr*, and its determinant, ***Δ*:**

$$Tr = -\left[(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) + \frac{\theta\varepsilon\ {}_0}{(1-\varepsilon)h^2} + [(s_C - s_H)k + s_H]\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} - \tau_1\mu\right] < 0$$

$$\boldsymbol{\Delta} = (s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\left[\frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2} + \frac{s_C\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} - \tau_1\mu\right] > 0$$

Given our assumptions that $s_C > s_H$ and $\Pi > \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}$, both conditions are satisfied if $J_{22} < 0$, which ensures that the $\hat{h} = 0$ isocline is negatively sloped at E_1, as shown in Figure 2. In the rest of the paper, we shall assume that this condition holds.[8] Inspection of the arrows in Figure 2, instead, shows that the equilibrium at E_2 is unstable.

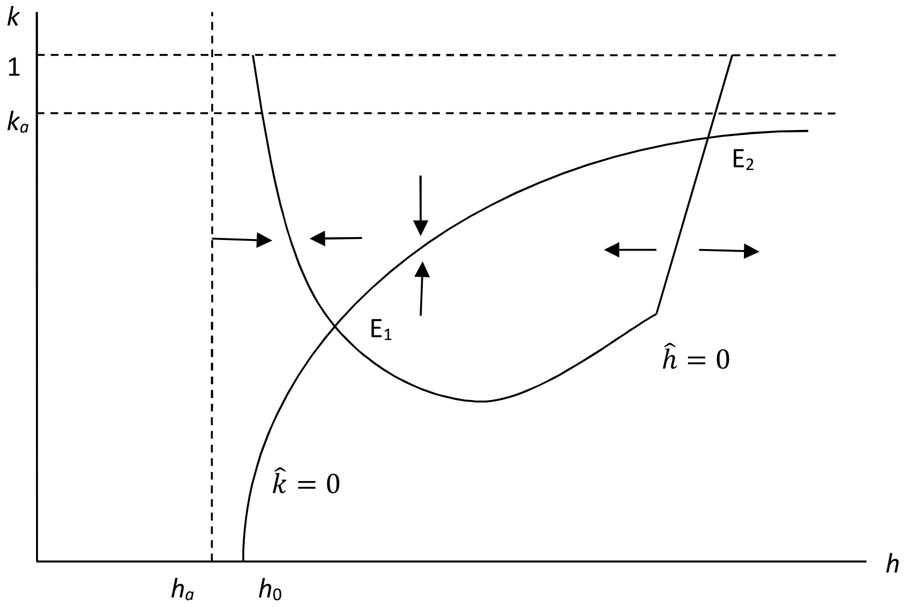

Figure 2. Long-run dynamics and equilibria

4. Political economy and the effects of parametric changes

We can now examine some broader political economy implications of our model. To do so we analyze the effects of some parametric shifts, focusing on the stable long-run equilibrium E_1. In this case, the implications for the long-run equilibrium levels of k and h can be obtained by totally differentiating equations (17)-(18). If $h \le \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$, then

$$\begin{bmatrix} -(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) & (s_C - s_H)(1-k)\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} \\ -(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) & \tau_1\mu - \frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2} - [(s_C - s_H)k + s_H]\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} \end{bmatrix}\begin{bmatrix} dk \\ dh \end{bmatrix}$$

$$= \begin{bmatrix} (1-k)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) + \frac{\lambda\alpha_0}{1-\varepsilon} \\ (1-k)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) + \frac{\lambda\alpha_0}{1-\varepsilon} \end{bmatrix} ds_H + \begin{bmatrix} -(1-k)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) \\ k\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) \end{bmatrix} ds_C +$$

$$\begin{bmatrix} 0 \\ -\left(\frac{\varepsilon\alpha_0}{(1-\varepsilon)h} - \varepsilon\sigma_m\right) \end{bmatrix} d\theta + \begin{bmatrix} \frac{\lambda\alpha_0}{(1-\varepsilon)^2}\left((s_C - s_H)(1-k)\left(1+\frac{\alpha_2}{h}\right) + s_H\right) \\ -\frac{\lambda\alpha_0}{(1-\varepsilon)^2}\left[(s_H + (s_C - s_H)k)\left(1+\frac{\alpha_2}{h}\right) - s_H\right] - \theta\left[\frac{\alpha_0}{h(1-\varepsilon)^2} - \sigma_m\right] \end{bmatrix} d\varepsilon$$

$$+ \frac{1}{1-\varepsilon}\begin{bmatrix} \left(\alpha_0 + (1-\varepsilon)\alpha_1 + \frac{\alpha_0\alpha_2}{h}\right)(s_C - s_H)(1-k) + s_H\alpha_0 \\ -[(s_C - s_H)k + s_H]\left((1-\varepsilon)\alpha_1 + \frac{\alpha_0\alpha_2}{h}\right) - (s_C - s_H)k\alpha_0 \end{bmatrix} d\lambda. \qquad (20)$$

If $h > \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$, the only difference is that the vector multiplying dθ is the null vector. Equation (20) can be used to examine the effects of policy and behavioral changes on the long-run equilibrium values of k and h.

We analyze the effects on growth and income distribution by examining, respectively, the effects on $\hat{K}$ and on the income shares of the three classes. Given the fixed output-capital ratio a_K, the growth rate of the capital stock coincides with the growth rate of output and since in long-run equilibrium $\hat{K} = \hat{K}_C$ it can be expressed as follows:

$$\hat{K} = s_C\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right), \qquad (21)$$

The income shares of the three classes, denoted by ι_i, are given by

$$\iota_C = \left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\frac{k}{a_K}, \tag{22}$$

$$\iota_H = \left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\frac{1-k}{a_K} + \frac{\alpha_0\lambda}{a_K(1-\varepsilon)}, \tag{23}$$

$$\iota_L = \frac{\lambda}{a_K}\left(\alpha_1 + \frac{\alpha_0\alpha_2}{(1-\varepsilon)h}\right), \tag{24}$$

with $\iota_C + \iota_H + \iota_L = 1$. The interests of the different classes may not, however, lie only in their income shares, but also in employment growth, the spread of education and the relative wage.

4.1 Change in saving rates

Consider first an increase in the saving rate of H-workers, s_H. This shifts the $\hat{k} = 0$ curve in Figure 2 down (equation (15)). The increase in s_H implies that H-workers save and add to their capital stock at a higher rate, reducing the growth rate of the share of capitalists, $\hat{k}$, making it negative. To increase $\hat{k}$ and bring it back to zero, *k* must decline. The $\hat{h} = 0$ curve also shifts down (equation (16)). This is because H-workers accumulate capital at a higher rate, which increases the overall rate of capital accumulation and therefore reduces $\hat{h}$. To restore $\hat{h}$ back to zero, *k* must decrease which, by distributing profit income from capitalists to H-workers with a lower propensity to save, reduces $\hat{K}$ and increases $\hat{h}$. Using equation (20):

$$\frac{dk}{ds_H} = -\frac{(1-k)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right) + \frac{\lambda\alpha_0}{1-\varepsilon}}{(s_C - s_H)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)} < 0$$

$$\frac{dh}{ds_H} = 0.$$

The long-run equilibrium value of k falls because when H-workers save at a higher rate, they increase their rate of capital accumulation while capitalists do not. The long-run equilibrium value of h remains constant because the rise in s_H affects $\hat{k}$ and $\hat{h}$ by exactly the same amount: it coincides with the change in $\hat{K}$ at a given k and h, and an identical change in k can restore the original value of $\hat{K}$ without any change in h.

The constancy of the long-run equilibrium value of h with respect to changes in s_H is very similar to the effect of a change in workers' saving propensity in Pasinetti's (1962) model, the so-called *Pasinetti paradox*. In our model, since an increase in s_H reduces k but leaves h unchanged, it has no effect either on the profit rate (equations (11)-(12)), or on the growth rates of capital and output (equation (21)). There is also no change in the growth rate of productivity (equation (5)). Thus, a change in the H-workers' saving rate has no effect on the rates of profit and growth, exactly as in Pasinetti's model. We obtain these results from a more general model than Pasinetti's, in that we allow for two kinds of workers, education, skill formation, and growth with unemployment of L-workers. However, the rates of profit and accumulation are unaffected for the same reason as in Pasinetti's model: in the steady state, the accumulation rate is determined entirely by capitalists' saving and accumulation decisions and the profit rate, and a change in the saving rate of H-workers only changes the share of capital they own.

Regarding income distribution, the H-workers' share increases and the capitalists' share decreases because of the increase in the share of capital owned by H-workers (equations (22)-(23)). The L-workers' share (equation (24)), the growth rates of employment of both kinds of labor, and the skill premium do not change.

It is interesting to examine the consequences of extreme changes in s_H. When s_H decreases to zero, so that H-workers do not save we have the scenario discussed in Dutt and

Veneziani (2010). Equation (17) implies that at any steady state $k = 1$. The equilibrium values of h can be obtained from equation (18) setting $s_H = 0$ and $k = 1$.

When s_H rises so much that $s_H = s_C$, at any steady state $k = 0$ (equation (15)): in the limit all capital is owned by H-workers, because although capitalists and H-workers save the same fraction of their profit income, H-workers also save part of their wage income. Instead, the dynamics of h is independent of k, since a redistribution of capital between capitalists and H-workers does not change the rate of capital accumulation (equation (16)). The long-run equilibrium values of h can be obtained by solving from the equation $\Omega + \tau_1 \mu h + \frac{\theta\varepsilon_0}{(1-\varepsilon)h} = s_H \left[\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right]$, which is qualitatively similar to the values in the case with $s_H = 0$, since $s_H = s_C$. Other than the fact that in the first case the capitalist share of capital goes to unity and in the second case it goes to zero, as long as the other parameters of the model are the same, the two economies have qualitatively similar long-run equilibrium properties.

These results have interesting implications for understanding the role of increases in what has been called luxury consumption among the richer segments of wage recipients. While this phenomenon of increasing luxury consumption in general has been widely discussed – and has been called *luxury fever* by Frank (1999) and *affluenza* by de Graaf et al (2002) – and its implications for happiness and wellbeing and for macroeconomic performance and increases in consumer indebtedness extensively analyzed (see Frank 1999, Dutt 2008, and Schor 2010), its implications for maintaining the position of the capitalist class by reducing the saving propensity of high-income wage recipients has received less attention. Our analysis shows why, in the classical-Marxian economy, a reduction in the saving rate of H-workers is in the interest of the capitalist class. This is not to argue that capitalists will actually attempt to reduce s_H to maintain their position as capital owners, since it is unclear why they will or can act collectively to do so.

The analysis does imply, however, that attempts to increase consumption, in particular of H-workers – through, for instance, sales promotion efforts, product innovation and financial innovation – have the effect of maintaining the share of capitalists in the ownership of capital.

Our analysis also suggests that if attempts to keep the saving rate of H-workers low fail, and they eventually come to dominate capital ownership, then unless some other parameters change, nothing of substance changes in the economy: it behaves qualitatively in the same manner as the economy in which a separate capitalist class owns all capital. Capitalism functions in much the same way. Not all workers become human capitalists, because access to education is restricted, and income distribution can remain as unequal as in the world with capitalists, because human capitalists also accumulate capital.

H-workers, however, may not come to dominate capital ownership if capitalists also increase their own saving rate, s_C, for example by increasing the retained earnings of firms. An increase in s_C shifts the $\hat{k} = 0$ curve up because it increases $\hat{k}$ as capitalists accumulate more and increase their share of capital, so that k must increase to bring $\hat{k}$ back to zero. It also shifts the $\hat{h}$ = 0 curve down because it increases capital accumulation overall and therefore reduces $\hat{h}$ below zero, so that k must fall to reduce the rate of capital accumulation and restore $\hat{h}$ to zero. Using equation (20)

$$\frac{dk}{ds_C} = \frac{1}{\Delta}(1-k)\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\left[\frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2} + \frac{s_H\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} - \tau_1\mu\right] > 0,$$

$$\frac{dh}{ds_C} = -\frac{(s_C - s_H)}{\Delta}\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)^2 < 0.$$

An increase in capitalist saving increases their long-run share of capital, and reduces h by speeding up capital accumulation more than human capital formation. Thus, as one would expect, capitalists can maintain their dominance in capital ownership by increasing their saving rate. This change, by reducing h, lowers the profit rate, r (equations (11)-(12)). The effect on the long-run equilibrium growth rate is given by

$$\frac{d\hat{K}}{ds_c} = (s_C - s_H)(1-\varepsilon)h^2\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)^2\left[\frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2} - \tau_1\mu\right].$$

This expression cannot be definitely signed, even if $J_{22} < 0$. A negative growth effect is possible if θ and ε are small and τ_1 is large, that is the effect of the increase in h on technological change is stronger than the effect on the skill premium and the incentive for acquiring education, and the long-run equilibrium value of h is high. Given the signs of the above derivatives, the share of income of H-workers falls both by reducing the profit rate and by reducing their share of capital (equation (23)), and the share of L-workers increases with the fall in the profit rate (equation (24)). Equation (22) implies, using the above derivatives,

$$\frac{d\iota_C}{ds_C} = \frac{1}{a_K\Delta}\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)^2\left[\left(\frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2} + \frac{s_C\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2} - \tau_1\mu\right)(1-k) - (s_C - s_H)\frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h^2}\right].$$

Even with $\Delta > 0$, this expression is not necessarily positive: capitalists cannot be assured that their income share will increase when their saving rate increases (since the rise in the L-worker share can offset the fall in the H-worker share), But if $(s_C - s_H)$ is not too large, then an increase in capitalist saving will increase their income share.

4.2 Change in the openness of the education system

An increase in the openness of the educational system,[9] as represented by an increase in the parameter θ, leaves the $\hat{k} = 0$ curve unchanged, but shifts the $\hat{h} = 0$ curve upwards by increasing $\widehat{H}$, as long as $h < \frac{(1-\varepsilon)\alpha_0}{\sigma_m}$, at the initial equilibrium. As a result, the long-run equilibrium levels of k and h both increase. The growth rate of the economy rises (equation (21)), the capitalists' income share rises because both their share of capital and the profit rate increase (equations (22)), the L-workers' share falls because of the substitution in production away from them towards H-workers (equations (23)), and the effect on the H-workers' income share is ambiguous since they gain from a higher profit rate but lose from a smaller capital share (equations (24)). The skill premium declines.

If $h \geq \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$ at the initial equilibrium, however, the increase in θ has no effect on the two curves, and none of the results just noted occurs. In that case, as shown in Dutt and Veneziani (2010) in a model where H-workers do not save, the skill premium is not sufficient to induce people to obtain more education and the economy is caught in a low-skill trap. One way to expand education in this case is to reduce the opportunity cost of obtaining education, by reducing σ_m, for instance, by reducing tuition fees, or by increasing public investment and the share of H-workers in the education sector .

4.3 Change in the share of high-skilled labor devoted to education

An increase in ε has the effect of reducing the number of H-workers devoted to production or increasing the total demand for H-workers, which increases the skill premium in the short run which in turn squeezes profits both directly and by increasing the employment of L-workers (because of the low level of substitution between the two types of workers).

Thus, starting from an initial situation with $\hat{k} = 0$, an increase in ε reduces $\hat{k}$; because the profit rate falls, thereby reduceing the capitalists' accumulation rate and, possibly, total accumulation. Since the H-workers' income increases with the skill premium, total accumulation rises or falls less than capitalist accumulation, implying a reduction in $\hat{k}$. Therefore k must fall to restore $\hat{k}$, so that the $\hat{k} = 0$ isocline shifts downwards. Similarly, starting from an initial situation with $\hat{h} = 0$, the increase in ε increases $\hat{h}$ by increasing the number of educators and the skill premium, and very likely by reducing the profit rate and the accumulation rate. Therefore, *k* must increase to bring $\hat{h}$ back to zero, implying an upward shift in the $\hat{h} = 0$ curve. The shifts of the two curves imply that the long-run equilibrium value of *h* increases, but the effect on *k* is ambiguous, as confirmed by equation (20):

$$\frac{dh}{d\varepsilon} = \frac{(s_C - s_H)}{\Delta}\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\left\{\theta\left(\frac{\alpha_2}{(1-\varepsilon)^2 h} - \sigma_m\right) + \frac{\lambda\alpha_0}{(1-\varepsilon)^2} s_C\left(1 + \frac{\alpha_2}{h}\right)\right\} > 0,$$

$$\frac{dk}{d\varepsilon} = \frac{\lambda\alpha_0}{(1-\varepsilon)\Delta}\left\{\left(\tau_1\mu - \frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2}\right)\frac{1}{1-\varepsilon}\left[(s_\mathrm{C} - s_\mathrm{H})(1-k)\left(1 + \frac{\alpha_2}{h}\right) + s_\mathrm{H}\right] + \theta\left(\frac{\alpha_0}{(1-\varepsilon)^2 h} - \sigma_m\right)(s_\mathrm{C} - s_\mathrm{H})(1-k)\frac{\alpha_2}{h^2} - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)^2 h^2} s_C s_H\right\}.$$

The latter expression cannot be definitely signed (even if $J_{22} < 0$). The sign of the first term within curly brackets depends on the relative strengths of the effect of *h* on technological change and the growth of education by changing the wage premium. The second effect is positive and depends on the speed of human capital accumulation (provided $\sigma > \sigma_m$) and the third term is negative given $s_C > 0$ and $s_H > 0$.

Equations (21)-(24) show that the effects on growth and distribution cannot be definitely signed. They are determined in large part by the change in (1-ε)*h*, the ratio of effective units of high-skilled production workers to capital which, in general, cannot be signed since the rise in ε reduces it but the resultant increase in *h* increases it.[10] A strong expansion in *h* tends to increase the long-run growth rate of the economy, although an increase in ε also lowers Π because of the increase in the demand for L-workers induced by the increase in the skill premium. The capitalists' income share depends also on the change in *k*, which may offset any changes in the profit rate. The H-workers' income share receives a positive boost from the effect on their wage, but if k rises they can lose out because of the reduction in their share of capital. The L-workers' income share depends only on (1-ε)h; if this expression rises with ε their share falls. These ambiguous effects on the capitalist share suggest that the interests of the capitalist class may lie in limiting the size of the education sector.[11]

4.4 Change in the state of class struggle

We may motivate the discussion of a change in the state of class struggle by recalling that our analysis gives a multidimensional role to more educated labor. Education transforms L-workers into H-workers, and H-workers have a more complex role in the economy than just being a (qualitatively different) input into production, that is, they increase the efficiency of *all* workers through the process of innovation, and help in the process of education, as family members, mentors or educators. Yet, as argued in the classical-Marxian and radical tradition, education also plays an important role in affecting social stability and the relation between workers, capitalists and the state; in weakening workers' position by dividing them into groups based on their education; in creating and strengthening the perception of upward social mobility thereby increasing tolerance for income inequality, indoctrination and socialization, and easing the process of the extraction of labor (and hence labor productivity and profits) from labor power (Bowles and Gintis 1975, 1976). Some of these political-economy issues can be addressed in our model by examining the effects of parametric changes which capture changes, for instance, in the workers' bargaining position at least partly originating in the education system.[12]

Starting from an initial situation with $\hat{k} = 0$, an increase in λ reduces $\hat{k}$, and so k must fall to restore $\hat{k}$, so that the $\hat{k} = 0$ curve shifts downwards. Starting with an initial situation with $\hat{h} = 0$, the increase in λ increases $\hat{h}$ by reducing the rate of capital accumulation, which must be compensated by an increase in k to increase the rate of capital accumulation and restore $\hat{h}$ to its original value, so that the $\hat{h} = 0$ curve shifts up. An increase in λ therefore increases the long-run equilibrium value of *h*. Figure 2 suggests however that the effect on *k* cannot be unambiguously signed. Using equation (20):

$$\frac{dk}{d\lambda} = \frac{\left[\left(\alpha_0 + \alpha_1(1-\varepsilon) + \frac{\alpha_0\alpha_2}{h}\right)(s_C - s_H)(1-k) + s_H\alpha_0\right]\left(\tau_1\mu - \frac{\theta\varepsilon\ _0}{(1-\varepsilon)h^2}\right) - s_C s_H {\alpha_0}^2 \frac{\lambda\alpha_2}{(1-\varepsilon)h^2}}{(1-\varepsilon)\Delta},$$

$$\frac{dh}{d\lambda} = \frac{(s_C - s_H)}{(1-\varepsilon)\Delta}\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\left(\alpha_0 + \alpha_1(1-\varepsilon) + \frac{\alpha_0\alpha_2}{h}\right)s_C > 0.$$

Thus, *k* falls if $\tau_1\mu < \frac{\theta\varepsilon\alpha_0}{(1-\varepsilon)h^2}$, a condition we have met before. But if this condition is not satisfied, then $\frac{dk}{d\lambda} > 0$ is possible. The shift in the $\hat{h} = 0$ isocline occurs here because of the lower growth rate of capital, which increases the growth rate of effective human capital (as a ratio of the capital stock). If this effect is small (for instance, if the lower accumulation rate also reduces the growth of training due to learning by doing), we can assume that the $\hat{h} = 0$ curve hardly shifts. In this case, given the downward shift of the $\hat{k} = 0$ curve, in Figure 2, *k* falls and *h* rises.

The growth rate of the economy falls if the direct effect of the increase in λ on the profit rate is not offset by the increase in *h* which decreases high-skilled labor costs. Concerning income shares, the increase in λ has a direct positive effect on the L-workers' share , but it has a negative indirect effect by increasing *h*, thus lowering the skill premium, which leads firms to substitute H-workers for L-workers. The capitalist share will decrease if the direct negative effect of the increase in λ on the profit rate is stronger than the indirect positive effect via the skill premium. The effect on the high-skilled income share is ambiguous because a number of forces are at play: H-workers gain from the increase in the base wage and from the increase in employment due to the decrease in the skill premium when *h* increases, but lose due to the decrease in the skill premium itself. The overall effect on their profit income is difficult to predict, especially if $\frac{dk}{d\lambda} < 0$ which might lead to an increase in the H-workers' share of capital.

This analysis has important implications for the bargaining power of workers vis-à-vis capitalists. The fact that H-workers receive wages as well as profits implies that they can have divided loyalties. The "class struggle" variable provides a floor to the wage of H-workers but also affects the profit share, and it has a negative impact on the equilibrium skill premium. On the one hand, H-workers are interested in increasing λ to increase the floor for their wages, and possibly to increase their share of capital (if $\frac{dk}{d\lambda} < 0$ holds), and in this way share interests with L-workers. On the other hand, as owners of both physical and human capital, they are interested in higher profits and might prefer a higher skill premium (depending on the employment effects), and so, given $\frac{dh}{d\lambda} > 0$, prefer a lower level of λ, sharing interests with capitalists. Hence H-workers might be less interested in supporting a higher value of λ than if they did not hold capital, and this may weaken the workers' bargaining position. This exacerbates the ideological effects of education and the belief in higher social mobility, as well as a decrease in working class solidarity as a result of the emergence of the labor aristocracy, all of which are likely to reduce the workers' bargaining power (see Dutt and Veneziani 2010).

5. Conclusion

This paper has developed a classical-Marxian model of growth and distribution in which all saving is invested; education transforms low-skilled workers into high-skilled ones; and H-workers save and hold capital (as do capitalists), therefore receiving both high-skilled wages and profit income. The model is able to address questions about the implications for class divisions, growth and distribution, of the transformation of the modern capitalist economy from one in which the only class division is between capitalists who own capital and workers who only receive wage income into one in which education and human capital play a major role.

We show that an expansion in education – as reflected by an increase in the openness of education or by a fall in the cost of education – has a positive effect on the growth rate of the economy. However, this is not due to the increase in the growth rate of fully-employed effective labor as in neoclassical growth models. In the classical-Marxian model with unemployed labor, an increase in the growth rate of labor productivity does not, by itself, increase output growth. The effect on output depends on the distributional consequences of the expansion of education: in our model it results in a rise in the profit rate due to a fall in labor costs, which increases saving and investment, and also in a shift in capital ownership towards capitalists with a higher propensity to save.

Regarding classes and distribution, the increasing importance of education and skills produces some changes in capitalist economies, but need not alter their essential features. Although education may have positive externalities by increasing the productivity of all workers, inequality does not fall. The spread of education can be limited by the relative lack of openness of the educational system which can keep the skill premium high.[13] H-workers become not only human capitalists but also owners of capital and recipients of profits. Hence, although H-workers do not become richer by continuing to acquire more education and skills that increase their productivity other than due to overall technological change – in fact in our model education only converts L-workers into H-workers – they can increase their income by saving. Moreover, their interests may become aligned to those of capitalists. To the extent that H-workers identify themselves more as owners of capital than workers, the workers' bargaining power gets weakened, which likely results in a fall in the share of capital owned by H-workers because of the decline in their labor income. Capitalists can also protect their interests in maintaining their share of capital by saving at a higher rate – perhaps by increasing retained earnings – *and* by encouraging the growth of conspicuous consumption among H-workers. All of this implies that the capitalist-worker divide continues to have a major role in capitalist economies, despite the growing importance of education and skill formation. Even if they save at a high rate and come to dominate capitalists as capital owners, H-workers can maintain their dominance by keeping the bargaining position of workers at a low level, by restricting entry into education, and by becoming the new capitalists who also happen to work.

The ethical case against inequality remains strong to the extent that human capitalists also become owners of capital *and* because obtaining education is not simply a matter of choice, given difficult environmental circumstances, a low-quality basic education, and a relatively closed higher education system. Yet, the possibility of change induced by policies is likely to be weakened by the fact that high income workers who are human capitalists may see their interests being aligned to capitalists, rather than to the rest of the workers.

To be sure, our model is based on a number of simplifying assumptions which should be relaxed – for instance, to take into account the role of financiers and managers, and the importance of expectations and effective demand – in order to gain a more thorough understanding of the current trends in capitalist economies. Nevertheless, we hope that it provides an important first step towards such an understanding.

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6. Appendix: the $\hat{h} = 0$ isocline

Let the curves defined by equations (18) and (19) be denoted, respectively, HH' and MM'. It is immediate to prove that: HH' lies above MM' for all $h < \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$ and below it for all $h > \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$, with the two curves intersecting at $h = \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$; both curves have an h-asymptote at $h_a = \frac{\lambda\alpha_0\alpha_1}{(1-\varepsilon)\Pi}$; the slope of HH' is smaller than that of MM', provided that $h_a < \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$; and both curves are U-shaped. Further, let h_H and h_M denote the values of h at which HH' and MM' reach their minimum, respectively: if $h_a < \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$ then $h_H > h_M$.

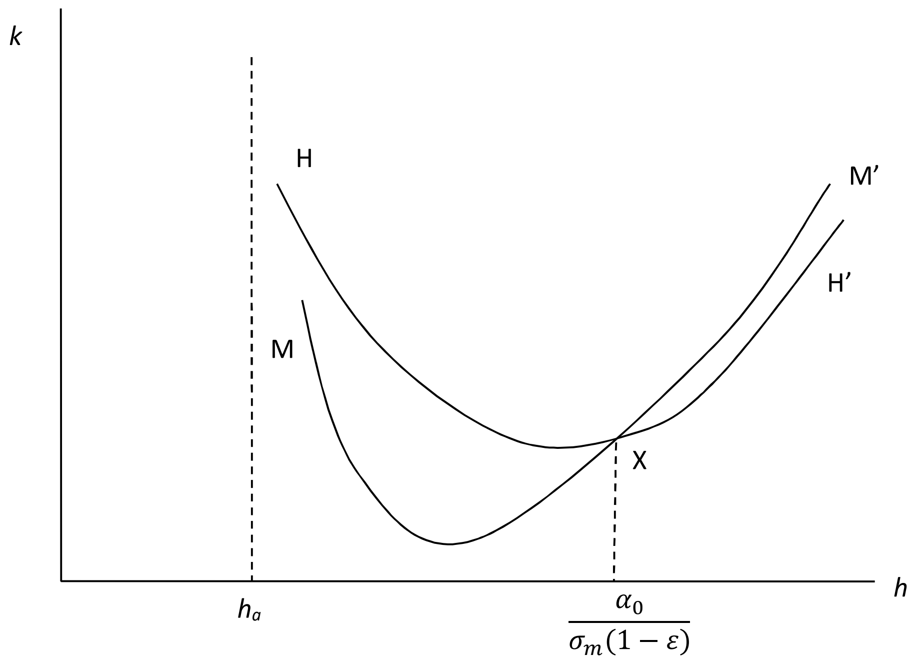

Figure A1. The $\hat{h} = 0$ curve

The HH' and MM' curves are depicted in Figure A1. Since the HH' curve is valid for $h \leq \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$ and the MM curve is valid for $h \geq \frac{\alpha_0}{(1-\varepsilon)\sigma_m}$, the $\hat{h} = 0$ curve comprises the two segments HX and XM', where X is where the HH' and MM' curve intersect. Note that from equations (18) and (19) it follows that $[\Omega + \tau_1 \mu h_a - s_H \Pi] > 0$ is sufficient for the $\hat{h} = 0$ curve to lie entirely in the positive quadrant, as shown.

NOTES

[1] The discussion here deliberately emphasizes the differences between systems with and without human capital. Not all those who stress the importance of human capital argue along the lines that follow. For instance, Lindsey (2013) stresses the unequalizing effects of human capitalism.

[2] Although not mentioned by Kuznets (1955) himself, explanations of the decrease in income inequality when per capita income continues to grow after attaining a high level, often emphasize the role of the spread of education in reducing inequality. See, for instance, Acemoglu and Robinson (2002).

[3] Some classical-Marxian models do introduce more than two classes, for instance, landlords (Pasinetti 1960, Petith 2008), rentiers or financial capitalists (Dutt 1989), and managers (Dutt 2016). Here, we implicitly incorporate managers and financiers into the capitalist class, despite the fact that they do some form of work.

[4] To assume that μ is constant seems reasonable (if not necessary) in a steady state, such that if any loss of generality occurs, this only has to do with the analysis of the transition path.

[5] Unger (2007, p. 96-97), who distinguishes such roles in terms of an idea about the mind, expresses it as follows: "We know how to repeat some of our activities, and we do not know how to repeat others. As soon as we learn how to repeat an activity we can express our insight in a formula and embody the formula in a machine … The not yet repeatable part of our activities – the part for which we lack formulas and therefore also machines – is the realm of innovation, the front line of production. In this realm, production and discovery become much the same thing."

[6] Some Marxian scholars incorporate effective demand issues by introducing an independent investment function. However, such investment functions are usually the hallmarks of post-Keynesian growth models, and taken to be the crucial difference from neo-Marxian or classical-Marxian models. See, e.g., Marglin (1984) and Dutt (1990).

[7] We confine our discussion to the case with $h \leq \frac{\alpha_0}{\sigma_m(1-\varepsilon)}$. When this condition does not hold, the terms involving θ below must be set equal to zero and the analysis otherwise remains the same.

[8] This is only for clarity and yields no loss of generality. All of our results can be extended to the case with $J_{22} > 0$ provided the relevant stability conditions hold. (The case with $J_{22} > 0$ is described in the Addendum.)

[9] The expansion of education refers to that of higher, not basic education, the latter assumed as being available to all.

[10] Defining $\Sigma = (1-\varepsilon)h$ and using the expression for $dh/d\varepsilon$ we find that

$$\frac{d\Sigma}{d\varepsilon} = -h + (1-\varepsilon)\frac{(s_C - s_H)}{\Delta}\left(\Pi - \frac{\lambda\alpha_0\alpha_2}{(1-\varepsilon)h}\right)\left\{\frac{\lambda\alpha_0}{(1-\varepsilon)^2}s_C\left(1+\frac{\alpha_2}{h}\right) + \theta\left(\frac{\alpha_0}{(1-\varepsilon)^2 h} - \sigma_m\right)\right\},$$

which could be positive or negative depending on the initial value of h.

[11] If the substitution between H-workers and L-workers is small, then if $\frac{d\Sigma}{d\varepsilon} > 0$ and $\frac{dk}{d\varepsilon} > 0$ growth increases, capitalists gain and L-workers lose out. The opposite holds if $\frac{d\Sigma}{d\varepsilon} < 0$ and $\frac{dk}{d\varepsilon} < 0$. The effect on H-workers is mixed in both cases, though, due to their mixed class allegiances. They unambiguously gain if $\frac{d\Sigma}{d\varepsilon} > 0$ and $\frac{dk}{d\varepsilon} < 0$.

[12] See Dutt and Veneziani (2010) for an explicit model of the influence of education on the labor market, which also considers the possibility that education may create more informed political participants and a discerning electorate.

[13] We do not need to appeal to questionable cultural characteristics that prevent low income groups from acquiring higher skills through education, as argued by Lindsey (2013).