A Marx ‘Crises’ Model: The Reproduction Schemes Revisited

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Abstract: This paper builds upon the Marxian reproduction schemes. It aims to test the impact of some of the most apparent 'stylised facts' which characterise the current phase of capitalism on an artificial two-sector growing economy. It is shown that, simplified though they are, the Marxian reproduction schemes allow framing a variety of radical and other 'dissenting' renditions of the recent economic and financial crises of early-industrialised countries with a flexible and sound analytical model.

Keywords: Marx, Crisis, Reproduction Schemes, Post-Keynesian Economics

JEL classifications: B5, E11, E12

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1. Introduction

The US financial crisis of 2007-2008 and the crisis of the Euro Area which has been taking place since the end of 2009 have arguably been triggered and then fostered by a multiplicity of factors. Several radical and other 'dissenting' renditions of the historical causes of western countries' economic and financial distress have been provided ever since. This is not surprising. Quite a few, alternate, theories of crisis can be found or ground in Marx's works (see, among others, Shaikh 1978, and Clarke 1990). The long-run fall in the rate of profit (resulting either from the rising organic composition of capital or from the depletion of the reserve army of labour), the thinning of the costing margin due to class struggle (either over distribution or over production), the lack of aggregate demand (meaning the tendency to overproduction that may result in a 'realisation' crisis), and the rise of sectoral imbalances (or 'disproportionalities'), are all mentioned by Marx as inner forces or tendencies of capitalism.

This paper aims neither to endorse explicitly any of the explanations above nor to provide a brand-new interpretation or theory of crisis. Rather it builds upon the Marxian enlarged reproduction schemes to test the effects of the most apparent 'stylised facts' of the current phase of capitalism on an artificial two-sector (or 'two-department') growing economy. It shows that, simplified though they are, the Marxian reproduction schemes may allow redefining and comparing different theories of crisis within a coherent analytical framework.

The paper is organised as follows. Sections 2 and 3 set up the benchmark model and define the reproduction (or balanced growth) conditions, respectively. The resemblance of the Marxian approach to the Cambridge School of Economics and other recent post-Keynesian theories is briefly discussed. In section 4 an extended Marxian enlarged reproduction model is developed, aiming to account for the effect of financial markets and institutions on the creation of social value and surplus value. In section 5 a number of experiments are performed to test the impact of some 'stylised facts' (distilled from recent developments in real-world highly-financialised countries) on an artificial two-department growing economy. Key findings are discussed further in section 6.

2. The benchmark model

The view of the economic system as a circular flow of interconnected acts of production and circulation of commodities and money is deeply rooted in the history of economic thought. Its inception can be traced back to the pioneering work of François Quesnay and other French Physiocrats of the eighteenth century.[1] The Physiocrats (and, at least to some extent, David Ricardo and the Classical political economists) focused on the process of creation, circulation, and consumption of the *produit net* of an agriculture-based economy. Marx built upon that line of research and focused on the process of creation, circulation, and destruction of the monetary surplus value of a manufacturing-based capitalist system (Veronese Passarella 2016). The so-called 'reproduction schemes' are developed in the second volume of *Capital* (Marx 1885, chapters 20 and 21), where Marx defines the theoretical equilibrium conditions of the economy in terms of interdependences between 'departments' – meaning the net flows of commodities that must be produced and circulated among the productive macro-sectors to meet the respective demands of inputs.

[1] See Marx (1885), chapter 19, pp. 435 ff., and chapter 20, pp. 509-13.

While Marx never engaged with a formal enlarged reproduction model, he provided several notes and numerical examples that may well be turned into a system of difference (or differential) equations. In fact, there is a well-established tradition of dynamic modelling carried out by Marxist economists since the 1970s, inspired by the Marxian reproduction schemes (see, among others, Harris 1972, Bronfenbrenner 1973, Morishima 1973; more recently, Olsen 2015 and Cockshott 2016). This section draws from that tradition and cross-breeds it with other current heterodox approaches, particularly with post-Keynesian macro-monetary modelling.[2] This allows setting up the formal benchmark model of a growing economy that moves forward non-ergodically in time, t, and is made up of two sectors (or departments): a sector producing capital or investment goods (called 'department I' by Marx), defined by the subscript 'i'; and a sector producing consumption goods (named 'department II'), defined by the subscript 'c'.[3] For the sake of simplicity, it is assumed that each production process takes a fraction $1/n_j$ (with $j = c, i,$) of the reference period, t, where n_j is a parameter accounting for the sectoral intra-period turnover rate.[4] Commodities are produced by means of capital goods and labour inputs. Labour supply is plentiful and does not form a binding constraint on the level of employment (see Appendix 1). A net product arises (both in real and monetary terms) in each sector and is distributed as wages to workers and surplus value (or profit) to capitalists. The rate of depreciation of capital is unity, that is, only circulating capital is used.

As is well known, Marx's analysis of value relies upon the distinction between the variable component of capital and its constant component. The former roughly corresponds to the wage bill paid by the industrial capitalists to the workers in exchange for their labour power. This sum covers the part of the total working day that is devoted to the production of 'subsistence' for workers.[5] Under a growing economy, the sectoral investments in variable capital inputs are, respectively:

$$V = V_{i,-1} + \frac{S_{i,-1} \cdot \theta_i}{1+q_i} \quad (1)$$

and

$$V_c = V_{c,-1} + \frac{S_{c,-1} \cdot \theta_c}{1+q_c} \quad (2)$$

where S_j is the surplus value created in the j-th department (with $j = i, c$), θ_j is the sectoral rate of saving or retention of capitalists, $q_j = C_j/V_j$ is the sectoral organic composition of capital (OCC), and C_j is the sectoral constant capital, meaning the amount of capital inputs (i.e. circulating capital net of wages in this simplified model) invested in the j-th department.

[2] The resemblance of the Marxian approach to the current post-Keynesian macro-monetary literature shows up particularly when an 'endogenist' rendition of Marx's monetary theory is adopted (Hein 2006).

[3] Notice that upper case letters are associated with endogenous variables expressed in monetary units, unless otherwise stated. Lower case letters stand either for percentages or for parameter values expressed in monetary units. The key to symbols is provided by Table 1.

[4] This possibly controversial assumption is discussed further later on, particularly in footnote 7. Notice that $n_j = 1$ in the baseline model. As a result, each production process takes exactly one period, unless otherwise stated.

[5] Actually this should be better defined as the 'unallocated purchasing power' of workers (Duménil and Foley 2008), meaning the quantity of direct labour expressed by the commodities bought by the wage earners on the market. For the sake of simplicity, this issue is neglected hereafter. The reader is referred again to Appendix 1.

Equations (1) and (2) show that a share θ_j of the surplus value created in the j-th sector is re-invested in the same sector in the subsequent period.[6] The ratio of constant capital to variable capital is defined by the OCC, which is taken as an exogenous of the model. Accordingly, the investment in constant capital can be worked out as:

$$C_i = V_i \cdot q_i \tag{3}$$

and

$$C_c = V_c \cdot q_c \tag{4}$$

According to Marx, it is only the variable capital that valorises in the production sphere, as the wage-earners work well beyond the time necessary to cover the exchange value of their own labour power. As a result, the 'masses' of surplus value created in each sector in a certain period are, respectively:

$$S_i = V_i \cdot \varepsilon_i \cdot n_i \tag{5}$$

and

$$S_c = V_c \cdot \varepsilon_c \cdot n_c \tag{6}$$

where $\varepsilon_j = S_j/V_j$ is the sectoral rate of surplus value (mirroring the composition of the total working day and hence the rate of exploitation of workers) and n_j is a parameter reflecting the sectoral intra-period turnover rate.[7]

Equations (5) and (6) show that the mass of surplus value created in the j-th sector across a period – say, a quarter or a year – is a direct function of the variable capital invested in that sector, the rate of surplus value, and the turnover rate, meaning the number of times the same capital is reinvested within the period. In principle, capitalists can either consume the non-retained surplus value or divert it towards their own personal saving. Accordingly, the capitalists' unproductive expenditures are, respectively:

$$F_i = (1 - \theta_i) \cdot S_i \cdot (1 - \sigma_{i1}) + (1 - \sigma_{i2}) \cdot H_{i,-1} \tag{7}$$

and

$$F_c = (1 - \theta_c) \cdot S_c \cdot (1 - \sigma_{c1}) + (1 - \sigma_{c2}) \cdot H_{c,-1} \tag{8}$$

where σ_{j1} and σ_{j2} (with $j = i, c$) are the marginal propensities to save out of income and wealth, respectively, and HH_j is the stock of wealth amassed by j-sector capitalists. The latter can defined as follows:

$$HH_i = HH_{i,-1} + \sigma_{i1} \cdot (1 - \theta_i) \cdot S_{i,-1} \tag{9}$$

[6] Marx neglects cross-sector investment instead (see Marx 1885, chap. 21, pp. 568-77, 577-81). This is a strong simplifying assumption. In fact, it seems at odds with the hypothesis of competition which requires free mobility of capital between sectors (Robinson 1951, Harris 1972). However, that simplification is maintained here, as it does not affect the main findings of the paper.

[7] Under an enlarged reproduction regime, the mass of surplus value created in a certain period should be better defined as: $S_j = V_j \cdot \varepsilon_j \cdot \sum_{\tau=1}^{n_j} (1 + \vartheta_j \cdot \varepsilon_j)^{\tau-1}$, where the subscript τ defines the sub-periods, n_j is the number of turnovers, and ϑ_j is the intra-period retention rate. This expression accounts for the reinvestment of variable capital within the same period (see Veronese Passarella and Baron 2015). However, such a complication is ignored hereafter. Notice that the expression above collapses to $S_j = V_j \cdot \varepsilon_j \cdot n_j$ under simple reproduction (i.e. for $\vartheta_j = 0$). Consequently, enlarged accumulation takes place *across* periods but not *within* periods in this simplified model.

and

$$HH_c = HH_{c,-1} + \sigma_{c1} \cdot (1 - \theta_c) \cdot S_{c,-1} \tag{10}$$

Accordingly, the realised total values of sectoral outputs are, respectively:

$$Y_i = C_i + V_i + \theta_i \cdot S_i + F_i \tag{11}$$

and

$$Y_c = C_c + V_c + \theta_c \cdot S_c + F_c \tag{12}$$

If capitalists spend their incomes all, either through productive investment or through consumption, then $Y_j = C_j + V_j + S_j$, meaning that the overall monetary value realised (by the capitalists) on the market matches or 'validates' the overall value created *in potentia* (by the workers) in the production sphere. Similarly, the realised sectoral profit rate are, respectively:

$$r_i = \frac{\theta_i \cdot S_i + F_i}{V_i + C_i} \tag{13}$$

and

$$r_c = \frac{\theta_c \cdot S_c + F_c}{V_c + C_c} \tag{14}$$

Finally, the sectoral rates of accumulation can be defines as:

$$g_i = \frac{\frac{S_i \cdot \theta_i \cdot n_i}{1+q_i}}{V_i} = \varepsilon_i \cdot \theta_i \cdot n_i \cdot \frac{1}{1+q_i} \tag{15}$$

and

$$g_c = \varepsilon_c \cdot \theta_c \cdot n_c \cdot \frac{1}{1+q_c} \tag{16}$$

Each sectoral rate of growth is a direct function of the saving rate, the exploitation rate, and the intra-period turnover rate parameter, and an indirect function of the organic composition of capital of the j-th sector.[8]

3. The reproduction conditions

3.1 *Simple reproduction* - As has been mentioned, Marx (1885) defines the equilibrium conditions for a capitalist economy in terms of the necessary interdependences between macro-sectors, meaning the theoretical requirements allowing the overall system to reproduce smoothly over time.[9] Marx analyses the equilibrium conditions under a simple reproduction regime (namely, a stationary-state economy) and then under an enlarged or expanded reproduction regime (meaning a growing economy). Capitalists' savings are assumed away, so that $\sigma_{1j} = 0$ and hence $H_j = 0$. In addition, sectoral rates of saving are all null under a simple reproduction regime ($\theta_j = 0$), and so are accumulation rates ($g_j = 0$). Investment and consumption goods markets clear when:

$$Y_i = C_i + C_c$$

and

[8] See Appendix 2 for a development of the model aiming to account for stocks and financial assets.

[9] Notice, however, that the equilibrium interpretation of the Marxian reproduction schemes is anything but uncontroversial (see Fine 2012).

$Y_c = V_i + V_c + F_i + F_c$

Using equations (11) and (12) in the equalities above, one gets the well-known Marxian reproduction condition for a stationary-state economy:

$$C_c = V_i + F_i = V_i + S_i \tag{4B}$$

After some manipulation, one obtains also:

$$\frac{V_i}{V_c} = \frac{q_c}{1+\varepsilon_i} \tag{4C}$$

Equation (4B) shows that the neo-value of output of the i-sector (right-hand component) must match investment plans of c-sector's capitalists (left-hand component). Equation (4C) shows that the equilibrium distribution of variable capital across sectors depends on the c-sector OCC and the i-sector exploitation rate. When this condition is met, the economy finds itself in the simple reproduction equilibrium position. By contrast, if $V_i + S_i > C_c$ there is a lack of demand for capital goods. According to Marx, market prices of capital goods will tend to fall short of reproduction values. As a result, both the (expected) profit rate and the real investment fall. Similarly, if $V_i + S_i < C_c$ there is an excess of demand for investment goods. Market prices exceed reproduction values. Both the profit rate and the real investment rise. Sooner or later the lack (excess) of demand for capital goods ends up reducing (raising) the supply of capital goods.[10]

However, Marx does not advocate any inner adjustment mechanism of capitalist economies. Under a free market regime, nothing ensures that the change in the supply of capital goods – resulting from capitalists' individual decisions – exactly matches the supply-demand gap. For Marx, once capitalists' investment plans are out of equilibrium, individual expectations and behaviours (meaning competition between capitalists) drag prices and quantities away from their own reproduction values. In real-world capitalist economies, 'supply and demand never coincide, or if they do so, it is only by chance and not to be taken into account for scientific purposes: it should be considered as not having happened' (Marx 1894, p. 291). In principle, the equilibrium condition may be regarded as a long run attractor, but cyclical fluctuations and crises are an inherent feature of capitalism.[11]

3.2 *Enlarged reproduction* - Things get slightly more complicated when considering a growing economy. Now the reproduction conditions are met if and only if capitalists' production and investment plans are mutually consistent. Following Marx, one can assume that it is the rate of accumulation in the consumption goods sector that varies to ensure the smooth reproduction of the system (see Olsen 2015). In other words, the c-sector demand for investment goods is assumed to adjust to match the net supply by the i-sector. In formal terms, the accumulation of constant capital in the c-sector is:

[10] Notice that variables are all 'expressed in terms of value aggregates and as such can provide only the conditions for aggregate equilibrium' (Harris 1972, p. 190). To discuss the effect of the disequilibrium conditions on prices and physical magnitudes (such as real supplies and employment levels), respectively, it is necessary to refine further the analysis – see Appendix 3.

[11] This could prepare the ground for a radical undermining of the system. However, the final collapse is anything but *necessary*. The contradictions of capitalism, including the one between the drive towards the unlimited expansion of production and limited consumption, 'testify to its historically transient character, and make clear the conditions and causes of its collapse and transformation into a higher form; but they by no means rule out either the possibility of capitalism' (Lenin 1908, chapter I, section VI, p. 57). In other words, the reproduction schemes show that capitalism 'proceeds though crises rather than being rendered an impossibility because of them' (Patnaik 2012, p. 374).

$$S_c \cdot \theta_c \cdot \frac{q_c}{1+q_c} + C_c = Y_i - C_i - S_i \cdot \theta_i \cdot \frac{q_i}{1+q_i}$$

Similarly, the accumulation of variable capital in the c-sector is:

$$S_c \cdot \theta_c \cdot \frac{1}{1+q_c} = \left[Y_i - C_i - S_i \cdot \theta_i \cdot \frac{q_i}{1+q_i} - C_c\right] \cdot \frac{1}{q_c}$$

Finally, the equilibrium rate of growth of the c-sector can be worked out as follows:

$$g_c = \frac{S_c \cdot \theta_c \cdot \frac{q_c}{1+q_c}}{C_c} = \frac{Y_i - C_i - S_i \cdot \theta_i \cdot \frac{q_i}{1+q_i}}{C_c} - 1$$

The condition above ensures the consistency of c-sector capitalists' investment plans with i-sector capitalists' production (and investment) plans. In other words, it assures the long-run gravitation of the economy towards the 'reproduction equilibrium'. Such a state is extremely unlikely to be matched (and maintained) in practice. In fact, the reproduction schemes allow Marx to argue that real-world capitalist economies always work in disequilibrium.[12] They also allow shedding light on the adjustment paths of key variables of the model. This is the reason equation below in used hereafter:

$$g_c = \left[\frac{Y_i - C_i - S_i \cdot \theta_i \cdot \frac{q_i}{1+q_i}}{C_c} - 1\right] + g_\xi \tag{16B}$$

where g_ξ is a random component accounting for broadly-defined 'exogenous shocks' to the c-sector growth rate.[13]

Notice that g_c may well differ from g_i in the short run. However, the former converges towards the latter in the long run (due to the constancy of OCCs), i.e. $\lim_{t \to +\infty} (g_{c,t}) = g_i$. As a result, the economy-wide 'balanced growth' rate is:

$$g = g_c = g_i = \varepsilon_i \cdot \theta_i \cdot n_i \cdot \frac{1}{1+q_i} = \theta_i \cdot r_i \tag{17}$$

Using equation (16) in (17), the equilibrium solution can be redefined as:

$$\frac{\theta_c}{\theta_i} = \frac{\varepsilon_i}{\varepsilon_c} \cdot \frac{n_i}{n_c} \cdot \frac{1+q_c}{1+q_i} \tag{17B}$$

Equation (17B) is the dynamic counterpart of equation (4C), meaning that the equilibrium requires that the ratio between sectoral saving rates be a direct function of the ratio between sectoral OCCs – given turnover and exploitation rates. Since these ratios are independent each other, there is nothing to ensure that the (17B) holds true in fact. A balanced growth is a theoretical possibility, as the expansion of production in one sector enlarges the market for the other. However, 'the rate of growth of production in the various branches of production is determined primarily by the uneven development of the conditions of production, rather than by the different rates of growth of the markets for their products' (Clarke 1990, p. 458). This leads to a disproportional development of the two sectors, which is the form taken by the inner tendency of capitalism to overaccumulation and crisis.

Similarly, equation (17) shows that enlarged reproduction conditions are matched if sectors grow all at the same pace. It bears a strong resemblance to the Cambridge distributive

[12] Balance growth 'is itself an accident' (Marx 1885, p. 571).

[13] It is worth noticing that the adjective 'exogenous' should be only referred to the formal model (i.e. the system of difference equations), not to its 'subject' (i.e. capitalist economies).

equation, interpreted as a dynamic investment function and rearranged for a two-sector economy (see Lavoie 2014). Equation (17) shows that the economy-wide rate of growth is a direct function of both the saving rate of i-sector capitalists and the i-sector rate of profit. This means that, while the i-sector saving rate is an exogenous, the saving rate of the c-sector is determined endogenously as follows:

$$\theta_c = g_c \cdot (1 + q_c)/(e_c \cdot n_c) \tag{18}$$

Equation (18) shows that the saving rate of the c-sector adjusts endogenously to guarantee the enlarged reproduction of the system.[14] The convergence of sectoral growth rates and the necessary adjustment of the c-sector saving rate are shown by charts A and B (in Figure 1), respectively. Chart C shows the (increasing) trend in sectoral outputs and hence in total output. Finally, chart D shows the sectoral profit rates and the general rate of profit of the economy. While the sectoral profit rates do no converge towards a uniform rate, the general or average profit rate of the economy declines in the first few periods and then stabilises, because of the asymmetric adjustment in the sectoral stocks of capital.

4. The amended model

Simplified though they are, the Marxian reproduction schemes provide a refined explanation of the fragility of unregulated capitalist economies. In fact, Marx's grim predictions fit well with the economic, political, and social instability that marked early-industrialised countries from the end of the Victorian Era to the Second World War. They also implicitly account for the stabilising function that has historically been performed by the government sector since the 1930s. However, there is no room in the reproduction schemes for the effect of the development in the banking and financial sector on the creation of social value and surplus value. In addition, they do not take into consideration the long-run impact of the competition between capitalists on sectoral profit rates and prices. In fact, no price setting mechanism is established, as prices are just assumed to be proportional to labour contents of commodities. The fact is that the reproduction schemes are discussed in the second volume of *Capital*, whereas the so-called 'equalisation' of the profit rate and the formation of production prices are covered by Marx in the third volume. While the manuscripts that comprise the third volume were written by Marx before those comprising the second one, the former logically follows the latter as the degree of abstraction gets lower as the analysis proceeds. The effect of competition (and market forces) on reproduction conditions can only be discussed after those conditions have been worked out under the hypothesis of exchange of equivalent values.

The current section aims to bridge these gaps. For this purpose, three amendments are made to the benchmark model. First, it is assumed that the saving rate and hence the investment undertaken by i-sector capitalists are a non-linear function of the expected rate of profit. Drawing from Robinson (1962), it is assumed that any increase in the propensity to invest (i.e. capitalists' rate of saving in this simplified model) requires ever larger increases in

[14] When the State is included in the analysis, the government sector may well be regarded as the 'buffer' of the economy. Economic planning to eliminate cross-sector disproportionalities and crises was advocated historically by Tugan-Baranowsky and Hilferding (see Shaikh 1978). Today the stabilisation function of government is advocated by the post-Keynesians and other heterodox economists. However, this view was criticised by Luxemburg and is still questioned by most Marxists. The reason is that disproportionalities are not regarded as 'the contingent result of the "anarchy of the market," which can be corrected by appropriate state intervention; they are the necessary result of the social form of capitalist production' (Clarke 1990, p. 459).

the expected rate of profit. If adaptive expectations are hypothesised, the equation defining the rate of saving in the i-sector can be defined as:

$$\theta_i = \theta_{i0} + \theta_{i1} \cdot \ln(1 + r_{i,-1} + r_\xi) \tag{19}$$

where r_ξ is a random component of profit expectations incorporating capitalists' 'animal spirits', whereas parameters θ_{i0} and θ_{i1} are defined in such a way that: $0 \leq \theta_i \leq 1$.

Similarly, it is assumed that the parameter defining the sectoral intra-period turnover rate is a function of the share of surplus value which is diverted from productive scopes to financial assets and services (Veronese Passarella and Baron 2015). More precisely, F_j (with $j = i, c$) is re-defined to include both the amount of (unproductive) capital invested in financial assets and the expenditure for financial services. This is the second amendment to the benchmark reproduction model. If a positive but decreasing impact of finance on the turnover is assumed, sectoral turnover rates can be defined as follows:

$$n_i = n_{i0} + n_{i1} \cdot \ln(F_{i,-1}) \tag{20}$$

and

$$n_c = n_{c0} + n_{c1} \cdot \ln(F_{c,-1}) \tag{21}$$

where $n_{i0}, n_{i1}, n_{c0}, n_{c1} \geq 0$. Equations (20) and (21) state that any increase in the sectoral rate of turnover requires ever larger increases in the past expenditure for financial assets and services. Notice that now θ_i defines i-capitalists' preference for productive investment against non-productive expenditure, while parameters σ_{1j} and σ_{2j} in equations (7) and (8) define the speed or pace of 'financialisation'.

Furthermore, competition between capitalists under a *laissez faire* regime entails the cross-sector levelling of profit rates in the long run (Marx 1894). While profit equalisation should be only regarded as a tendency, it allows pointing out: first, the dominance of capital-intensive sectors over labour intensive sectors (as the former 'steal' surplus value from the latter); second, the consistency of the general law of creation of value (meaning that social value arises from the exploitation of living labour in the production sphere) with the specific law of distribution of value (meaning the prevailing price setting, including the one defined by the competition hypothesis). Notice that the general rate of profit can be split into two components, notably the profit share of net income and (the inverse of the) total capital to net output ratio.[15] In formal terms, the wage share of net income is:

$$\omega = \frac{V_i + V_c}{Y_i + Y_i - (C_i + C_c)} \tag{22}$$

The profit share is:

$$\pi = \frac{\theta_i \cdot S_i + \theta_c \cdot S_c + F_i + F_c}{Y_i + Y_i - (C_i + C_c)} = 1 - \omega \tag{23}$$

Finally, the total capital (including the wage-bill) to net output ratio is:

$$a = \frac{V_i + V_c + C_i + C_c}{Y_i + Y_c - (C_i + C_c)} \tag{24}$$

[15] In principle, each sectoral capital to net output ratio could be expressed, in turn, as the product of the inverse of the sectoral actual rate of utilisation of plants and the sectoral capital to full-capacity net output ratio. For the sake of simplicity, and in line with the Marxian tradition, both rates of utilisation are assumed to be constant.

The general (realised) rate of profit is therefore:

$$r = \frac{\theta_i \cdot S_i + \theta_c \cdot S_c + F_i + F_c}{V_i + V_c + C_i + C_c} = \frac{\pi}{a} \tag{25}$$

As is well known, this is the profit rate that would prevail across sectors if capitalists were free to invest their own capitals wherever it is more convenient for them. Sectoral outputs can now be expressed in terms of prices of production. They are, respectively:

$$\hat{Y}_i = C_i + V_i + r \cdot (C_i + V_i) = C_i + V_i + \theta_i \cdot P_i + F_i \tag{11B}$$

and

$$\hat{Y}_c = C_c + V_c + r \cdot (C_c + V_c) = C_c + V_c + \theta_c \cdot P_c + F_c \tag{12B}$$

where $P_j = r \cdot \left(C_j + V_j\right)$ is the total mass of profit realised in the j-th sector.[16]

Notice that sectoral OCCs do not converge to a uniform ratio, as they depend on a variety of sector-specific technological and institutional factors. As a result, sectoral production prices usually differ from sectoral values. Growth rates, in contrast, still converge in the long run to meet the criteria for a balanced growth, and the same goes for sectoral saving rates (see charts E and F in Figure 1). In formal terms:

$$g_i = \frac{\theta_i \cdot P_i}{C_i + V_i} = \theta_i \cdot r \tag{15B}$$

and

$$g_{c,t} = \frac{\hat{Y}_i - C_i - \theta_i \cdot P_i - C_c}{C_c + V_c} = g = g_i \text{ for } t \to +\infty \tag{16C}$$

where P_j is the mass of profit realised by j-sector capitalists and r is the general rate of profit arising from the competition between capitals.

Since both the accumulation rate and the profit rate are uniform across sectors in the long run, sectoral saving rates must converge too:

$$\theta_{c,t} = \theta = \theta_i \text{ for } t \to +\infty$$

and hence:

$$g = g_c = g_i = \theta \cdot r \tag{17B}$$

where θ is the long-run uniform rate of retention on profits (or rate of saving out of capital incomes).

In addition, using equation (25) in equation (17B) one gets:

$$g = \frac{\theta}{a} \cdot \pi \tag{17C}$$

The latter calls to mind a familiar result in Keynesian macroeconomic dynamics of the 1930-40s, that is, the Harrod-Domar warranted rate of growth (Harrod 1939, Domar 1946).[17] Given the profit share, the economy-wide equilibrium growth rate depends on the capitalists' saving rate and (the inverse of) the capital to output ratio.

[16] Notice that now P_j replaces S_j in equations (7) and (8).

[17] That resemblance has been stressed by many authors, notably Robinson (1951), Harris (1972), and more recently Olsen (2015).

Notice, finally, that charts G and H confirm the well-known Marx's finding that capital-intensive sectors 'steal' surplus-value from labour-intensive sectors. Given the sectoral demand schedules, production prices of investment goods are higher than (or more than proportional to) values, whereas production prices of consumption goods are lower than (or less than proportional to) values. This happens because a higher OCC has been assumed in the i-sector compared to the c-sector.

5. Some experiments: shocking Marx

In this section some comparative dynamics exercises are performed. The aim is to see how the main endogenous variables of the amended model react following a shock to key exogenous variables and parameter values. The adjustment process from the old equilibrium position (meaning the initial balanced growth rate) to the new one is then analysed. Such a methodology is akin to the current post-Keynesian approach to macro-monetary modelling (e.g. Lavoie 2014). In particular, the impacts of the following shocks are tested:

a) An increase in the OCC. This is the standard Marxian assumption underpinning the alleged tendency for the general profit rate to fall.
b) A fall in the economy-wide propensity to consume,[18] leading to a lack of aggregate demand and hence to a realisation crisis.
c) A fall in the rate of saving out of profits, reflecting a fall in capitalists' propensity to invest in productive assets, or a higher reliance on financial markets to fund production plans, or a higher pressure to pursue shareholder value maximisation in the short run.
d) A change in the rate of turnover of capital, reflecting the 'reverse U-shaped' impact of the developments in banking and financial sectors on the 'manner' of extraction of living labour from workers in the production sphere (Veronese Passarella and Baron 2014).
e) The rise (or the worsening) of imbalances between departments, roughly mirroring the effect of external imbalances between national economies.

While experiments (a) and (b) have been the focus of long-lasting debates among the Marxists and between the Marxists and other economists, experiments (c) and (d) are somewhat original. They are meant to echo the recent developments in highly-financialised economies, preparing the ground for the US financial crisis of 2007-2008. Similarly, experiment (e) can be regarded as a first step towards a formal Marxian model aiming to account for the impact of external imbalances between the members of a certain economic area. The model tested is made up of equations (1)-(15), (16B), and (18)-(25). Equation (16B) provides the long-run attractor of the system. The analysis is focused on the medium-run re-adjustment dynamics triggered by specific shocks to exogenous variables and parameter values. Consequently, the profit equalisation effect generated by competition between capitalists is assumed away.[19] Shocks are all ran in period 20.

Focusing on the first experiment, figure 3 shows the impact of a 10% increase of i-sector OCC on growth rates, profit rates, and income shares. As one would expect, the impact is negative on both the economy-wide accumulation rate (chart I) and the i-sector rate of profit (chart L). The average rate of profit declines as well, but this does not affect the c-sector rate of profit if cross-sector capital movements are not allowed. Finally, the relative reduction of wages

[18] Since the Classical hypothesis is adopted (meaning that the propensity to consume out of wages is unity), this entails a reduction in the propensities to consume out of non-labour incomes ($1 - \sigma_{1j}$).

[19] This does not affect the main qualitative findings of the model anyway.

paid in the i-sector is obviously associated with a reduction in the economy-wide wage share and hence with an increase in the profit share in net income (chart M). As is well known, the fall in the rate of profit due to the increase in the organic composition of capital is regarded by Marx as the most important inner law of motion of capitalism. In fact, some contemporary Marxists regard financialisation as a result of the fall in profitability of western economies since the 1970s. Yet, that trend is regarded by other Marxist authors as a long-run secular tendency (acting as the economic equivalent of the law of gravitation) that does not provide the ground for a theory of crisis – meaning that can neither explain the *necessity* of crisis nor account for each specific cyclical turn.

So, unsurprisingly, only a few authors have traced the recent crises back to the tendency for the profit rate to fall. Most Marxist, radical and post-Keynesian economists (and also some New Keynesians) have focused on income inequality and other financial factors as the main triggers of the US crisis of 2007-2008 and the current crisis of Euro Area's member-states. Figure 4 shows the impact of a fall in i-sector propensity to consume on growth rates, profit rates, and income shares.[20] The negative effect on the accumulation rate of the c-sector is apparent, though temporary (chart N). The fall in aggregate demand, in turn, affects negatively the economy wide profit rate and the profit share in net income (charts O and P). In other words, the realisation crisis turns into a profitability crisis for the capitalist class.[21] Notice that the lack of demand (and the overproduction) may well be the outcome of an increase in income inequality, involving a rise in the economy-wide marginal propensity to save, as is usually claimed by the Keynesians.

As mentioned, the possible link between income inequality and crisis has been stressed by many heterodox and 'dissenting' orthodox economists since the start of the US financial crisis. Popular though it is, the 'inequality' interpretation neglects some of the most notable developments of highly-financialised capitalist economies in the last few decades. Two of them are worth stressing here: the fall in the rate of retention on corporate profits, and the impact of the financial sector on the turnover rate of capital. A fall in the saving rate of capitalists depresses the economy-wide accumulation rate, even though the initial impact on the c-sector growth rate is positive (chart Q in Figure 5), because of the increase in current consumption. Sectoral profit rates remain unchanged, but a somewhat paradoxical positive effect on the average profit rate arises, because of the increasing weight of the c-sector (chart R). Finally, the impact on income distribution is such that wage earners are worse off and capitalists are better off under the new theoretical steady state (chart S).

The association between growing income inequality and increasing short-termism of corporations has been one of the most important features of highly-financialised Anglo-Saxon economies since the 1990s. However, the analysis of the causes of the initial success of such a finance-led capitalism is as important as the examination of its own flaws. Notice that, from a Marxian perspective, the amount of capital invested in financial assets and businesses is unproductive. Finance may well circulate the already-created value, but cannot add up a (macroeconomic) surplus value to it. However, financial markets, banks, and other financial institutions are all but unnecessary. In fact, they allow the industrial capitalists to fund their

[20] The reader is referred again to footnote 18.

[21] Notice that here the fall in the rate of profit is the result of the realisation crisis, as claimed by the 'underconsumptionist' branch of Marxism. By contrast, experiment (a) assumes that the fall in the rate of profit (following a rise in the OCC) is the cause of the crisis, as advocated by most Marxist theories of the 1970s (see Clarke 1990).

own production and investment plans.[22] In addition, financialisation (meaning the stronger and stronger dominance of financial markets, agents, motives, and culture) ends up affecting the 'form' of the extraction of surplus labour from workers, leading to a 'real subsumption of labour to finance' (Bellofiore 2011). It is not coincidence that the increasing weight of finance is usually associated with 'reforms' of the labour market and a change in the corporate governance.

Such an indirect impact of finance on the creation of surplus value is captured by the turnover rate of capital in the Marxian theory. Particularly, it seems to be reasonable to assume that the absolute impact on the (intra-period) turnover rate of the investment in financial assets or services is positive, at least during 'normal times', whereas its marginal impact is negative.[23] The effect on accumulation, profitability, and net income distribution, of an increase in the autonomous component of the i-sector turnover rate function – n_{i0} in equation (20) – is shown in Figure 6. The growth rate of the economy increases (chart T) and so does the average profit rate (chart U). These effects arise, in turn, from the increase in the mass of social surplus value. The profit share in net income augments too (chart V), thereby confirming the negative influence of financialisation on distributive equality. The opposite happens in 'times of distrust', when the impact of finance on capital valorisation fades away or becomes even negative.[24] These features are all consistent with the available empirical evidence about the effect of financialisation on advanced economies in the last three decades.[25]

The last experiment deals with the effect of a positive but temporary shock to the c-sector autonomous accumulation on sectoral growth rates and the output gap (meaning the difference between i-sector output value and c-sector one). It shows that the readjustment process can be rather painful for the 'dependent' sector or economy (chart W in Figure 8). A catching up process initially shows up, but the output gap keeps on increasing in absolute terms and remains unchanged in relative terms in the long run (chart X). Clearly, the current model is too simplified to be applied to the analysis of real-world capitalist economies. However, this simple experiment shows that a further refinement of the Marxian reproduction schemes could allow accounting for the impact of external imbalances between national economies, or between an individual country (which is likened to the dependent sector, i.e. the c-sector) and the rest of the world (which is likened to the i-sector). In fact, the limits to domestic growth arising from the state of world-wide demand for import may well be regarded as a natural extension of the Marx's two-sector model, that bears a resemblance with current post-Keynesian balance of payments constrained growth models (Thirlwall 2014).

[22] For a thorough analysis of the different functions performed (within a financially-sophisticated capitalist economy) by banks and other financial institutions, respectively, see Sawyer and Veronese Passarella (2015).

[23] 'The rationale is that the higher the degree of development of the banking and finance sector […], the higher the speed at which manufacturing firms (or their owners/shareholders) could re-invest the initial capital. At the same time, beyond a given historically determined threshold at least, 'diseconomies' are expected to arise as the (relative) dimension of the banking and finance sector increases' (Veronese Passarella and Baron 2014, p. 1435-36).

[24] If, following Veronese Passarella and Baron (2014), a parabolic turnover function is adopted then both accumulation and profitability collapse in the long run, whereas income shares fluctuate (see Figure 7).

[25] A full review of recent literature about financialisation is out of the purpose of this contribution. The reader is referred to FESSUD Studies in Financial Systems (available at: http://fessud.eu/deliverables/).

6. Final remarks

The aim of this paper is to recover and develop the reproduction schemes to test the impact of some of the most apparent ‘stylised facts’ of current capitalism on an artificial two-sector growing economy. For this purpose, the key features of the Marx’s schemes have been pointed out and discussed. The strong family resemblance to early and current post-Keynesian models of growth has been highlighted and discussed as well. In addition, some simple amendments have been made to Marx’s benchmark framework in order to make it suit for the analysis of the impact of finance on accumulation, profitability, and income distribution. It has been shown that the Marxian reproduction schemes allow framing a variety of radical, post-Keynesian and other dissenting theories of crisis of advanced countries with a flexible and sound analytical model. Clearly the preliminary findings presented in section 5 are just of qualitative nature. The model is still too simplified to provide a quantitative assessment of recent developments in real-world capitalist economies. Besides, some analytical aspects should be further discussed and refined (particularly, the functional form of turnover and saving rates). Finally, numerical simulations should be coupled with a sensitivity analysis (or an empirical estimate of parameter values) to check the robustness of results. However, the preliminary findings look consistent with the available empirical evidence and they may well open the way to future research.

Table 1. Key to symbols and values

Symbol	Description	Kind	Value	Symbol	Description	Kind	Value
a	Net output to total capital ratio	En		r_c	Rate of profit in consumption sector	En	
C_c	Constant capital in consumption sector	En		r_i	Rate of profit in investment sector	En	
C_i	Constant capital in investment sector	En		r_ξ	Random component of profit expectations	X	ξ
F_c	Unproductive spending from consumption sector	En		S_c	Surplus value in consumption sector	En	
F_i	Unproductive spending from investment sector	En		S_i	Surplus value in investment sector	En	
g	Economy-wide rate of accumulation			V_c	Variable capital in consumption sector	En	750*
g_c	Rate of accumulation in consumption sector			V_i	Variable capital in investment sector	En	1000*
g_i	Rate of accumulation in investment sector			Y_c	Value of output of consumption sector	En	
g_ξ	Random comp. of consumption sector growth rate	X	ξ**	$\hat{Y}_c$	Price of production of output of consumption sector	En	
HH_c	Consumption sector capitalists' wealth (stock)	En		Y_i	Value of output of investment sector	En	
HH_i	Investment sector capitalists' wealth (stock)	En		$\hat{Y}_i$	Price of production of output of investment sector	En	
L	Total direct labour spent by workers	En		α	Propensity to consume out of wages	X	1.00
L_c	Direct labour in consumption sector	En		δ	Depreciation rate of constant capital	X	1.00
L_i	Direct labour in investment sector	En		ε_i	Rate of exploitation in investment sector	X	1.00
m	Monetary expression of labour time			ε_c	Rate of exploitation in consumption sector	X	1.00
n_c	Turnover rate in consumption sector	En		θ_c	Saving rate in consumption sector	En	
n_{c0}	Parameter in consumption sector turnover function	X	1.00	θ_i	Saving rate in investment sector	En	
n_{c1}	Parameter in consumption sector turnover function	X	0.00	θ_{i0}	Parameter of investment sector saving function	X	0.50**
n_i	Turnover rate in investment sector	En		θ_{i1}	Parameter of investment sector saving function	X	0.00
n_{i0}	Parameter in investment sector turnover function	X	1.00	π	Profit share of total net income	En	
n_{i1}	Parameter in investment sector turnover function	X	0.00**	σ_{c1}	Cons. sector capitalists prop. to save out of income	X	0.00
P_c	Mass of profit in consumption sector	En		σ_{c2}	Cons. sector capitalists prop. to save out of wealth	X	0.95
P_i	Mass of profit in investment sector	En		σ_{i1}	Invest. sector capitalists prop. to save out of income	X	0.00**
q_c	OCC in consumptions sector	X	2.00	σ_{i2}	Invest. sector capitalists prop. to save out of wealth	X	0.95
q_i	OCC in investment sector	X	4.00**	ω	Wage share of total net income	En	
r	General rate of profit						

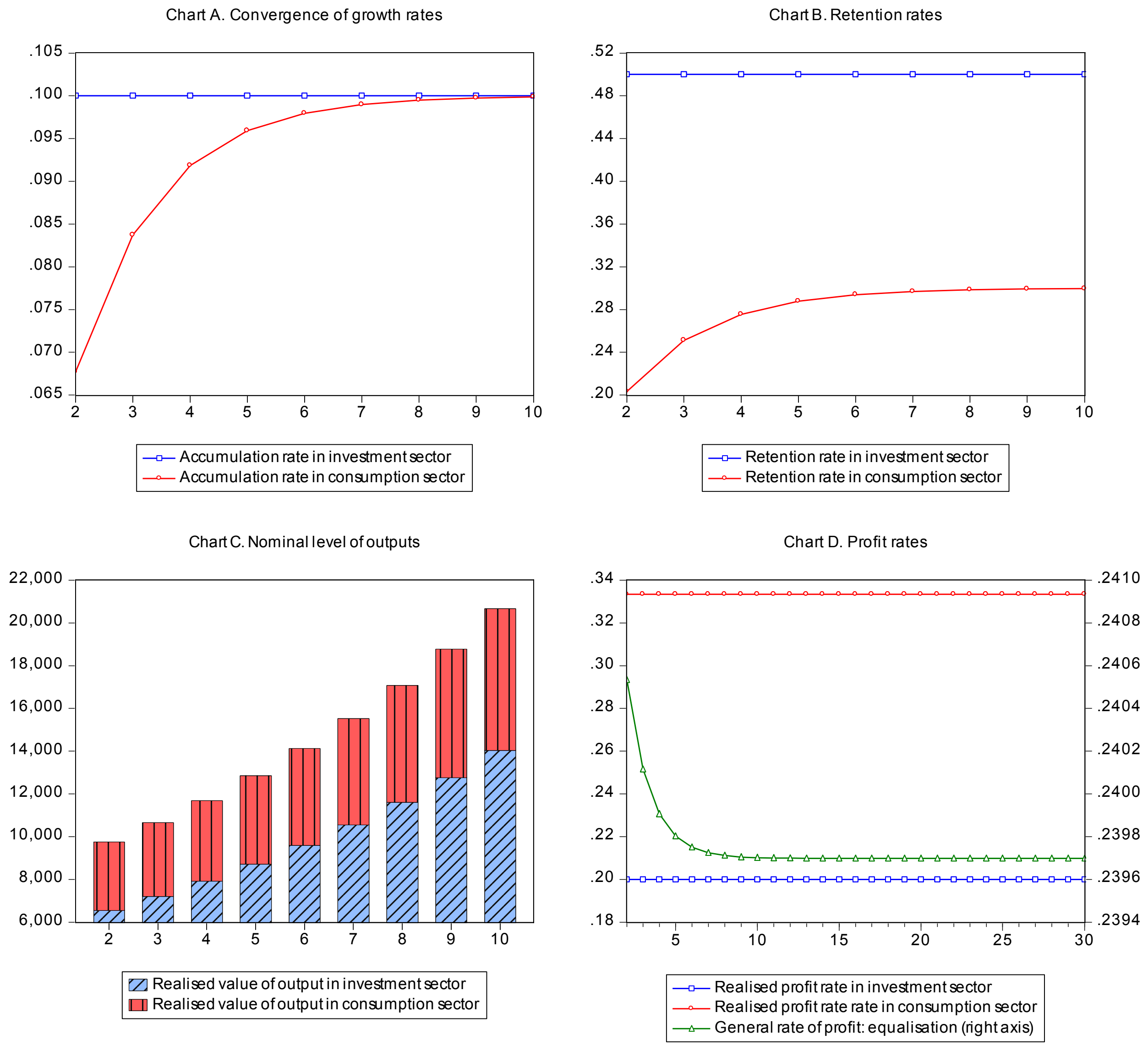
Notes: En = endogenous variable. X = exogenous variable or parameter. * Starting values for stocks and lagged endogenous variables. ** Shocked parameters: $\theta_{i0} = -50\%$ (scenario 1); $q_i = +10\%$ (scenario 2); $\sigma_{i1} = 0.01$ (scenario 3); $n_{i1} = 0.001$ (scenario 4); $g_\xi = +0.01$ (scenario 5).

Table 2. Transactions-flow matrix of the two-sector economy

	Workers	**Consumption Sector Capitalists**		**Investment Sector Capitalists**		**Financial Sector Capitalists**	Σ
		Current account	Capital account	Current account	Capital account		
1. Consumption of workers [and capitalists]	$-\alpha \cdot (V_i + V_c)$	$\alpha \cdot (V_i + V_c)$ [$+ F_i$]		[$-F_i$]			0
2. Investment in constant capital ($C_{i,c} = \Delta CC_{i,c}$)			$-C_c$	$C_c + C_i$	$-C_i$		0
3. Variable capital (payment of wage bill)	$V_i + V_c$	$-V_c$		$-V_i$			0
4. Amortisation funds = Deprec. allowances		$-\delta \cdot C_{c,-1}$	$\delta \cdot C_{c,-1}$	$-\delta \cdot C_{i,-1}$	$\delta \cdot C_{i,-1}$		0
5. Return on financial assets		$+r_{F,-1} \cdot FF_{c,-1}$		$+r_{F,-1} \cdot FF_{i,-1}$		$-r_{F,-1} \cdot (FF_{i,-1} + FF_{c,-1})$	0
6. Return on financial liabilities		$-r_{B,-1} \cdot BB_{c,-1}$		$-r_{B,-1} \cdot BB_{i,-1}$		$+r_{B,-1} \cdot (BB_{i,-1} + BB_{c,-1})$	0
7. Retained surplus value		$-(S_c - F_c)$	$S_c - F_c$	$-(S_i - F_i)$	$S_i - F_i$		0
8. Δ Financial liabilities ($B_{i,c} = \Delta BB_{i,c}$)			B_c		B_i	$-(B_i + B_c)$	0
9. Δ Financial assets ($F_{i,c} = \Delta FF_{i,c}$)			$-F_c$		$-F_i$	$F_i + F_c$	0
Σ	0	0	0	0	0	0	0

Notes: A '+' before a magnitude denotes a receipt or a source of funds, whereas '–' denotes a payment or a use of funds. α is the (average and marginal) propensity to consume out of wages, and δ is the depreciation rate of capital. However, it is assumed that $\alpha = \delta = 1$ in the model defined by (1)-(15), (16B), and (18)-(25). Similarly, faded areas are zero-sum games for the capitalist class considered as a whole and are not explicitly modelled.

Figure 1 Adjustment to the balanced growth path: baseline (no equalisation)



Notes: Initial conditions are: $V_i = 1000$, $V_c = 750$, $q_i = 4$ (or $C_i = 4000$), $q_c = 2$ (or $C_c = 1500$), $\varepsilon_i = \varepsilon_c = 1$, and $\theta_i = \theta_{i0} = 0.5$ (see Table 1). These values are drawn from Marx (1885) and are commonly used in the literature on the enlarged reproduction schemes (cfr. Luxemburg 1913, Olsen 2015).

Figure 2 Adjustment to the balanced growth path: profit equalisation

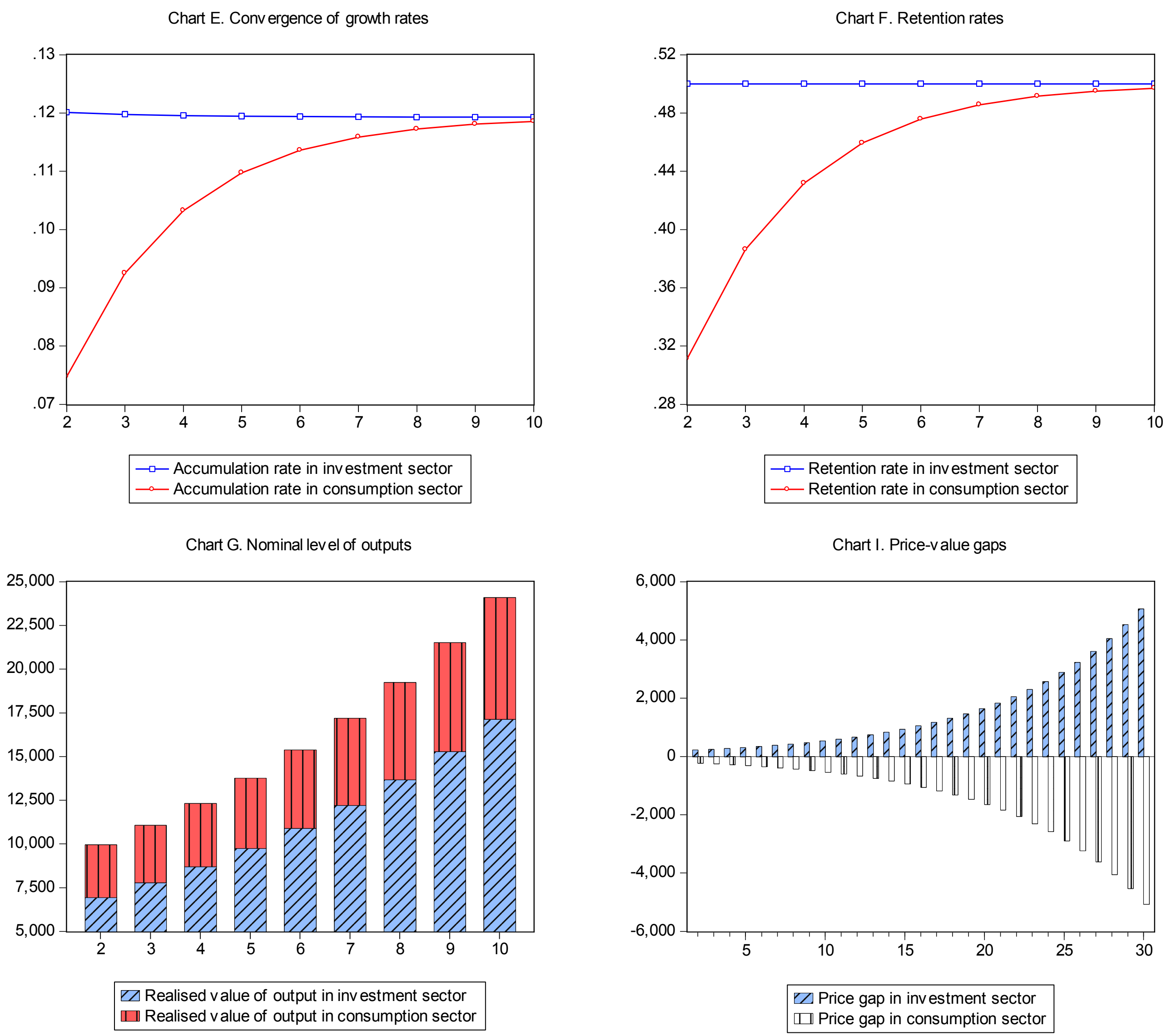


Figure 3 An increase in the organic composition of capital invested in ***i***-sector

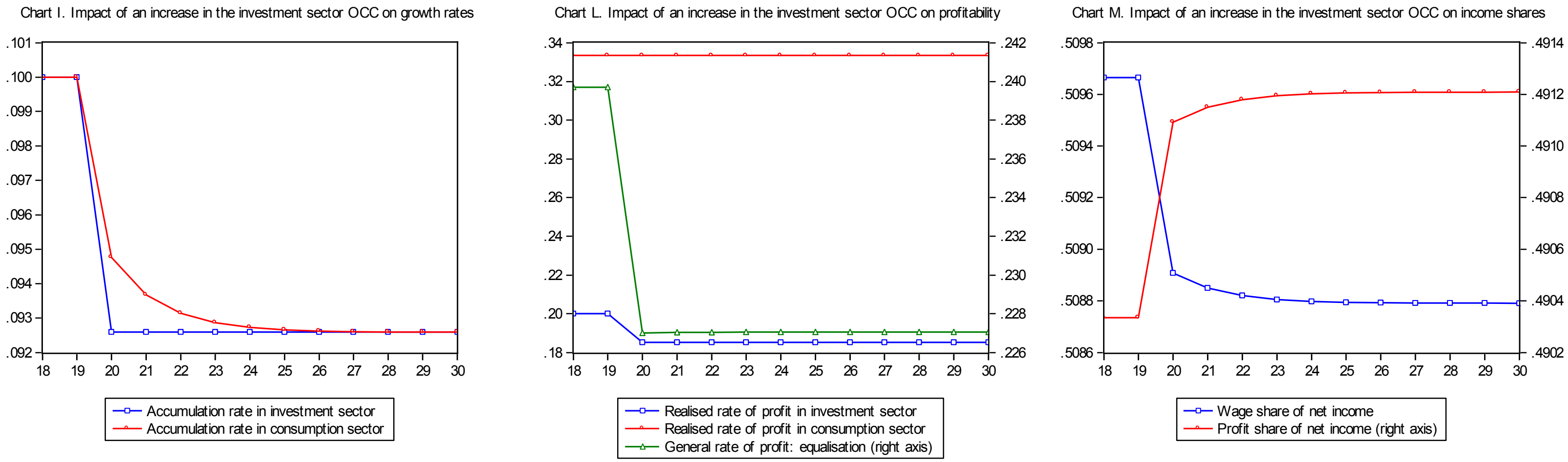


Figure 4 A fall in *i*-sector capitalists' propensity to consume

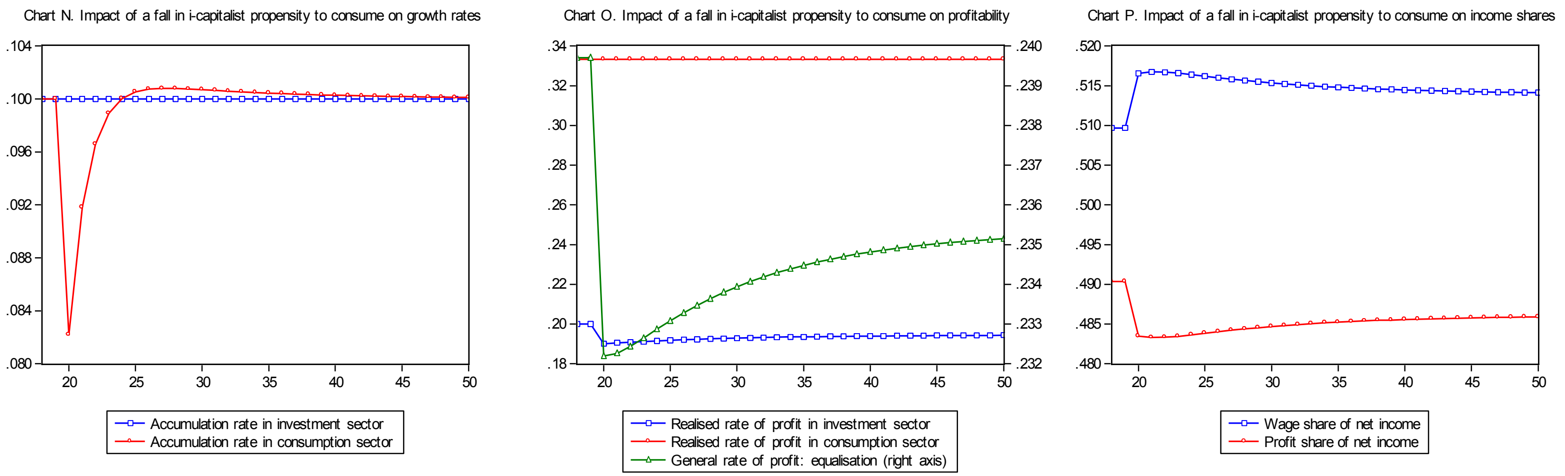


Figure 5 A fall in i-sector capitalists' saving rate

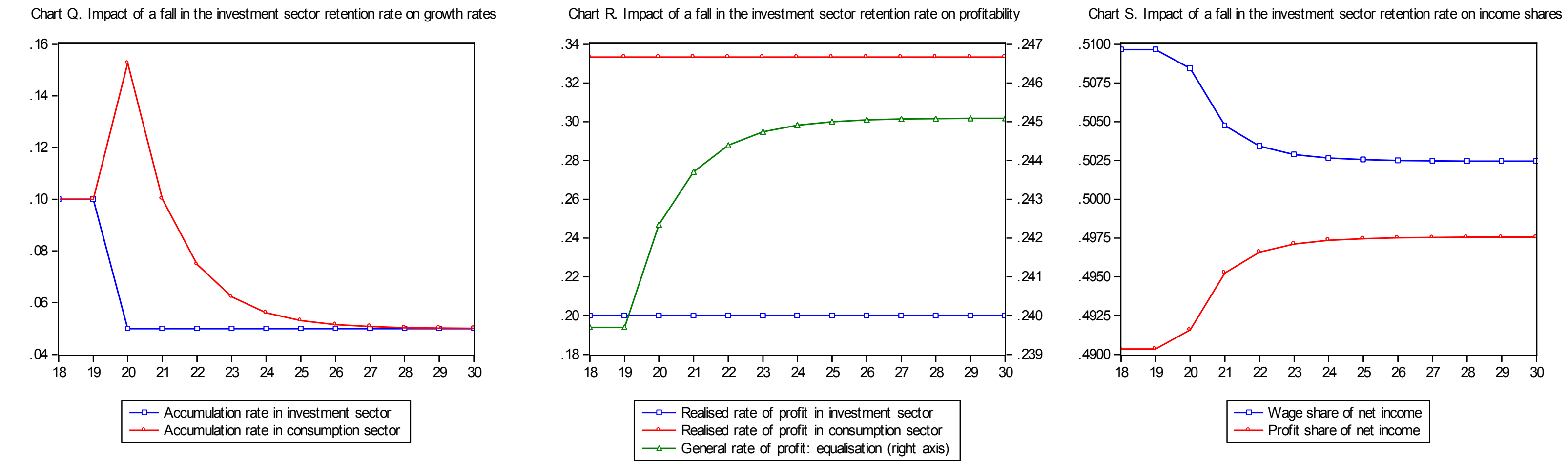


Figure 6 An increase in finance sensitivity of i-sector turnover rate

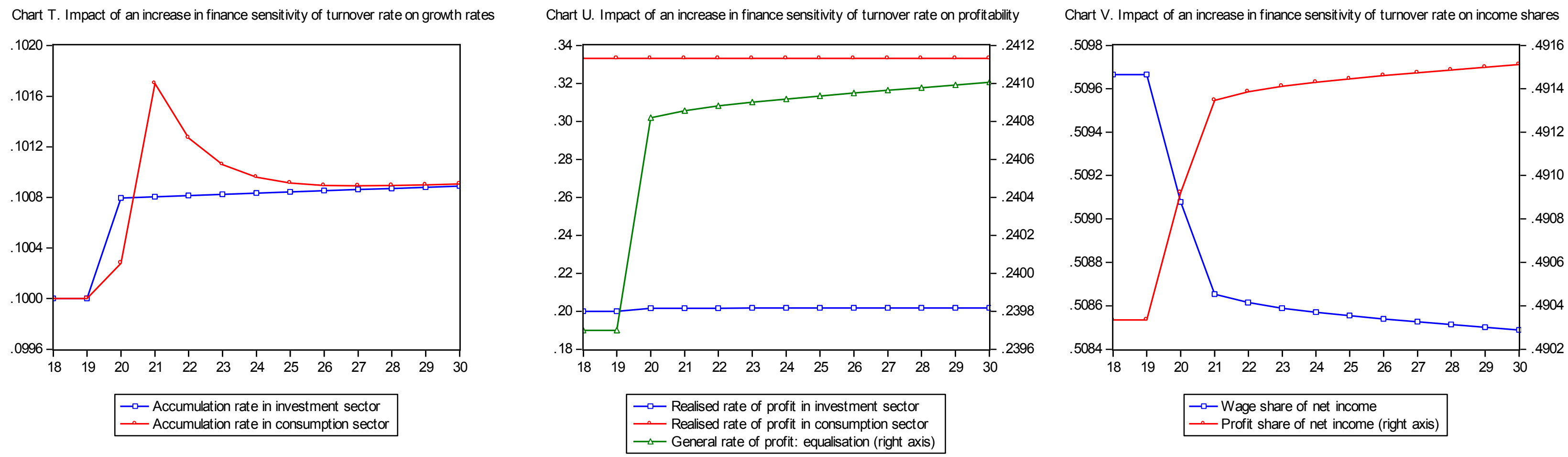


Figure 7 Long-run impact of an increase in finance sensitivity of i-sector turnover rate when a parabolic turnover function is used

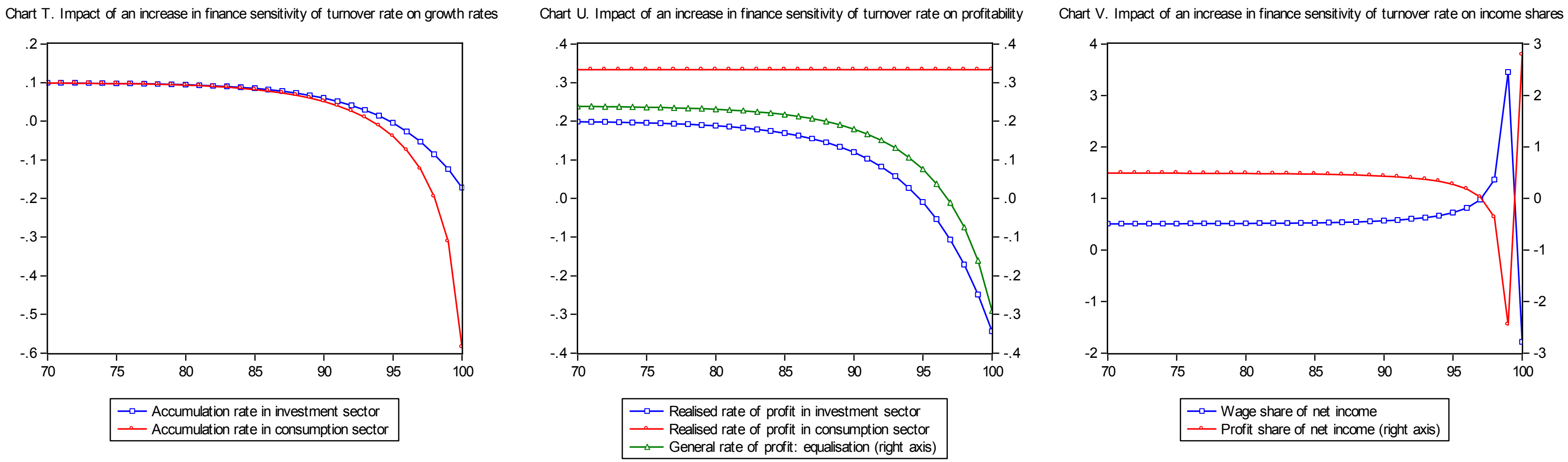
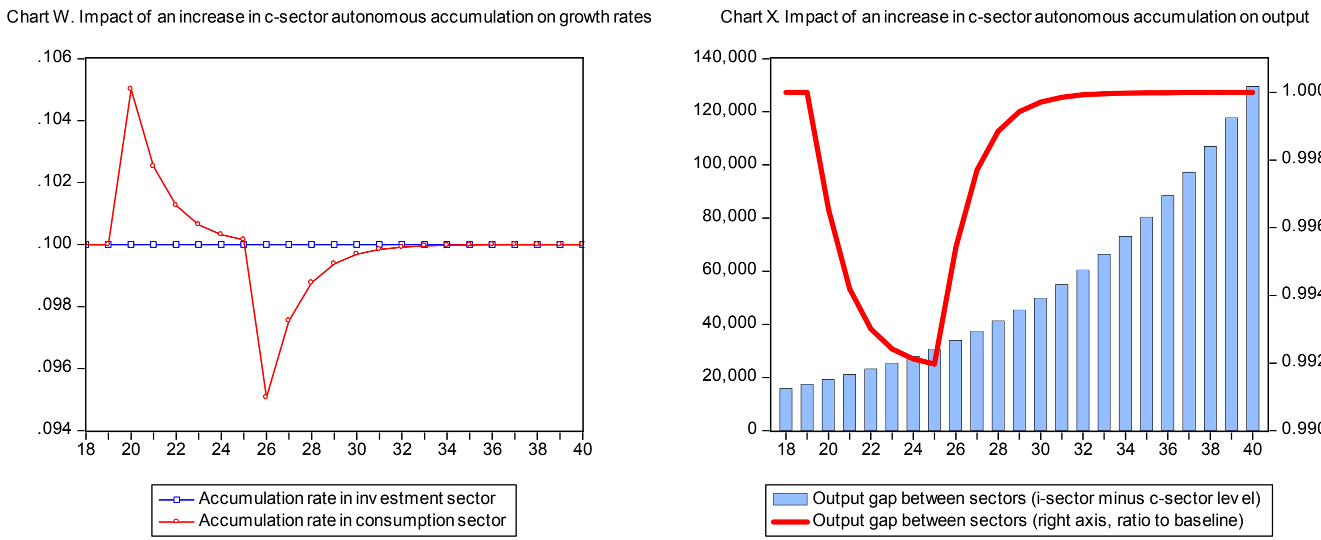


Figure 8 Impact of a temporary (i.e. 5-periods) increase in the autonomous component of c-sector accumulation rate



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Appendix 1. The monetary expression of social labour time

In this paper a 'simultaneous' and 'single-system interpretation' of the Marxian labour theory of value is implicitly adopted, in the wake of Duménil and Foley (2008). As a result, a fixed ratio between units of money and units of direct social labour is assumed. This ratio, named 'the monetary expression of labour time', is defined as the ratio of the monetary value added of the economy (say, the domestic net product at current prices) to the direct productive labour expended in the production process over a certain period. In formal terms, one gets:

$$m \equiv \frac{V_i+V_c+S_i+S_c}{L} = \bar{m} \tag{A1}$$

The main strength of the hypothesis above is that it allows equating the monetary accounting with the labour accounting, in spite of the specific price-setting system of the economy. In addition, since m is given, equation (A1) defines the quantity of labour inputs (say, the number of working hours or the employment level) demanded by the capitalists:

$$L = \frac{V_i+V_c+S_i+S_c}{\bar{m}} \tag{A2}$$

For the sake of simplicity, it is assumed that the supply of labour is plentiful and does not form a binding constraint on the level of employment. In other words, the capitalist class can count on an abundant 'reserve army' of unemployed workers. Accordingly, the allocation of labour inputs across sectors mirrors their own relative weights:

$$L_i = L \cdot \frac{V_i+S_i}{V_i+V_c+S_i+S_c} \tag{A3}$$

and:

$$L_c = L - L_i \tag{A4}$$

where L_j (with $j = i, c$) is the sectoral employment level determined by the autonomous production plans of the capitalists.

Appendix 2. Adding up stocks and financial assets

The reproduction schemes describe a pure-flow economy. Only current expenditures and circulating components of constant capital are taken into consideration. In principle, this gap could be bridged by considering the stock of constant (fixed) capital and the accumulation of financial assets and liabilities. In formal terms, the sectoral stocks of constant capital are, respectively:

$$CC_i = CC_{i,-1} \cdot (1-\delta) + C_i \tag{A5}$$

and

$$CC_c = CC_{c,-1} \cdot (1-\delta) + C_c \tag{A6}$$

where δ is the depreciation rate of fixed capital (and $0 < \delta \leq 1$).

If F_j (with $j = i, c$) is defined as the sectoral investment in financial business, the stock of financial assets held by j-sector capitalists (FF_j) could be worked out in a similar fashion. A more realistic rendition of how capitalist economies work would also require to take into consideration the process of creation of money and other financial liabilities. Industrial capitalists need monetary means (lent by bankers or monetary capitalists) to get the production process started, and issue other financial liabilities – call them BB_j – to cover residually their own investment plans. Clearly the stock of financial assets does not match the overall stock of liabilities when the formation of fixed capital (that is, a stock of productive assets) is taken into consideration. In the simplified model made up of equations (1)-(15), (16B), (18)-(25), no fixed capital is accounted for ($\delta = 1$), and the positive return rate on financial assets is implicitly assumed to match the negative interest rate on liabilities ($r_F = r_B$). As a result, the share of surplus value that turns into 'financial rent' is null (see lines 5-6 and 8-9 in Table 2). In the real world, individual capitalists may well wish to hold financial assets when their return rate is higher than the return rate on productive assets. However, the rationale for the capitalist class (considered as a whole) to divert resources from the productive sector to the financial one is to increase the rate of turnover of capital. Such a macroeconomic rationale is the one considered here.

Appendix 3. Simple reproduction condition: a disaggregated formulation

Once Marx's equations are conveniently disaggregated, the two-fold clearing condition of goods markets can be redefined as follows:

$$\lambda_i \cdot y_i = \lambda_i \cdot (c_i + c_c)$$

and

$$\lambda_c \cdot y_c = v \cdot (L_i + L_c) + F_i + F_c$$

where λ_i is the unit value of capital goods (say, inventories or one-period lasting machines), λ_c is the unit value of consumption goods, and v is the unit value of the labour power (corresponding to the money wage rate). Notice that both output, y_j, and constant capital (homogenous) inputs, c_j, are expressed in real terms (with $j = i, c$).

Similarly, the reproduction values of sectoral outputs are:

$$\lambda_i \cdot y_i = \lambda_i \cdot c_i + v \cdot L_i + \theta_i \cdot S_i + F_i \quad (11B)$$

and

$$\lambda_c \cdot y_c = \lambda_i \cdot c_c + v \cdot L_c + \theta_c \cdot S_c + F_c \quad (12B)$$

where: $S_j = \varepsilon_j \cdot v \cdot L_j$ (with $j = i, c$).

The Marxian reproduction condition for a stationary-state economy becomes:

$$\lambda_i \cdot c_c = v \cdot L_i + S_i \quad (4D)$$

and hence:

$$\frac{L_i}{L_c} = \frac{q_c}{1+\varepsilon_i} \quad (4E)$$

Equation (4E) shows that the equilibrium distribution of labour across sectors depends on the c-sector OCC and the i-sector exploitation rate. Finally, equation (4D) redefines the equilibrium condition in terms of equilibrium values (or prices), allowing for three possible scenarios:

a) $\lambda_i = (v \cdot L_i + S_i)/c_c = p_i$, the demand for capital goods matches the supply, so that the market price of capital goods (call it p_i) equals the reproduction value (λ_i) and the system reproduces smoothly;
b) $\lambda_i = (v \cdot L_i + S_i)/c_c > p_i$, there is lack of demand for capital goods, so that market prices tend to fall short of reproduction values, thereby leading to a reduction in the production of capital goods;

c) $\lambda_i = (v \cdot L_i + S_i)/c_c < p_i$, the demand for investment goods exceeds the supply, so that market prices tend to exceed reproduction values, thereby leading to an increase in the production of capital goods.

Notice that here the adjustment affects market prices in the short run, whereas it involves a change in quantities in the long run (through a change in profit expectations).