Bridging Kalecki and Kaldor with cost share-induced technological change

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Abstract

While much post-Keynesian analysis uses comparative statics to analyze the impact of changes in distribution on level of output, several authors have built dynamic models that determine distribution and output simultaneously. Typically, the resulting growth rates are incompatible with Harrod's natural rate. In this paper a dynamic post-Keynesian model is introduced whose equilibrium features growth at Harrod's natural rate. When combined with cost share-induced technological change, the equilibrium also features Harrodian technological change and Kaldor's stylized facts of constant cost share and constant rate of profit. The results depend on the pricing regime. The stability conditions of the equilibria are discussed and a comparative statics analysis of pricing regimes proposed.

Keywords: Kaleckian, Kaldorian, Harrod, growth, distribution
JEL: E11, E12, O33, O41

1. Introduction

The research program to which this paper contributes seeks to understand the macroeconomics of a sustainable economy, including the transition from our current, unsustainable economies. That analysis involves short-run processes, long-run outcomes, and the pathways from one to another. These topics have been explored in the post-Keynesian and classical literature. Among the contributions, some focus on the simultaneous determination of the level of output, given by capacity utilization, and of distribution (see, e.g., Bruno, 1999; Bhaduri, 2008). Others seek to bridge (short-run) post-Keynesian theory with (long-run) classical theory (e.g., Duménil and Lévy, 1999). The context of a sustainability transition gives this work new

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relevance, and even urgency. In this paper it will be framed in terms of bridging two post-Keynesian traditions: short-run Kaleckian analysis and long-run Kaldorian analysis.

Focusing on a sustainability transition brings to the fore the importance of technological change. Most questions have to do with intensity of use of energy and materials. However, in this paper the analysis is limited to capital and labor productivity. Two mechanisms are explored: the standard post-Keynesian Kaldor-Verdoorn mechanism (Kaldor, 1966) and cost share-induced technological change (Dutt, 2013; Duménil and Lévy, 2010; Kemp-Benedict, 2022).

The strategy pursued in this paper is familiar: dynamic post-Keynesian models are proposed, their equilibria are identified, and the stability of those equilibria explored. To be sure, local stability is not strictly necessary; "Harrodian" models allow for local instability (Skott, 2010). However, local stability will be imposed in this paper and the implications explored. In total, four models are considered, distinguished by the pricing regime and the presence or absence of cost share-induced technological change: a standard Kaleckian fixed-markup model; a wage-conflict model with Kaldor-Verdoorn technological change; a target-return model with cost share-induced technological change; and the prior wage-conflict model to which cost share-induced technological change has been added.

The main reason to present multiple models is to isolate the impacts of different assumptions on the results. Two results are key: the equilibrium value of the investment rate under conflict pricing is equal to Harrod's natural rate; and cost share-induced technological change produces, at the equilibrium, Harrod-neutral technological change and Kaldor's stylized facts of constant cost share and constant profit rate (Kaldor, 1961). Both through the choice of models and through these finding, this paper overlaps with that of Bhaduri (2006). Bhaduri similarly considered the interaction between technological change and distribution under different pricing regimes within a dynamic post-Keynesian model. As in this paper, he found an equilibrium at which the growth rate aligns with Harrod's natural rate. However, the mechanisms differ in some details, and Bhaduri's paper can be seen as partly overlapping but mainly complementary to the present paper.

A second reason to present multiple models is to motivate a particular type of comparative statics analysis. In a conventional Kaleckian comparative statics analysis, distribution is determined exogenously through firms' markups. The goods market equilibrium then determines capacity utilization. Equilibrium utilization thus depends on the profit share; if utilization rises when the profit share rises, the economy is said to be "profit-led" and in the opposite case to be "wage-led." However, aside from the fixed-markup Kaleckian model, the equilibria of the models treated in this paper fully determine both utilization and distribution. (The cost share-induced technological change models determine capital productivity as well.) However, the equilibrium point depends on the pricing regime: the equilibrium under target-return pricing is not the same as under conflict wage determination. Thus, rather than comparing equilibrium utilization for different values of the profit share, the analysis compares equilibrium utilization and profit share (or profit rate) for different pricing regimes.

Pricing regimes are also important for reasons of political economy. Maintaining a high rate of return under target-return pricing creates pressure on the labor supply. From the 1970s onward, considerable effort was expended in suppressing labor power, a process that accelerated in the 1980s. Target-return pricing will generally be inconsistent with Harrod's natural rate of growth, but the pressure on the labor force can be relieved in any of a number of ways, including: offshoring (Crinò, 2009); immigration (Rodriguez, 2004); extending working lives (Vickerstaff, 2010); and bringing children into the workforce (Pollack et al., 1990). These are all highly salient, including efforts to erode child labor laws.[1] However, each of these strategies has a limit, and each is politically fraught. At some point the contradictions will be forced to the front of political debate. This is also salient today, as unionization is arguably beginning to expand, 40 years after the de-unionization efforts of the 1980s.

Section 2 introduces "reference models" without cost share-induced technological change: one with fixed markup and one with conflict-based wage determination. Section 3 introduces models with cost share-induced technological change: one featuring target-return pricing and the other an extension of the wage conflict model. Section 4 discusses the results; Section 5 concludes.

[1]See https://www.theguardian.com/us-news/2023/oct/20/republican-child-labor-law-death.

2. Reference models

This paper has two aims: bridging short-run and long-run processes in dynamic post-Keynesian models and exploring the implications of cost share-induced technological change on those models. This section focuses on the first of these two aims and offer as a reference point two models without cost share-induced technological change. The first, which features utilization dynamics alone, is a standard post-Keynesian model with a fixed markup (e.g., see Lavoie, 2014). The second features conflict wage determination. Despite prior work on such models (e.g., Rowthorn, 1977; Dutt, 1992), this paper finds an apparently novel result; at equilibrium, this pricing regime leads to growth at Harrod's natural rate.

2.1. Reference Model 1: Utilization dynamics

The first model introduces the equation of motion for capacity utilization used throughout the paper in the context of a model in which firms set prices using a fixed markup, which exogenously determines the profit share. The assumed functional form is a familiar dynamic extension to a standard Kaleckian model and is used to introduce some notation and definitions.

The investment function is specified only to the extent that it depends positively on the profit rate at full utilization $r = \pi\kappa$, where π is the profit share and κ is capital productivity, and capacity utilization u,

$$\text{investment function: } g^i(r, u), \qquad g^i_r, g^i_u > 0. \tag{1}$$

Note that, because $r = \pi\kappa$,

$$g^i_\pi = g^i_r \kappa, \qquad g^i_\kappa = g^i_r \pi. \tag{2}$$

In this expression, capital productivity is in principle the monetary value of output at firms' assessed level of full capacity operation divided by the monetary value of capital as assessed by firms. In practice it may be calculated as estimated potential output derived from time-series data divided by value of the capital stock as computed using the perpetual inventory method, but the difference is a practical one, not a difference in principle; the relevant values to use in behavioral relations are what firms assess them to be. That this is, indeed, a realistic approach is supported by the calculation of capital stock by managers in the pricing studies surveyed by Lee (1994) and the observation that plant managers indeed understand the potential output of their plant (Corrado and Mattey, 1997, p. 152).

The saving function is similarly general, specifying only that it depends on the profit share, capacity utilization, and capital productivity,

$$\text{saving function: } g^s(\pi, \kappa, u), \qquad g^s_\pi, g^s_\kappa, g^s_u > 0. \tag{3}$$

The only non-standard feature of these expressions is that capital productivity is usually taken to be a constant parameter. Indeed, it will be assumed constant for the "reference models" presented in this section. However, it will be endogenized in the next section on cost share-induced technological change. For that reason, it is shown explicitly.

Saving and investment must ultimately be reconciled at a macroeconomic level, but in the short run they can differ. The difference is the growth in inventories,

$$\frac{d}{dt}\text{inventory} = S - I = K\left(g^s - g^i\right). \tag{4}$$

Firms are assumed to adjust the utilization of their equipment in response to build-up or draw-down of inventories. Specifically,

$$\dot{u} = -\alpha\left(g^s - g^i\right). \tag{5}$$

This is a standard assumption; for example, see Lavoie (2014, p. 363). The difference $g^s - g^i$ will recur throughout the paper and will be denoted by Γ,

$$\Gamma \equiv g^s - g^i, \tag{6}$$

so that $\dot{u} = -\alpha\Gamma$.

The inverse of the rate parameter α sets a time scale for adjustment of capacity utilization to its equilibrium. Suppose that π and κ change on time scales much longer than $1/\alpha$. Then they can be treated as approximately constant. When the goods equilibrium is established at firms' desired profit share $\overline{\pi}$, utilization is at a level u^* that satisfies

$$g^* = g^i(\overline{\pi}\kappa, u^*) = g^s(\overline{\pi}, \kappa, u^*). \tag{7}$$

This equilibrium is stable if $g^s_u(\overline{\pi}, \kappa, u^*) > g^i_u(\overline{\pi}\kappa, u^*)$ or, equivalently, $\Gamma_u > 0$. This is the Keynesian stability condition. So, at least locally, if the Keynesian stability condition holds and the dynamics of the profit share and capital productivity are much slower than that of capacity utilization, then capacity utilization will reach an equilibrium that clears the goods market.

This is the essence of a Kaleckian comparative statics exercise. As is well known, the system can become unstable if the accelerator g_u^i is sufficiently strong. That may, indeed, be possible. In that case, the dynamics are locally unstable but can be globally stable (as in Kemp-Benedict, 2020) due to ceilings (e.g., labor, capacity, or inventory constraints), floors (e.g., limits on disinvestment), or corridors (Setterfield, 2019; Hartley, 2022). However, this paper will assume that the Keynesian stability condition is satisfied.

2.1.1. Incompatibility with Harrod's natural rate

In this conventional Keynesian model, technical coefficients are given and firms set prices through a fixed markup, which determines π. In general, the investment rate at equilibrium will not equal Harrod's natural rate of growth g_n, which is given by

$$g_n = \hat{\lambda} + n + \delta. \tag{8}$$

In this expression, λ is labor productivity, $\hat{\lambda}$ is its growth rate, n is the growth rate of the working age population, and δ is the depreciation rate (or, more correctly, the retirement rate). If $g^* > g_n$, then there will be increasing pressure on the labor force. As noted in the Introduction, firms can try to lower that pressure by increasing n, a task that may involve lobbying for laws to be changed over immigration, mandatory retirement, and child labor. However, it reflects a fundamental contradiction and the building pressure will result in efforts to alter the distribution, for example by strengthening the political voice of labor.

Often the rate n is given by the rate of growth of the labor force. However, as noted by Sawyer (2009, p. 28), the labor force depends on the participation rate, and labor participation depends on aggregate demand. Accordingly, the model assumes that when labor demand is growing faster than the growth of the working age population, more people of working age will be brought into the labor force. This dynamic is not explicitly modeled. Instead, the mismatch is presumed to be resolved through a rise in wages; both the attraction of the rising wage and the observation that jobs are more available leads to an influx that relieves the pressure. Similarly, when the growth in labor demand is slower than growth in the working age population, firms can offer lower wages, while discouraged workers exit the labor force.

2.1.2. The profit-led investment criterion

Intuitively, it makes sense that if the investment rate is higher than the natural rate, to bring it in line with the natural rate it is necessary to reduce the markup and therefore the profit share. Certainly the investment and saving rates by themselves are increasing in the profit share. However, the equilibrium investment rate g^* may or may not behave that way. Using standard methods from Kaleckian comparative statics, it is possible to show that

$$\frac{\Delta g^*}{\Delta \pi} = \frac{g_u^s g_\pi^i - g_\pi^s g_u^i}{g_u^s - g_u^i} = \frac{g_u^s g_\pi^i - g_\pi^s g_u^i}{\Gamma_u}. \tag{9}$$

When this is positive – as intuition would suggest – the economy may be said to feature *profit-led investment*. Assuming the Keynesian stability condition $\Gamma_u > 0$ is satisfied, profit-led investment holds when

$$g_u^s g_\pi^i - g_\pi^s g_u^i > 0. \tag{10}$$

This condition can be compared to the conventional criterion for profit-led or wage-led regimes, in which $\Delta u / \Delta \pi > 0$ when the economy is profit-led and $\Delta u / \Delta \pi < 0$ when it is wage-led. A comparative static analysis shows that

$$\frac{\Delta u^*}{\Delta \pi} = \frac{g_\pi^i - g_u^s}{g_u^s - g_u^i} = -\frac{\Gamma_\pi}{\Gamma_u}. \tag{11}$$

Through multiplication and cancelling like terms, it can be shown that

$$g_u^s \Gamma_\pi - g_\pi^s \Gamma_u = g_u^s g_\pi^i - g_\pi^s g_u^i. \tag{12}$$

This expression is positive when the economy features profit-led investment, so when the profit-led investment condition holds, the response of capacity utilization to the profit share satisfies the inequality

$$\frac{\Delta u^*}{\Delta \pi} = -\frac{\Gamma_\pi}{\Gamma_u} > -\frac{g_\pi^s}{g_u^s}. \tag{13}$$

This shows that the profit-led investment condition is weaker than the conventional criterion for profit-led and wage-led economies. When profit-led investment holds, both profit-led and wage-led behavior are possible in terms of the conventional definition. Note that for a particular family of saving functions of the form $g^s = s(\pi)\kappa u$, with $s(\pi) \geq \pi s'(\pi)$,[2] it follows that $g_\pi^s / g_u^s \leq u/\pi$, so for this family of saving functions it is also true that

$$\frac{\Delta u^*}{\Delta \pi} = -\frac{\Gamma_\pi}{\Gamma_u} > -\frac{u^*}{\pi}. \tag{14}$$

[2]This inequality holds, for example, if $g^s = [s_p \pi + s_w(1 - \pi)]\kappa u$.

This relationship will sometimes be used when checking stability conditions, noting that it only applies to a certain family of saving functions.

2.2. Reference Model 2: Conflict wage determination

The second reference model introduces a form of conflict wage determination. The assumption is that the growth in the real wage depends positively on the gap between the growth in labor demand and the (exogenous) growth in the working age population. The general form of the function is similar to that of Rowthorn (1977) or Dutt (1992).

One outcome of the model is that in equilibrium both the profit share and capacity utilization are fully determined. There is no profit-utilization trade off and therefore no concept of a "growth regime". Growth regimes will be reintroduced but in a different form in §2.3 on "conflicting claims."

2.2.1. Equations of motion

The growth of the real wage rate is assumed equal to the growth rate of productivity plus a term that depends positively on the gap between the growth rate of labor demand L and the working age population growth rate n,

$$\hat{w} = \hat{\lambda} + f(\hat{L} - n), \qquad f(0) = 0, f' > 0. \tag{15}$$

The growth in the wage share ω is therefore

$$\hat{\omega} = \hat{w} - \hat{\lambda} = f(\hat{L} - n). \tag{16}$$

This gives an expression for the time rate of change of the profit share $\pi = 1 - \omega$ as

$$\dot{\pi} = -(1 - \pi) f(\hat{L} - n). \tag{17}$$

Growth in labor demand $\hat{L}$ is equal to growth in output less the growth rate of labor productivity λ. The growth in output is the growth in potential output plus the growth in capacity utilization. The growth in potential output, in turn, is given by $g^i(r, u) - \delta + \hat{\kappa}$, where δ is the depreciation rate (or, more properly, the retirement rate). Putting that together,

$$\hat{L} = g^i(r, u) - \delta + \hat{\kappa} + \hat{u} - \hat{\lambda}. \tag{18}$$

Allowing for a Verdoorn's law expression for $\hat{\lambda}$,

$$\hat{\lambda} = l + (1 - a) g^i(r, u), \tag{19}$$

where l is the intercept term and $1 - a$ is Verdoorn's coefficient, with a typical value of around one-half. (The use of $1 - a$ rather than a is for later convenience.) Substituting gives

$$\hat{L} = (1-a)g^i(r,u) - \delta + \hat{\kappa} + \hat{u} - l. \tag{20}$$

The equation of motion for the profit share then becomes

$$\dot{\pi} = -(1-\pi)f\left(ag^i(r,u) - \delta + \hat{\kappa} + \hat{u} - l - n\right). \tag{21}$$

As a final step, the growth rate of u can be expressed in terms of the expression for $\dot{u}$ in Eq. (5). Recalling that the difference between the saving and investment rates is denoted by $\Gamma \equiv g^s - g^i$,

$$\dot{\pi} = -(1-\pi)f\left(ag^i - \delta + \hat{\kappa} - \frac{\alpha}{u}\Gamma - l - n\right). \tag{22}$$

When combined with the equation of motion for u, this completes the dynamic model.

2.2.2. Kaldorian equilibrium

Assuming as before that the capital productivity κ is fixed, at the equilibrium the argument of the function f must be zero, while $\hat{u} = 0$ as well. The equilibrium is therefore at a point (u^*, π^*) that satisfies

$$g^i(\pi^*\kappa, u^*) = g^s(\pi^*, \kappa, u^*), \tag{23a}$$

$$g^i(\pi^*\kappa, u^*) = \frac{l+n+\delta}{a}. \tag{23b}$$

Eq. (23a) by itself, without imposing Eq. (23b), is the equilibrium condition for the model in Lavoie (2014, p. 381), which in turn was based on Bruno (1999) and Bhaduri (2008). Taken by itself, it provides a schedule of values for π^* and u^*. The addition of Eq. (21) fixes a unique equilibrium point (π^*, u^*). Moreover, it ensures the growth rate of the economy is at Harrod's natural rate g_n,

$$g^i = g^s = \frac{l+n+\delta}{a} = \hat{\lambda} + n + \delta = g_n. \tag{24}$$

This system therefore gives an equilibrium investment rate compatible with Harrod's natural rate but at the expense of fixing both capacity utilization and the profit share. In this case the conventional notions of "profit-led" and "wage-led" do not make sense. However, as discussed in the next sub-section, the distinction is relevant when comparing pricing regimes.

Before discussing the stability of the system, note that for the particular case in which $g^s = s_p\pi^*\kappa u^*$,

$$r^*u^* = \pi^*\kappa u^* = \frac{g_n}{s_p}. \tag{25}$$

This is the Cambridge equation; see Hein (2014, p. 129). If the equilibrium is stable, this model bridges Kalecki and Kaldor, in that Kaleckian utilization dynamics, when combined with conflict wage-setting, leads at equilibrium to the Cambridge equation.

2.2.3. Stability

In the vicinity of the the equilibrium determined by Eqs. (23),

$$\Delta\dot{u} = -\alpha\left(\Gamma_u\Delta u + \Gamma_\pi\Delta\pi\right), \tag{26a}$$

$$\Delta\dot{\pi} = -(1-\pi^*)f'(0)\left[\left(ag^i_\pi - \frac{\alpha}{u^*}\Gamma_\pi\right)\Delta\pi + \left(ag^i_u - \frac{\alpha}{u^*}\Gamma_u\right)\Delta u\right]. \tag{26b}$$

Define $\beta \equiv (1-\pi^*)f'(0)$ for convenience, and write this in matrix form as

$$\begin{pmatrix}\Delta\dot{u}\\ \Delta\dot{\pi}\end{pmatrix} = \begin{pmatrix}-\alpha\Gamma_u & -\alpha\Gamma_\pi\\ -\beta\left(ag^i_u - \frac{\alpha}{u^*}\Gamma_u\right) & -\beta\left(ag^i_\pi - \frac{\alpha}{u^*}\Gamma_\pi\right)\end{pmatrix}\begin{pmatrix}\Delta u\\ \Delta\pi\end{pmatrix}. \tag{27}$$

From the Routh-Hurwitz stability criteria, this system is stable if the trace of the matrix of coefficients is negative and the determinant is positive. The determinant D, is given by

$$D = a\alpha\beta\left(g^i_\pi\Gamma_u - g^i_u\Gamma_\pi\right) = a\alpha\beta\left(g^i_\pi g^s_u - g^i_u g^s_\pi\right). \tag{28}$$

Note that the determinant is positive if the economy exhibits profit-led investment; see Eq. (10). That condition will be assumed to hold.

The trace T, is given by

$$T = -\alpha\Gamma_u - \beta a g^i_\pi + \beta\frac{\alpha}{u^*}\Gamma_\pi. \tag{29}$$

The first term is the standard Keynesian stability condition. The others, which are multiplied by $\beta = (1-\pi^*)f'(0)$, arise from the wage equation. If investment is more responsive than saving to a change in distribution, then $\Gamma_\pi < 0$ and all terms are negative. Otherwise there is a range of values for $\Gamma_\pi > 0$ over which the trace remains negative.

2.3. Conflicting claims

As demonstrated above, the basic dynamic post-Keynesian model can be closed in multiple ways that reflect different pricing regimes. Two were considered: a fixed markup and conflict wage setting. When the Keynesian stability condition and the profit-led investment condition of Eq.(10) are satisfied, the system is stable. For Reference Model 1 (§2.1), the profit share is exogenous. The equilibrium utilization rate then depends on the profit share. However, in the wage conflict model (Reference Model 2, §2.2), the profit rate and capacity utilization are fully determined at equilibrium, so the standard Kaleckian comparative static analysis cannot be applied.

From a political economy perspective, either pricing regime can create tensions, so the stability of the system is conditional. Under a fixed markup, there is no reason why the resulting growth rate should be compatible with Harrod's natural rate. As discussed above, if the imbalance places upward pressure on labor demand, then rather than give up profits, firms might try to escape the labor constraint by increasing the size of the working age population through offshoring and immigration, or expanding the definition of "working age" by postponing retirement and bringing children into the labor force. All of these options can be observed, and are indeed highly politically salient at the moment.

A further politically salient trend is resurgent interest in union membership. In the terms of the present model, that could lead to a change in pricing regime. If under conflict wage determination the profit rate is lower than the target that had been maintained previously by firms, then the profit share would begin to fall. The resulting equilibrium would be consistent with the growth rate of the working age population. However, this state is also politically unstable to the extent that disappointed firms and investors will seek to break labor's influence over distribution and return to a regime of higher profits.

The outcome of the conflicting claims reflected in alternating pricing regimes is wave-like changes in capacity utilization and the profit share. The timing is influenced by stresses created by one regime or the other and moderated by political processes. These typically play out over multiple decades, and so would appear as (irregular) "long wave" phenomena.

Because the conflicting claims dynamics are characterized by differences in equilibrium states, it is appropriate to carry out a comparative statics analysis. Following standard methods,

$$\frac{\Delta u^*}{\Delta \pi^*} = \frac{g^i_\pi - g^s_\pi}{g^s_u - g^i_u}. \tag{30}$$

Assuming the equilibria are stable (that is, both Keynesian stability and profit-led investment hold), then from the inequality in Eq. (13), $\Delta u^*/\Delta \pi^* > -g^s_\pi/g^s_u$. This allows for both profit-led and wage-led regimes as conventionally defined. Under a profit-led regime, shifting from a target-return pricing regime to a conflict wage determination one, with a lower profit rate, would lower the equilibrium rate of capacity utilization (assuming that the investment and saving functions retain their form). Under a wage-led regime, the shift would result in higher capacity utilization.

3. Cost share-induced technological change

With cost share-induced technological change, productivity growth rates depend on distribution. This is a classical, not neoclassical, assumption; productivities are constant in the (very) short run but are continually changing due to formal and informal processes of innovation. In some models, such as the classical models of Dutt (2013) or the post-Keynesian models of Cassetti (2003) or Hein and Tarassow (2010), only one productivity is cost share-dependent. In other theories, including the classical model of Foley et al. (2019) and the classical-evolutionary model of Duménil and Lévy (2010), if one productivity depends on cost shares then so must at least one other. While the author favors the classical-evolutionary theory of cost share-induced technological change (Kemp-Benedict, 2022), the present paper places very few demands on the model beyond the assumption that the capital productivity growth rate depends positively on the profit share. We therefore assume, quite generally, that

$$\hat{\kappa} = k(\pi), \qquad k' > 0, \tag{31}$$

and

$$\hat{\lambda} = l(\pi) + ag^i(r, u), \qquad l' \leq 0. \tag{32}$$

Thus capital productivity is assumed to always depend on the cost share, while labor productivity might or might depend on the cost share.

We present two models, one featuring target-return pricing and the other featuring conflict wage determination. Note that in Reference Model 1 (§2.1), which featured a fixed markup, because the profit share is exogenous at a level $\overline{\pi}$, under cost share-induced technological change the capital productivity will have a trend as determined by Eq. (31), equal to $k(\overline{\pi})$. In contrast, under target-return pricing distribution depends on capital productivity, generating a feedback that tends towards a constant cost share and constant capital productivity.

3.1. Target-return pricing

In the simplest formulation of target-return pricing, firms respond immediately to changes in capital productivity by altering their markups so as to maintain a constant target profit rate $\overline{r}$. Because utilization is endogenously determined, the target profit rate should be evaluated at the equilibrium rate of capacity utilization. However, because firms do not know that level *a priori*, they are assumed to adjust their markups taking current capacity utilization into account,

$$\pi \kappa u = \overline{r}. \tag{33}$$

This equation can be applied with an exogenously changing target profit rate. However, assuming for simplicity that $\overline{r}$ is not changing over time, this produces an equation of motion for π,

$$\dot{\pi} = -\pi \left(\hat{\kappa} + \hat{u} \right) = -\pi \left(k(\pi) - \frac{\alpha}{u} \Gamma \right). \tag{34}$$

In the final expression, the equation for $\dot{u}$ from Reference Model 1, Eq. (5), has been applied. Combining this equation with utilization dynamics from Reference Model 1 gives an equilibrium characterized by constant profit share π^* at which $\hat{\kappa} = k(\pi^*) = 0$, and a constant utilization u^* that clears the goods market when the profit share is at π^* and capital productivity is $\kappa^* = \overline{r}/\pi^* u^*$. The equilibrium is therefore characterized by Harrod-neutral technological change and two of Kaldor's stylized facts: constant cost shares and constant profit rate (Kaldor, 1961).

In the vicinity of the equilibrium,

$$\Delta \dot{u} = -\alpha \left(\Gamma_u \Delta u + \Gamma_\pi \Delta \pi \right), \tag{35a}$$

$$\Delta \dot{\pi} = -\pi^* \left(k' - \frac{\alpha}{u^*} \Gamma_\pi \right) \Delta \pi + \pi^* \frac{\alpha}{u^*} \Gamma_u \Delta u. \tag{35b}$$

The stability conditions are

$$\text{negative trace: } \alpha\left(\Gamma_u - \frac{\pi^*}{u^*}\Gamma_\pi\right) + \pi^* k' > 0, \tag{36a}$$

$$\text{determinant: } \alpha\pi^* k'\Gamma_u > 0. \tag{36b}$$

Under cost share-induced technological change, $k' > 0$. Furthermore, using the inequality in Eq. (14), we find that the term that appears in parentheses in the trace equation, $\Gamma_u - (\pi^*/u^*)\Gamma_\pi$, is positive for the class of saving functions of the form $g^s = s(\pi)\kappa u$. The negative trace condition is therefore satisfied, at least for this class of saving functions.

The positive determinant condition shows that stability requires $\Gamma_u > 0$, just as in the case for utilization dynamics alone. The stability conditions are thus satisfied when the Keynesian stability condition is satisfied. The difference from Reference Model 1 is that the profit share and capital productivity are no longer exogenous. Rather, their product multiplied by capacity utilization is exogenous (equal to $\overline{r}$), while they are endogenously determined by cost share-induced technological change.

As with the case of a fixed markup, under target-return pricing the trajectory of the economy need not align with the natural rate of growth. This conflict will be taken up in §2.3, but first the model with conflict wage determination and cost share-induced technological change will be discussed.

3.2. Conflict wage determination

Cost share-induced technological change adds a further equation to the system for Reference Model 2 (in §2.2). From Eq. (31),

$$\dot{\kappa} = \kappa k(\pi). \tag{37}$$

When combined with Reference Model 2, all the variables π, κ, and u are endogenous. The equilibrium is a point (π^*, κ^*, u^*) that satisfies

$$k(\pi^*) = 0, \tag{38a}$$

$$g^i(\pi^*\kappa^*, u^*) = g^s(\pi^*, \kappa^*, u^*), \tag{38b}$$

$$g^i(\pi^*\kappa^*, u^*) = \frac{l + n + \delta}{a}. \tag{38c}$$

Note that in addition to attaining Harrod's natural growth rate at equilibrium, as with target-return pricing the equilibrium is characterized by

Harrod-neutral technological change and Kaldor's stylized facts of constant cost shares and profit rate.

In the vicinity of the equilibrium,

$$\Delta\dot{\kappa} = \kappa^* k'(\pi^*)\Delta\pi, \tag{39a}$$

$$\Delta\dot{u} = -\alpha\left(\Gamma_\kappa \Delta\kappa + \Gamma_u \Delta u + \Gamma_\pi \Delta\pi\right), \tag{39b}$$

$$\Delta\dot{\pi} = -\beta \sum_{x\in\{\kappa,u,\pi\}} \left(a g^i_x - \frac{\alpha}{u^*}\Gamma_x\right)\Delta x. \tag{39c}$$

If utilization is held fixed, then taken by themselves the equations for π and κ are dynamically stable. Thus, as in the case of target-return pricing, the cost share-induced technological change mechanism generates stable dynamics, while utilization can still be destabilizing if the accelerator is strong enough.

Working through the algebra, the characteristic equation for this system can be shown to be

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \tag{40}$$

where (suppressing dependence of functions on κ^*, u^*, and π^*),

$$a_2 = \alpha\Gamma_u + \beta\left(a g^i_\pi - \frac{\alpha}{u^*}\Gamma_\pi + k' - l'\right), \tag{41a}$$

$$a_1 = a\alpha\beta\left(g^i_\pi g^s_u - g^s_\pi g^i_u\right) + \kappa^* k'\beta\left(a g^i_\kappa - \frac{\alpha}{u^*}\Gamma_\kappa\right), \tag{41b}$$

$$a_0 = \kappa^* k' a\alpha\beta\left(g^s_u g^i_\kappa - g^s_\kappa g^i_u\right). \tag{41c}$$

From the Routh-Hurwitz criteria, the system is stable if $a_0, a_1, a_2 > 0$ and $a_1 a_2 > a_0$.

For purposes of comparison, it can be useful to write these equations in terms of the determinant D and trace T of the two-variable system for Reference Model 2. The trace condition becomes

$$T < 0 \quad \rightarrow \quad -a_2 = T - \beta\left(k' - l'\right) < 0. \tag{42}$$

The determinant condition becomes

$$D > 0 \quad \rightarrow \quad a_1 = D + \kappa^* k'\beta\left(a g^i_\kappa - \frac{\alpha}{u^*}\Gamma_\kappa\right) > 0. \tag{43}$$

The condition for a_0 remains as in Eq. (41c). In the expression for a_2, $k' > 0$ and $l' \leq 0$, so in terms of the trace, the cost share-induced technological change dynamics serve to further stabilize the system when compared to the two-variable system. The other expressions are more ambiguous, and require additional elaboration.

First, recall that because g^i depends on $r = \pi\kappa$,

$$\kappa g^i_\kappa = \pi g^i_\pi = r g^i_r. \tag{44}$$

Further, for the class of saving functions considered in §2.1.2, $s(\pi) > \pi s'(\pi)$. We therefore have, for that family,

$$\kappa g^s_\kappa \geq \pi g^s_\pi. \tag{45}$$

This means that

$$a_0 \leq \pi^* k' a\alpha\beta \left(g^s_u g^i_\pi - g^s_\pi g^i_u\right) = \pi^* k' D. \tag{46}$$

Similarly,

$$a_1 \leq D + \pi^* k'\beta \left(a g^i_\pi - \frac{\alpha}{u^*}\Gamma_\pi\right) = D - \pi^* k' \left(T + \alpha\Gamma_u\right). \tag{47}$$

Taken together, this suggests that the condition $D > 0$ must still be satisfied. That is the profit-led investment criterion of Eq. (10). However, the condition on the trace is weaker than in the case of Reference Model 2. Cost share-induced technological change does not therefore alter the core conditions of Keynesian stability and profit-led investment, but it does modestly loosen some of the other conditions.

3.3. Conflicting claims

As with the reference models, the models with cost share-induced technological change invite a conflicting claims analysis. The result is similar: target-return pricing will generally be incompatible with the growth rate of the working age population. Efforts to preserve profits in the face of mounting pressure on labor can lead to a reaction and the establishment of a wage-conflict pricing regime, but disappointment with the profits available under conflict wage determination encourage political pressure to weaken unions and labor laws.

The introduction of cost share-induced technological change does lead to some changes. When comparing the reference cases of a fixed markup to a wage-conflict model without cost share-induced technological change, it is the profit share that adjusts to bring the profit rate to a level compatible with the growth of the working age population. However, with cost share-induced technological change added to the dynamics, the equilibrium profit share is the same in both pricing regimes; it is the value π^* that satisfies $k(\pi^*) = 0$ under both target-return pricing and wage-conflict pricing with cost share-induced technological change. Rather, it is the capital productivity that adjusts.

While the equilibrium profit share is unchanged, that is not true along the trajectory between pricing regimes. At the start of the transition from a target-return regime to a wage-conflict regime, the profit share begins to fall. Capital productivity growth becomes negative, and if labor productivity growth depends on distribution, it will rise. However, by the end of the transition the cost share is the same and the investment rate is lower. Capital productivity is lower, and labor productivity growth is also lower due to the Verdoorn term.

4. Discussion

This paper contributes to a strand of post-Keynesian analysis in which distribution and capacity utilization are simultaneously determined in dynamic models. One novel contribution is a model with conflict wage determination and cost share-induced technological change whose equilibrium features Harrod's natural rate of growth, Harrodian technological change, and Kaldor's stylized facts of constant cost share and constant profit rate. Another model, incorporating target-return pricing, yields Harrodian technological change and Kaldor's stylized facts, but not Harrod's natural rate of growth. Rather than reject the model, the paper proposes to treat both target-return pricing and wage-conflict pricing as two possible pricing regimes, neither of which is politically stable.

For all but one of the models presented in the paper, both the profit share and capacity utilization are fully determined at the equilibrium. Thus, the notion of "profit-led" and "wage-led" loses meaning. However, by carrying out a comparative statics analysis in terms of pricing regimes – for example, target-return and wage-conflict – the profit-led and wage-led distinction regains its meaning. Moreover, comparison of pricing regimes is politically salient. Struggles over distribution play out over long times as the contradictions raised by one regime or another – incompatibility with the working age population growth rate for target-return pricing and disappointing returns for wage-conflict pricing – lead to political opposition. Repeatedly transitioning from one pricing regime to the other can lead to the appearance of "long waves" in equilibrium capacity utilization, profit rates, and distribution.

The analysis was simplified by assuming that structural features of the economy remained the same. In fact, the passage from one pricing regime to another could change saving or investment behavior, as well as available technologies. Furthermore, while stability was assumed, it is not always observed and is not strictly necessary. When it is absent, the analysis should shift towards global rather than local stabilizing mechanisms, such as ceilings, floors, or corridors.

The motivation for the study is the analysis of a sustainability transition. For such an analysis, technological change is an important dynamic, but mainly in terms of use natural resources. This paper instead chose to connect to the post-Keynesian literature by focusing on labor and capital inputs. However, the paper did focus on transitions between regimes, which is an important topic for sustainability analysis. Importantly, distinct pricing regimes give rise to distinct equilibria. Changing behavior can destabilize an economy and trigger a transition to a new equilibrium.

5. Conclusion

In contrast to the comparative statics Kaleckian analysis that constitutes much of post-Keynesian analysis, some authors have built dynamic post-Keynesian models to simultaneously determine both distribution and output. This paper is in that tradition. It adds new models and mechanisms. A mechanism for conflict wage determination yields an equilibrium that is plausibly stable at which the investment rate coincides with Harrod's natural rate. The mechanism of cost share-induced technological change yields equilibria characterized by Harrodian technological change and Kaldor's stylized facts of constant cost share and constant rate of profit.

The models fully determine capacity utilization and the profit share at equilibrium. Compared to a standard Kaleckian comparative statics analysis, this can seem unduly constraining. Typically, changes in the equilibrium capacity utilization at different levels of the profit share are analyzed. This paper proposes that the equilibria for different pricing regimes be compared. The ones treated in the paper include constant markup, target-return, and wage-conflict pricing. Each pricing regime can generate pressures that are relieved through a switch to a different pricing regime. The switch is conditioned by political economy considerations and can give rise to decades-long cycles.

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