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Abstract

This study analyses the interaction of demand, income distribution, and natural output level in a dynamic Kaleckian model with output hysteresis. Hysteresis means that the natural output level depends on the path of the demand-driven actual output level. We consider wage-led and profit-led demand regimes and goods market-led and labour market-led income distribution regimes. We find that the stability of the steady state is closely related to hysteresis in certain regimes. Limit cycles can arise when the strong flexibility of either prices or wages to the output gap is combined with a moderate degree of natural output hysteresis. We make the persuasive case that a Kaleckian model with a wage-led demand regime and anticyclical profit share is less unstable and that pseudo-Goodwin cycles can arise in the profit-led demand regime with a procyclical profit share.

Keywords: Hysteresis, Natural output level, Demand regime, Income distribution

JEL Classification: E12, E32, D33

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1 Introduction

Hysteresis in the natural output level has attracted renewed interest since the global financial crisis. Ball (2014), Blanchard et al. (2015), DeLong and Summers (2012), and Blanchard (2018) document that the crisis of 2008 has left long-lasting scars on the potential growth rate and natural output level. While this may come as a surprise to mainstream economists, who typically conceive long-run performance as anchored by supply-side conditions, Kaleckians (post-Keynesians) have asserted that demand and the income distribution matter beyond the short run. The crisis indeed seems to have demonstrated that the natural output level is influenced by aggregate demand. In other words, the natural output level is neither a unique nor a supply-side-determined attractor of the actual output level, but rather the causality is the inverse. However, most Kaleckian studies have thus far insufficiently modelled hysteresis in the natural output level.

This paper presents a dynamic Kaleckian model of demand and the income distribution with hysteresis in the natural output level (i.e. supply-side adjustments to the effective demand dynamics). The natural output level is often used as synonymous to the non-accelerating inflation rate of unemployment (NAIRU). We define the natural output level as the level of potential output at which there is no acceleration in the price and wage inflation rates, thus establishing a stable income distribution. We use hysteresis to mean that the natural output level depends on the path of the actual output level, which is driven by effective demand. Thus, our terminology contrasts with the premise of most mainstream theories that regard it as purely supply-side-determined.

In this vein, we analyse (i) the macroeconomic dynamics (i.e. stability, instability, and cycles) generated by the interactions of output and the income distribution when there is hysteresis in the natural output level, (ii) how the hysteretic property of the natural output level is related to the transitional dynamics and steady state of the macroeconomic variables, and (iii) the macroeconomic consequences of a change in the income distribution.

Despite the increasing importance of the hysteresis phenomenon since the global financial crisis, few Kaleckian models have focused on the issue of output hysteresis. Lavoie (2006), Stockhammer (2008, 2011), and Michl (2018) are exceptional in this regard. Lavoie (2006) presents a post-Keynesian growth model in which the natural growth rate is driven by the actual growth rate. However, the role of the income distribution, which remains a central topic for Kaleckians, is not analysed. Similarly, Michl (2018) examines demand-led hysteresis in the natural output level. However, he also prevents changes in the income distribution from affecting demand. In contrast to these studies, our model introduces endogenous changes for both the natural output level and the income distribution, while sharing the idea of demand-driven hysteresis with them. Considering the wage-led demand (WLD) regime, Stockhammer (2008, 2011) describe the different views on why the natural output level tends to be endogenous. Meanwhile, we analytically and numerically explain the nature of an economy in which the natural output level varies by hysteresis. Besides, our model also differs from that of Carlin and Soskice (2015, 2018), who present a New Keynesian three-equation model of output, inflation, and the monetary policy rule. The natural output level in their study is ruled by supply-side conditions through labour market institutions. Although we do not explicitly cover monetary policy, the current study builds a model in which it responds to the historical path of effective demand and the income distribution.[1]

Highlighting demand-led hysteresis, we build a Kaleckian model that can comprehensively analyse output levels and the income distribution. To allow for endogenous change in the income distribution, we augment a conflicting claims model with distribution norms and wage and price spiral models. Workers' and capitalists' norms for the income distribution evolve to the gap between the actual and natural output levels, affecting pro- or anticyclical change in the profit share through wage and price spirals. Then, the actual output level varies depending on the WLD and profit-led demand (PLD) regimes, which feeds back to the natural output level through the hysteresis channel.

Our analytical framework advances the understanding on how macroeconomic dynamics are related to the magnitude of hysteresis, change in norms for the income distribution, and adjustment speed of the actual output level in alternative combinations of demand and distribution regimes. We find that when the profit share and actual output level change oppositely, the steady state is locally asymptotically stable. In this case, the degree of hysteresis for the natural output level does not matter for the transitional dynamics. However, when they positively react to each other, the degree of hysteresis as well as other adjustment parameters play a key role in preventing potential instability. In particular, we show when either workers' wage share norm or capitalists' profit share norm changes to the output gap markedly, a moderate magnitude of hys-

[1] In addition to these works, Arestis and Sawyer (2009) also provide a notable contribution that discusses the concept, theory, and application of hysteresis and path dependency. In particular, Setterfield (2009) and Dutt (2009) introduce some modelling of hysteresis. Our modelling is different from the former in that we explicitly introduce the dynamics of the income distribution. It also differs from the latter in that we consider demand-led hysteresis for the natural output level.

teresis induces limit cycles, generating either clockwise or anticlockwise cycles in the wage share and output level. The comparative statics analysis shows that the macroeconomic consequences of changes in the income distribution differ, especially depending on the demand regime.

The remainder of the paper is organised as follows. Section 2 defines a three-dimensional dynamic Kaleckian model. Section 3 analyses the dynamic relation between the income distribution and actual and natural output levels (employment rates). The conditions for stability, instability, and cycles are also presented in this section and the nature of the transitional dynamics is numerically confirmed. Further, this section briefly presents the results from the comparative statics analysis. Section 4 concludes. The appendix provides the mathematical explanations for the main propositions and results.

2 Model

This section builds a dynamic Kaleckian model that consists of the dynamics of the profit share, actual output level, and natural output level. The following are the basic notations used to set up the model: y_t: actual output level, l_t: actual employment level, n: labour supply, y^n_t: natural output level, k_t: capital stock, c_t: consumption demand, g_t: investment demand, p_t: output price, w_t: nominal wage rate, m_t: profit share, and t: time. Below, we do not explicitly denote time t for parsimony.

A closed economy with no government sector is supposed, in which workers supply labour to firms managed by capitalists. The former receives a wage bill and the latter receives a profit income. Firms in the economy operate with the following Leontief-type fixed coefficient production function using capital stock and labour:

$$y = \min(k, l), \tag{1}$$

where the capacity utilisation rate and labour productivity level are scaled to unity for the sake of simplicity. Labour demand is determined by $l = y$ based on Keynesian effective demand. Accordingly, we obtain $\dot{l} = \dot{y}$, where the dot symbol means the time derivative of the variable (i.e. $\dot{x} = dx/dt$).

The employment rate is defined by l/n and the unemployment rate is $1 - l/n$. Once we determine the employment rate, the unemployment rate is then found. The evolution of the labour supply is a social phenomenon on which we do not focus in our analysis. Therefore, we assume that the labour supply is exogenous and constant, and simply normalise it to unity at which a plentiful reserve army is available. Then, the change in the employment rate is $\dot{l} = \dot{y}$. At a steady state where $\dot{y}$ is zero, the employment rate and unemployment rate also remain constant over time. Hence, the actual output level can represent the actual (un)employment rate.

In our model, the price system determines the wage, price, and income distribution, while the quantity system determines the expenditure and income generation. Let us start with a definition of the price system, which is as follows:

$$py = wl + prk, \tag{2}$$

where the nominal income py is distributed to the wage bill wl and profit prk. The profit share m is derived from this accounting relationship as follows:

$$m = 1 - \frac{w}{p}. \tag{3}$$

To endogenously determine the dynamics of the income distribution, we augment the conflicting claims theory of the income distribution with models for the wage and price spirals and distribution norms of two classes.[2] First, we combine the conflicting claims theory with the wage and price dynamic spiral model baselined by Asada et al. (2006), Flaschel (2008) chapter 9, and Proaño et al. (2011). Then, the wage and price dynamics structurally take the following equations:

$$\hat{w} = \beta(m - m_{WN}) + \kappa_w \hat{p}, \tag{4}$$

$$\hat{p} = (1 - \beta)(m_{CN} - m) + \kappa_p \hat{w}, \tag{5}$$

where the hat symbol means the rate of change per time (i.e. $\hat{x} = \dot{x}/x$). Here, m_{WN} is workers' norm for the profit share and m_{CN} is capitalists' norm for the profit share. The magnitudes of β and $1 - \beta$ represent the relative strength of workers' and capitalists' bargaining power. As workers' bargaining power strengthens, they have a higher influence on the wage increase; on the contrary, as capitalists' bargaining power strengthens, they have a higher influence on the price increase. In addition, the pass-through rates $\kappa_w \in (0, 1)$ and $\kappa_p \in (0, 1)$ cause dynamic wage and

[2] Our model of the income distribution consists of three parts: the conflicting claims theory of the income distribution, wage and price spirals, and distribution norms of two classes. All three parts play a crucial role for the type of distribution regime (e.g. equation 10), the steady state (e.g. equation 19), and stability conditions (see Appendix 2) as the parameters building each part are concerned with them.

price spirals. The size of κ_w reflects the degree of the pass-through rate to the wage when there is a change in price inflation, while κ_p reflects that to the price when there is a change in wage inflation, determining the speed of the wage and price spirals in this system.

Second, to endogenise the distribution norms, we are inspired by Skott (2005), Stockhammer (2008, 2011), and Michl (2018). These studies suggest that workers and capitalists have prevalent wage and profit norms (targets), but may update their actual norms at a different pace when their aspirations are frustrated by certain shocks. The endogenous distribution norms are written as follows:

$$m_{WN} = \bar{m}_W - \gamma_w(y - y^n), \tag{6}$$

$$m_{CN} = \bar{m}_C + \gamma_p(y - y^n), \tag{7}$$

where $\bar{m}_W$ and $\bar{m}_C$ are the prevalent norms of workers and capitalists, respectively, which are exogenous and positive. We assume that $\bar{m}_C > \bar{m}_W$ for their norms because capitalists usually demand a higher profit share, whereas workers demand a higher wage share. The sign of γ_w is positive, representing the flexibility of the change in the wage share norm to the output gap. The sign of γ_p takes a positive value, representing the flexibility of the change in capitalists' profit share norm to the output gap.[3]

Here, workers' and capitalists' distribution norms change according to the output gap, meaning that they are updated with reference to the natural output level that establishes a stable income distribution. Equation (6) indicates that workers' wage share norm depends on their prevalent wage norms and positively depends on the output gap, which we formalise in terms of the profit

[3]These formalisations are in contrast to much of the NAIRU literature (Layard et al. (2005); Carlin and Soskice (2015)), which treats the NAIRU as exogenous where the natural unemployment rate does not affect the wage and profit shares over time. For example, since labour productivity is constant in the current model, equations (6) and (7) are similar to the wage- and price-setting curves, respectively in Carlin and Soskice (2015) chapter 2, which are defined in the real wage and actual output level space. In these equations, $\bar{m}_W + \gamma_w y^n$ and $\bar{m}_C - \gamma_p y^n$ compose the intercept of these curves in the distribution norm and actual output level space. In Carlin and Soskice (2015), since the natural output level is regarded as a constant term that is supply-side-determined, the intercepts never change over time. By contrast, in our model, the intercepts change over time because of endogenous changes in the wage norms. Moreover, the distribution norms in equations (6) and (7) never equilibrate with workers' and capitalists' norms—even if the output gap disappears—because the distributional conflict between capital and labour ($\bar{m}_C > \bar{m}_W$) remains over time. Consequently, capitalists receive a lower profit share than their norm at the steady state, while workers are subject to a lower wage share than their norm ($m_{CN} > \bar{m}^* > m_{WN}$), as equation (19) indicates.

share. Similarly, equation (7) indicates that capitalists' profit share norm depends on their prevalent profit norms and positively depends on the output gap. These two equations capture that when a positive output gap happens, it motivates workers that are still receiving a wage share below their norm to increase their wage share norm (decrease their profit share norm) and encourages capitalists that still receive a profit share less than their norm to increase their profit share norm.[4]

The dynamics of the profit share are

$$\dot{m} = (1 - m)\left(\hat{p} - \hat{w}\right), \tag{8}$$

from equation (3). By rearranging equations (4)–(8), we obtain the dynamics of the profit share:[5]

$$\dot{m} = (1 - m)[\theta_m(y - y^n) - (\theta_p + \theta_w)m + \theta_p\overline{m}_C + \theta_w\overline{m}_W], \tag{9}$$

where

$$\theta_m \equiv \frac{(1 - \beta)\gamma_p(1 - \kappa_w) - \beta\gamma_w(1 - \kappa_p)}{1 - \kappa_w\kappa_p}, \tag{10}$$

$$\theta_p \equiv \frac{(1 - \beta)(1 - \kappa_w)}{1 - \kappa_w\kappa_p} > 0, \tag{11}$$

$$\theta_w \equiv \frac{\beta(1 - \kappa_p)}{1 - \kappa_w\kappa_p} > 0. \tag{12}$$

[4]Lavoie (1992) in chapter 7 similarly introduces the distribution norms of different classes. However, the crucial difference between these works is that while he explains the change in norms based only on actual output (in level or growth terms), our study is based on the output gap. The idea is that distribution norms cannot be established when the wage and price keep accelerating. Hence, in changing the norms, a reference to the natural output level is more essential than one to the actual output level only. It is at the natural output level that stable wage and price inflation and the income distribution are realised in the present model.

[5]Summarising equations (4)–(7) gives the reduced form of the actual wage inflation rate and price inflation rate as follows:

$$\hat{w} = \frac{1}{1 - \kappa_w\kappa_p}\left[(\beta\gamma_w + (1 - \beta)\gamma_p\kappa_w)(y - y^n) + (\beta - \kappa_w(1 - \beta))m + (1 - \beta)\kappa_w\overline{m}_C - \beta\overline{m}_W, \right],$$

$$\hat{p} = \frac{1}{1 - \kappa_w\kappa_p}\left[((1 - \beta)\gamma_p + \beta\gamma_w\kappa_p)(y - y^n) + (\kappa_p\beta - (1 - \beta))m + (1 - \beta)\overline{m}_C - \kappa_p\beta\overline{m}_W, \right].$$

They are similar to the reduced forms of the wage and price Phillips curves in Asada et al. (2006), Flaschel (2008), and Proaño et al. (2011). However, the impacts of the output gap on workers' and capitalists' distribution norms and hysteresis kick in originally in our paper. Finally, substituting these equations into equation (8), we obtain equation (9).

Note that θ_m, θ_p, and θ_w are constants, since they are determined by constant exogenous variables. Although the signs of θ_p and θ_w are necessarily positive, that of θ_m is either positive or negative, especially depending on the relative size of the parameters concerning the conflicting claims, wage and price spirals, and distribution norms. These parameters thus play a vital role in the stability of the dynamic system.

We then define two types of income distribution regimes for equation (9).

Definition 1. *The labour market-led (LML) distribution regime refers to the case in which a rise in the output gap leads to a decrease in the profit share. That is,* $\theta_m < 0$ *is established in the LML distribution regime. By contrast, the goods market-led (GML) distribution regime refers to the case in which a rise in the output gap leads to an increase in the profit share. That is,* $\theta_m > 0$ *is established in the GML distribution regime.*

This definition follows Flaschel (2008) and Proaño et al. (2011). The income distribution regime refers to how the income distribution changes to the output gap. In the current model, a positive shock to the output gap raises the wage and price inflation rates at a different pace. In the LML distribution regime, the impact on the former is larger than that on the latter (e.g. a large value of γ_w). Consequently, the profit share is squeezed. This scenario is similar to Marxist models of the industrial reserve army, where the profit share moves anticyclically, whereas the wage share moves procyclical. Since the profit share is driven by nominal wage adjustments and therefore primarily by labour markets, this case is called the LML distribution regime. In the GML distribution, by contrast, the impact of a rise in the output gap on price inflation is stronger than that on wage inflation (e.g. a large value of γ_p). Consequently, the profit share moves procyclical, while the wage share moves anticyclically. This is akin to Keynesian and Kaleckian views of the cyclicity of the income distribution. Since the profit share is driven by the developments of price dynamics and therefore mainly of goods markets, this case is called the GML distribution regime.

Next, we define the quantity system. The consumption function and investment function are introduced into the system principally based on Bhaduri and Marglin (1990). Following them, we assume that workers spend everything they earn (wage bill) and capitalists spend a proportion of what they obtain (profit) on consumption. Then, the consumption function is as follows:

$$c = (1 - m)y + (1 - s)my, \tag{13}$$

where $s \in (0, 1]$ is the saving propensity of capitalists from their profit income.

Bhaduri and Marglin (1990)'s investment function employs the profit share and capacity utilisation rate as explanatory variables to highlight the demand effect of investment. The current model uses the output level instead of the capacity utilisation rate to determine investment demand as follows:

$$g = g(m, y), \tag{14}$$

where $g_m > 0$ and $g_y > 0$ are the derivatives of this function regarding the profit share and output, respectively. The former represents the profit effect and the latter tries to capture the accelerator effect on the investment determination.

The dynamics of the quantity system are defined based on the adjustment of the actual output level to excess demand (supply):

$$\dot{y} = \phi(c + g - y), \tag{15}$$

where the positive value of ϕ represents the adjustment speed of a change in the output level in response to disequilibrium in the goods market. By introducing equations (13) and (14) into (15), we obtain the dynamics of the output level:

$$\dot{y} = \phi[g(m, y) - smy]. \tag{16}$$

We impose the following assumption on the dynamics of the actual output level.

Assumption 1.

$$sm > g_y(m, y). \tag{17}$$

This is the Keynesian stability condition, meaning that saving reacts to changes in output more than in investment. Imposing this condition, we define a WLD or PLD regime according to the following criterion.

Definition 2. *At a steady state, we define* $sy^* - g_m < 0$ *as a PLD regime and* $sy^* - g_m > 0$ *as a WLD regime.*

The asterisk represents the steady-state value. The demand regime in an economy can be classified as a PLD or WLD regime based on whether the output level is an increasing or decreasing function of the profit share.

Lastly, we introduce hysteresis in the natural output level into the present Kaleckian model. When the actual output level stays below the natural output level, it could have the damaging effect that the natural output level is lowered. Conversely, when the actual output level stays above the natural output level, it could have the improving effect that the natural output level is raised. Thus, the path of the natural output level depends on the historical actual output level. In an adaptive manner, the dynamics of the natural output level are

$$\dot{y^n} = \delta(y - y^n). \tag{18}$$

This equation means that whenever the actual output level deviates from the natural output level, the latter begins to evolve accordingly. As the actual output level is driven by the demand dynamics in equation (16), the determination of the natural output level is also demand-led. Lavoie (2006, 2009) also examines the dynamics of the natural output level and growth rate and León-Ledesma and Thirlwall (2002) empirically confirm them. The parameter $\delta \in (0, 1)$ reflects the degree of hysteresis in the natural output level. If δ takes a small value, the evolution of the natural output level is sticky to the current position and its change caused by the historical gap between the two output levels is small. By contrast, when δ takes a large value, it changes quickly according to the historical gap. Recall that the output gap also shifts workers' and capitalists' distribution norms in equations (6) and (7), respectively. The gap not only changes the natural output level through demand-led hysteresis, but also affects the distribution norms of the two classes.

This study focuses on the macroeconomic effects of hysteresis and we do not explore why hysteresis arises in an economy. Therefore, we only refer to the existing literature that has analysed these mechanisms. Indeed, different arguments have been made about the causes of the hysteretic movement in the natural level. For example, while Lindbeck and Snower (1986) highlight the importance of insider/outsider effects, Ball et al. (1999) point to the different stances of macroeconomic policy, in particular monetary policy, during recessions and Rowthorn (1999) examines the role of capital accumulation on the hysteresis phenomenon. Furthermore, Storm and Naastepad (2017) and Fazzari et al. (2018) emphasise that productivity growth responds to demand growth and wage growth.

3 Analysis

3.1 Steady state

In the dynamic system, the demand-driven output gap induces changes in the natural output level through the hysteresis channel (18). The output gap also affects the change in distribution norms, thereby altering the wage and profit shares (9). Consequently, the actual output level is determined depending on the demand regime (16), which brings about the subsequent dynamics.

In this section, we first define the steady state in our system and check the comparative statics analysis. Second, we examine the local stability of the steady state. Third, we describe the persistent cycles using a numerical method.

The steady state is a situation in which $\dot{m} = \dot{y} = \dot{y^n} = 0$ is realised. Then, the profit share and the actual and natural output levels at a steady state must satisfy the following three equations:

$$m^* = \frac{\theta_p \overline{m}_C + \theta_w \overline{m}_W}{\theta_p + \theta_w}, \tag{19}$$

$$g(m^*, y^*) = sm^* y^*, \tag{20}$$

$$y^{n*} = y^*. \tag{21}$$

Since there are three endogenous variables and three equations, the system is complete. We assume that the steady state is unique. Equation (19) presents the actual profit share determined by the weighted average of the custom norms. Equation (20) means there is no excess demand for the goods in the economy. Equation (21) indicates that the natural output level is equalised to the actual output level. In addition, since the output gap disappears at the steady state, workers' and capitalists' distribution norms are equal to their custom norms according to equations (6) and (7). Finally, the steady state is independent of parameters γ_w, γ_p, ϕ and δ. These parameters, however, play a crucial role in determining the local stability of the steady state. Before considering these, we briefly note the comparative statics for the stable steady-state solutions.

Table 1

Table 1 summarises the main results of the comparative statics analysis and Appendix 1 details their calculation. The results are in line with standard Kaleckian models. Overall, the impact of the change in the parameters depends on the type of demand regime. If an economy has the WLD regime, a fall in the profit share arises with an expansion in output levels, while if it has the PLD regime, a fall in the profit share accompanies a fall in output levels.

First, a rise in (β) represents that workers' relative bargaining power increases, which results in a decrease in the profit share. This expands the actual and natural output levels in the WLD regime. By contrast, the profit squeeze reduces these output levels in the PLD regime. Second, a rise in the wage to price pass-through rate (κ_p) raises the profit share. In a WLD regime, this leads to a fall in the output levels. By contrast, in a PLD regime, it expands the actual and natural output levels. A rise in the price to wage pass-through rate (κ_w) raises the wage share, resulting in an output expansion (reduction) in the WLD (PLD) regime. Third, a change in the prevalent profit norms of capitalists ($\bar{m}_C$) and those of workers ($\bar{m}_W$) leads to a positive proportional change in the actual profit share, resulting in corresponding demand changes depending on the demand regime. Finally, a change in the saving rate of capitalists neither affects workers' and capitalists' distribution norms nor the profit share because the profit share is structurally determined by the parameters for the distribution dynamics. However, a rise in the saving rate decreases both the actual and the natural output levels regardless of the demand regime. Therefore, the thrift paradox holds in a Kaleckian model with hysteresis in the natural output level.

The natural output level moves in the same way as the actual output level in all cases. The comparative statics analysis demonstrates that the former falls (rises) when the latter falls (rises). Importantly, this clearly shows that the principle of effective demand may work inversely. Following a lack of effective demand (the actual output level) caused by an adverse distribution or demand shock, it creates its own lack of supply (the natural output level).

3.2 Stability, instability, and cycles

We can obtain the conditions for local asymptotic stability, instability, and the existence of limit cycles by Hopf bifurcation for this system. This section summarises these conditions in propositions first and then considers the economic interpretations. Since the proof for the proposition is lengthy, we provide it in Appendix 2.

Proposition 1. *In an economy that has a PLD regime and an LML distribution regime, the steady state is locally asymptotically stable for all the positive adjustment parameters γ_w, γ_p, δ, and ϕ.*

Proposition 2. *In an economy that has a WLD regime and a GML distribution regime, the steady state is locally asymptotically stable for all the positive adjustment parameters γ_w, γ_p, δ, and ϕ.*

In these two cases, the degree of hysteresis in the natural output level does not affect the stability of the steady state. As Flaschel (2008) and Proaño et al. (2011) explain, the combinations of the PLD and LML distribution regimes and that of the WLD and GML distribution regimes involve self-stabilising mechanisms.[6] For example, when there is a rise in the profit share, this increases the actual output level in the PLD regime. However, a rise in the output share decreases the profit share in the LML distribution regime. Thus, the profit share falls in economic booms, restraining divergence in the output level and the income distribution from the steady state. By contrast, a rise in the wage share stimulates the output level in a WLD regime, whereas a rise in the output level restrains the wage share in the GML distribution regime. Accordingly, the initial rise in the wage share is suppressed.

The degree of hysteresis together with the other adjustment parameters thus play an important role in stability, instability, and the emergence of cycles for other combinations of the demand and income distribution regimes. We therefore obtain the following propositions.

Proposition 3. *In an economy that has a PLD regime and a GML distribution regime:*

(1) There exists one positive value θ^*_{PG} such that the unique steady state is locally asymptotically stable for $\theta_m < \theta^*_{PG}$, locally asymptotically unstable for $\theta^*_{PG} < \theta_m$, and a limit cycle occurs by Hopf bifurcation for θ_m sufficiently close to θ^*_{PG}.

(2) Suppose that the positive impact of the output gap on the profit share is strong and $\theta_m > \underline{\theta}_{PG}$. Then, there exists one positive value δ^*_{PG} such that the unique steady state is locally asymptotically unstable for $\delta < \delta^*_{PG}$, locally asymptotically stable for $\delta^*_{PG} < \delta$, and that a limit cycle occurs by Hopf bifurcation for δ sufficiently close to δ^*_{PG}.

(3) Suppose that the positive impact of the output gap on the profit share is strong and $\theta_m > \tilde{\theta}_{PG}$. Then, there exists one positive value ϕ^*_{PG} such that the unique steady state is locally asymptotically stable for $\phi < \phi^*_{PG}$, locally asymptotically unstable for $\phi > \phi^*_{PG}$, and that a limit cycle occurs by Hopf bifurcation for ϕ sufficiently close to ϕ^*_{PG}.

Proposition 4. *In an economy that has a WLD regime and an LML distribution regime:*

[6] Bhaduri (2007) and Lavoie (2014) also obtain similar results. However, they consider neither the natural output level nor changes in the wage and price clearly. Our propositions are derived under different settings from theirs in that hysteresis in the natural output level is demand-led and the dynamics of the income distribution are more explicitly formalised.

(1) There exists one negative value θ^*_{WL} such that the unique steady state is locally asymptotically stable for $\theta^*_{WL} < \theta_m$, locally asymptotically unstable for $\theta_m < \theta^*_{WL}$, and a limit cycle occurs by Hopf bifurcation for θ_m sufficiently close to θ^*_{WL}.

(2) Suppose that the negative impact of the output gap on the profit share is strong and $\theta_m < \bar{\theta}_{WL}$. Then, there exists one positive value δ^*_{WL} such that the unique steady state is locally asymptotically unstable for $\delta < \delta^*_{WL}$, locally asymptotically stable for $\delta^*_{WL} < \delta$, and a limit cycle occurs by Hopf bifurcation for δ sufficiently close to δ^*_{WL}.

(3) Suppose that the negative impact of the output gap on the profit share is strong and $\theta_m < \tilde{\theta}_{WL}$. Then, there exists one positive value ϕ^*_{WL} such that the unique steady state is locally asymptotically stable for $\phi < \phi^*_{WL}$, locally asymptotically unstable for $\phi > \phi^*_{WL}$, and a limit cycle occurs by Hopf bifurcation for ϕ sufficiently close to ϕ^*_{WL}.

Proposition 3 (1) states that given the adjustment speed for excess demand and magnitude of hysteresis, the stronger flexibility of the change in capitalists' norm to a change in the output gap compared with that in workers' norm causes destabilisation in our Kaleckian model. The mechanism of destabilisation can be explained as follows. Suppose that the state of the economy is originally at a steady state and that there is a sudden rise in the actual output level following some shock. In this situation, the actual output level receives a negative feedback from equation (16), while the profit share increases from equation (9) under the GML distribution regime. As this procyclical change in the profit share works strongly, the increase in the profit share is proportionally large. Consequently, the further effect on the output expansion in the PLD regime is also large. Thus, a significant variation in capitalists' distribution norm to a change in the output gap plays a role in destabilisation.

Proposition 3 (2), by contrast, implies that even if capitalists' distribution norm varies rather flexibly to a change in the output gap, when the natural output level hardly changes because of weak hysteresis, this prevents instability in an economy with the PLD regime and GML distribution regime. The process of stabilisation can be explained as follows. When there is a sudden rise in the actual output level following some shock, the actual output level receives a negative feedback from equation (16), whereas the profit share increases from equation (9) under the GML distribution regime. The rise in the profit share in turn stimulates the actual output level on the one hand from equation (16). As the change in the natural output level is quick because of the strong hysteretic effect, a rise in the actual output level leads to a significant increase in the natural output level from equation (18), which decreases the profit share comparably from equation (9) on the other hand. Then, the fall in the profit share decreases the actual output level in turn. As the impact of the natural output level on the profit share works strongly, this stabilising effect is sufficient. Hence, when hysteresis on the natural output level works strongly, it stabilises an economy with the PLD regime and GML distribution regime.

Proposition 3 (3) similarly states that a slower quantity adjustment prevents the instability caused by more flexible capitalists' norm than workers' norm in this combination. The process of stabilisation can be explained as follows. Suppose that there is a sudden rise in the actual output level following some shock on the initial steady state. Then, the actual output level falls from equation (16), whereas the profit share increases from equation (9). The rise in the profit share expands output, whereas its impact on the increase in the output level is small when the quantity adjustment to excess demand is slow. This slow adjustment in the actual output prevents an excessive explosion in the output dynamics. Put differently, when capitalists' distribution norm moves faster than workers' norm, as far as the quantity adjustment is sufficiently slow, the stability of the steady state is ensured. However, if the quantity adjustment is sufficiently fast, then the stability of the steady state is violated.

Since Proposition 4 is a mirror image of Proposition 3, we only briefly summarise the economic implications. Proposition 4 (1) states that the flexibility in workers' distribution norm is a source of instability for the economy with the WLD regime and LML distribution regime. As the procyclical impact on the wage share works strongly, the further effect on the output expansion in the WLD regime is proportionally large, causing unstable dynamics. Propositions 4 (2) and (3) confirm that even if the wage moves much more flexibly to a change in the output gap than the price does, the strong effect of hysteresis or slower quantity adjustment prevents instability in an economy with the WLD regime and LML distribution regime. By contrast, the weak effect of hysteresis induces potential instability caused by the positive feedback between the actual output level and wage share.

Our model augments the wage and price dynamic spiral models presented by Flaschel (2008) chapter 9 and Proaño et al. (2011) by allowing for hysteresis. We briefly compare the results obtained in the current study with those in their studies by summarising them in Table 2, where P. refers to the proposition number of the current paper. Appendix 2 explains the stability conditions in their baseline model as well.

Table 2

First, both sets of results show that when a rise in the profit share positively (negatively) stimulates the actual output level, while the wage moves faster (slower) than the price moves because of a change in norms, the income distribution and actual output level are stable. Therefore, the combinations of the PLD regime and LML distribution regime and of the WLD regime and GML distribution regime are stable. Second, since the combinations of the WLD regime and LML distribution regime and of the PLD regime and GML distribution regime involve positive feedback mechanisms between distribution and demand, there is potential instability. However, third, our model endogenises the natural output level through the hysteresis channel, which affects the distribution norms and actual profit share. Therefore, even if there is such potential instability, the hysteresis dynamics in the natural output level may prevent this instability. In addition, when a moderate magnitude of hysteresis is combined with the flexible dynamics of workers' and capitalists' norms, the economy experiences endogenous and perpetual cycles. Such mechanisms do not exist in Flaschel (2008) and Proaño et al. (2011). Thus, the current paper contributes to the literature by finding that an economy may also realise stable or cyclical dynamics through hysteresis in the natural output level.

3.3 Numerical simulations

The previous section showed that the combinations of the PLD regime and GML distribution regime and of the WLD regime and LML distribution regime may give rise to limit cycles due to Hopf bifurcation. According to Propositions 3 (2) and 4 (2), this depends on the combination of workers' and capitalists' norms and a moderate degree of hysteresis in the natural output level/moderate adjustment speed of the actual output level. This section confirms the configuration of cycles for these combinations by focusing on the degree of hysteresis.[7]

For the numerical simulation, we need to specify the investment demand function. Let us spell out (14) as $g = g_0 m^{g_m} y^{g_y}$ with a Cobb–Douglas-type function, where $g_0 > 0$, $g_m > 0$, and

[7]The aim of this numerical study is not to calibrate a real economy but rather to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, the values introduced below are set for these purposes to obtain economically meaningful outcomes under the assumption that the labour supply is normalised to unity.

$g_y \in (0, 1)$ are imposed. Using a Cobb–Douglas-type function facilitates distinguishing the WLD and PLD regimes without any loss of generality: $g_y \in (0, 1)$ ensures the Keynesian stability condition, $g_m < 1$ establishes the WLD regime, and $g_m > 1$ establishes the PLD regime.

Then, the following parameters are set to build an economy with the PLD regime and GML distribution regime:

$$\gamma_p = 1.2,\ \gamma_w = 0.4,\ \phi = 0.1,\ \beta = 0.5,\ \kappa_w = 0.5,\ \kappa_p = 0.5,$$
$$\bar{m}_W = 0.3,\ \bar{m}_C = 0.4,\ s = 0.2,\ g_0 = 0.5,\ g_m = 2,\ g_y = 0.5.$$

The unique steady state values generated in the PLD regime and GML distribution regime are as follows:

$$m^* = 0.35,\quad y^* = 0.765625,\quad y^{n*} = 0.765625.$$

The precondition for the existence of the limit cycle that $\theta_m > \bar{\theta}_{PG}$ is also satisfied. We set the hysteresis parameter δ to 0.0026, which is sufficiently close to the Hopf bifurcation value in this parameter configuration. Setting the initial values to $m_0 = 0.3$, $y_0 = 0.8$, and $y_0^n = 0.8$, we project the solution path in the three dimensions of all the variables (Figure 1) and in the two dimensions of the actual output level and distribution (Figure 2) and of the natural output level and distribution (Figure 3), where the distribution share is shown in terms of the wage share to allow comparison with the implications in the existing literature. Each figure draws the solution path from $t = 5000$ to $t = 10000$ and shows that each variable traces a cyclical path.

Figures 1, 2, and 3

Similarly, the following parameters are set to build an economy with the WLD regime and LML distribution regime:

$$\gamma_p = 0.4,\quad \gamma_w = 1.2,\quad \phi = 0.1,\quad \beta = 0.5,\quad \kappa_w = 0.5,\quad \kappa_p = 0.5,$$
$$\bar{m}_W = 0.3,\ \bar{m}_C = 0.4,\ s = 0.3,\ g_0 = 0.125,\ g_m = 0.2,\ g_y = 0.5.$$

These parameters are set to the same value as in the PLD regime and GML distribution regime except $\gamma_p, \gamma_w, s, g_0$ and g_m to obtain economically meaningful values. In this setting, the unique steady-state values generated by them are as follows:

$$m^* = 0.35,\quad y^* = 0.931255,\quad y^{n*} = 0.931255,$$

where $\theta_m < \bar{\theta}_{WL}$ is also satisfied. We set the hysteresis parameter δ to 0.00369, which is sufficiently close to the Hopf bifurcation value in this parameter configuration. Setting the initial values to $m_0 = 0.35$, $y_0 = 0.9$, and $y_0^n = 0.9$, we project the solution path in the three dimensions of all the variables (Figure 4) and in the two dimensions of the actual output level and distribution (Figure 5) and of the natural output level and distribution (Figure 6). Each figure draws the solution path from $t = 5000$ to $t = 10000$ and shows that each variable traces a cyclical path.

Figures 4, 5, and 6

Let us compare the configurations in the PLD and GML distribution regimes with those of the WLD and LML distribution regimes. First, as these figures clearly show, there is a Hopf bifurcation value for the hysteresis parameter δ in each combination, meaning that an economy involving these regimes and parameter undergoes perpetual fluctuation. In both cases, the natural output level fluctuates with the actual output level in a synchronised manner with some delay. The peak and trough of the actual output level come first, and then those of the natural output level follow. Previous studies have explained the effect of hysteresis on changes in the trend path (Carlin and Soskice (2015); Michl (2018)); however, the numerical simulation here reveals the possibility that this induces a cyclical fluctuation as well.

Second, there is a contrasted movement between the two cases in terms of the income distribution and output levels. The wage share and output levels move in an anticlockwise manner in the PLD regime with the GML distribution regime (Figures 2 and 3). On the contrary, their evolution is clockwise in the WLD regime with the LML distribution regime (Figures 5 and 6).

Although the anticlockwise cycles in the PLD regime with the GML distribution regime look like a Goodwin cycle, the mechanism is different. The Goodwin model is based, in the terminology of our model, on the PLD regime with the LML distribution regime (e.g. Barbosa-Filho and Taylor (2006); Flaschel (2008); von Arnin and Barrales (2015)). In contrast to these studies, Figure 2 shows that the anticlockwise cycles in the PLD regime may also arise without the LML distribution regime. Such cyclical behaviours can indeed be produced by the GML distribution regime, where the Marxian profit squeeze mechanism does not work dominantly. Therefore, the observed cycles are a form of what Stockhammer and Michell (2016) call the pseudo-Goodwin cycle, namely anticlockwise movements in the output/wage share space that are not due to the Goodwin mechanism.

Here, the degree of hysteresis in the natural output level is closely related to the emergence of these anticlockwise cycles. A low degree of hysteresis cannot prevent the instability inherent in the PLD and GML distribution regimes, whereas a high degree of hysteresis can by sufficiently shifting the natural output level. When a shock to the actual output level occurs, thereby changing the natural output level, they oppositely change the profit share as Proposition 3 (2) implies. When the degree of hysteresis is moderate, the profit share first rises following an increase in the actual output level and then falls following a rise in the natural output level in a lasting manner. Thus, a cyclical fluctuation between the income distribution, actual output level, and natural output level arises.

Finally, Figures 5 and 6 indicate that in an economy where hysteresis in the natural output level is moderate, when the Kaleckian WLD regime meets the Marxian reserve army effect (i.e. the LML distribution regime), clockwise cycles arise. As Proposition 4 (2) implies, moderate hysteresis has a lasting effect in that there is a fall and a rise in the profit share following rises in the actual output and natural output levels, respectively. These 'anti-Goodwin cycles' in the Kaleckian world with a reserve army distribution function have not thus far been demonstrated. Therefore, the current study is the first to show that clockwise cycles between the wage share and output levels are also theoretically plausible.[8] This also raises interesting challenges for the empirical analysis of the Goodwin cycle. Because most of the empirical literature does not consider the natural output level and its hysteresis (Harvie (2000); Mohun and Veneziani (2008)), the possibility of clockwise cycles might have been overlooked.[9] On the contrary, the current paper theoretically finds that fluctuations in the actual output level are affected not only by the

[8]For example, Sasaki (2013) combines the Kaleckian WLD regime and Marxian profit squeeze mechanism and numerically shows a cyclical solution path (in his case 1). He shows clockwise limit cycles in the wage share and capacity utilisation rate plane, but the cycles for the wage share and employment rate are still anticlockwise. By contrast, our model presents the clockwise (anti-Goodwin) limit cycles for both relationships in the WLD regime and LML distribution regime. Besides, von Arnin and Barrales (2015) shows a PLD regime and Marxian profit squeeze mechanism with the Harrodian instability generate anticlockwise limit cycles in wage share and capacity utilisation rate plane. Therefore, clockwise cycles between the wage share and output levels cannot be observed in their study.

[9]For instance, Mohun and Veneziani (2008) try to detect short-run Goodwin cycles based on detrended variables. However, the natural output level and hysteresis are very much concerned with the trend component, as Ball (2014) and Blanchard et al. (2015) show. Therefore, by doing so, they overlook the role of the natural output level that might have affected the trend as well as been affected by the actual output level.

income distribution but also by hysteretic changes in the natural output level.

4 Conclusion

Although hysteresis has been recognised as increasingly important since the 2008 global financial crisis, Kaleckians have insufficiently explored its macroeconomic effects so far. To bridge this gap, we theoretically investigated the dynamic interaction between the income distribution and output determination while introducing natural output (and employment) hysteresis in a Kaleckian model.

The main results for macroeconomic performance can be summarised as follows. The comparative statics analysis reveals that depending on the combination of demand and the distribution regime, the principle of effective demand works either positively or negatively. When it works positively, a rise in the actual output level driven by the redistribution of income or demand expansion creates its own increase in the natural output level. In the opposite case, a lack of effective demand creates its own lack of supply.

In an economy that has the PLD regime and LML distribution regime or the WLD regime and GML distribution regime, the steady state is locally asymptotically stable for any positive parameters for the distribution norms, degree of hysteresis in the natural output level, and adjustment speed of the actual output level. The steady state of these combinations of demand and distribution regimes is principally stable as Propositions 1 and 2 summarise. The degree of hysteresis does not qualitatively change dynamic macroeconomic performance in these regimes.

By contrast, in an economy with the PLD regime and GML distribution regime or in one with the WLD regime and LML distribution regime, instability and perpetual fluctuation can occur. The degree of hysteresis as well as the distribution norms and adjustment speed of the actual output level matter for stability in these regimes. Importantly, limit cycles can arise when either workers' wage share norm or capitalists' profit share norm respond strongly to the output gap and the degree of hysteresis is moderate.

Our analysis demonstrates that hysteresis may give rise to cyclical behaviour in an economy with the PLD regime and GML distribution regime or the WLD regime and LML distribution regime (i.e. in those regimes with reinforcing feedback mechanisms), which has been overlooked by the existing literature. This result is important for two reasons. First, many Kaleckians regard a WLD regime and an LML distribution regime as a plausible scenario in the medium to long run. At the same time, they may have overstated the unstable dynamics in this combination (Stockhammer (2004); Flaschel (2008); Proaño et al. (2011)). However, we show that if hysteresis works, the WLD regime with the LML distribution regime is not as unstable as the literature emphasises. In other words, an important implication of hysteresis for Kaleckian models is that instability is *less* likely to occur as the hysteresis effects increase.

Second, it also has important implications for the empirical literature explaining Goodwin (i.e. anticlockwise) cycles. Our simulation shows that a reserve army effect in the income distribution (LML distribution regime) is not necessary for a Goodwin-like cycle to arise in the PLD regime. It can also be caused by the GML distribution regime, where the degree of hysteresis in the natural output level plays a crucial role for the emergence of cycles. This demonstrates the existence of pseudo-Goodwin cycles driven by different mechanisms from Stockhammer and Michell (2016). In addition, the current study has newly demonstrated that clockwise cycles between the wage share and output levels are also theoretically plausible in the WLD regime and LML distribution regime. As these cycles involve changes in the natural output level, these would concern long-run cycles. An implication of these findings for business cycle research is that fluctuations in the actual output level may not only be led by the income distribution but also be directly or indirectly affected by hysteretic changes in the natural output level.

Appendix 1

This section describes the calculation for the comparative statics analysis. The sign of $sm^* - g_y$ is positive according to the Keynesian stability condition and that of $sy^* - g_m$ is positive for the WLD regime and negative for the PLD regime. In addition, $\bar{m}_C > \bar{m}_W$ is imposed because of the distributional conflict between capitalists and workers. We denote inequality only when the sign is uniquely determined.

- The impacts of a change in the relative bargaining power of capitalists and workers (β) are as follows:

$$\frac{\partial m^*}{\partial \beta} = -\frac{(1-\kappa_p)(1-\kappa_w)(\bar{m}_C - \bar{m}_W)}{(1-\kappa_w(1-\beta)-\kappa_p\beta)^2} < 0, \tag{22}$$

$$\frac{\partial y^*}{\partial \beta} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \beta}, \tag{23}$$

$$\frac{\partial y^{n*}}{\partial \beta} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \beta}. \tag{24}$$

- The impacts of a change in the degree of wage to price pass-through (κ_p) are as follows:

$$\frac{\partial m^*}{\partial \kappa_p} = \frac{(1-\kappa_w)(\bar{m}_C - \bar{m}_W)\beta(1-\beta)}{(1-\kappa_w(1-\beta)-\kappa_p\beta)^2} > 0, \tag{25}$$

$$\frac{\partial y^*}{\partial \kappa_p} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \kappa_p}, \tag{26}$$

$$\frac{\partial y^{n*}}{\partial \kappa_p} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \kappa_p}. \tag{27}$$

- The impacts of a change in the degree of price to wage pass-through (κ_w) are as follows:

$$\frac{\partial m^*}{\partial \kappa_w} = -\frac{(1-\kappa_p)(\bar{m}_C - \bar{m}_W)\beta(1-\beta)}{(1-\kappa_w(1-\beta)-\kappa_p\beta)^2} < 0, \tag{28}$$

$$\frac{\partial y^*}{\partial \kappa_w} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \kappa_w}, \tag{29}$$

$$\frac{\partial y^{n*}}{\partial \kappa_w} = -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \kappa_w}. \tag{30}$$

- The impacts of a change in capitalists' prevalent profit norms ($\bar{m}_C$) are as follows:

$$\frac{\partial m^*}{\partial \bar{m}_C} = \frac{(1-\kappa_w)(1-\beta)}{1-\kappa_w(1-\beta)-\kappa_p\beta} > 0, \tag{31}$$

$$\frac{\partial y^*}{\partial \bar{m}_C} = -\frac{(sy^* - g_m)(1-\kappa_w)(1-\beta)}{(sm^* - g_y)(1-\kappa_w(1-\beta)-\kappa_p\beta)} \equiv -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \bar{m}_C}, \tag{32}$$

$$\frac{\partial y^{n*}}{\partial \bar{m}_C} = --\frac{(sy^* - g_m)(1-\kappa_w)(1-\beta)}{(sm^* - g_y)(1-\kappa_w(1-\beta)-\kappa_p\beta)} \equiv -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \bar{m}_C}. \tag{33}$$

- The impacts of a change in workers' prevalent profit norms ($\bar{m}_W$) are as follows:

$$\frac{\partial m^*}{\partial \bar{m}_W} = \frac{(1-\kappa_p)\beta}{1-\kappa_w(1-\beta)-\kappa_p\beta} > 0, \tag{34}$$

$$\frac{\partial y^*}{\partial \bar{m}_W} = -\frac{(sy^* - g_m)(1-\kappa_p)\beta}{(sm^* - g_y)(1-\kappa_w(1-\beta)-\kappa_p\beta)} \equiv -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \bar{m}_W}, \tag{35}$$

$$\frac{\partial y^{n*}}{\partial \bar{m}_W} = -\frac{(sy^* - g_m)(1-\kappa_p)\beta}{(sm^* - g_y)(1-\kappa_w(1-\beta)-\kappa_p\beta)} \equiv -\left(\frac{sy^* - g_m}{sm^* - g_y}\right)\frac{\partial m^*}{\partial \bar{m}_W}. \tag{36}$$

- The impacts of a change in the saving rate (s) are as follows:

$$\frac{\partial m^*}{\partial s} = 0, \tag{37}$$

$$\frac{\partial y^*}{\partial s} = -\frac{m^* y^*}{sm^* - g_y} < 0, \tag{38}$$

$$\frac{\partial y^{n*}}{\partial s} = -\frac{m^* y^*}{sm^* - g_y} < 0. \tag{39}$$

Appendix 2

This section provides the proof of Propositions 1, 2, 3, and 4. Further, to compare the baseline model of Flaschel (2008)'s chapter 9 and Proaño et al. (2011), we briefly examine the stability conditions when the dynamic system consists of the income distribution and actual output level with the exogenous natural output level, dropping the hysteresis dynamics.

To investigate the local asymptotic stability of the steady state, the system of differential equations (9), (16), and (18) is linearised around the steady state. The linearised system is given by

$$\begin{bmatrix} \dot{m} \\ \dot{y} \\ \dot{y}^n \end{bmatrix} = \underbrace{\begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & 0 \\ 0 & j_{32} & j_{33} \end{bmatrix}}_{\boldsymbol{J}^*} \begin{bmatrix} m - m^* \\ y - y^* \\ y^n - y^{n*} \end{bmatrix}, \tag{40}$$

where $\boldsymbol{J^*}$ is the Jacobian matrix. The non-zero elements of the Jacobian matrix are given as follows:

$$j_{11} \equiv \frac{\partial \dot{m}}{\partial m} = -(1 - m^*)(\theta_p + \theta_w), \tag{41}$$

$$j_{12} \equiv \frac{\partial \dot{m}}{\partial y} = (1 - m^*)\theta_m, \tag{42}$$

$$j_{13} \equiv \frac{\partial \dot{m}}{\partial y^n} = -(1 - m^*)\theta_m, \tag{43}$$

$$j_{21} \equiv \frac{\partial \dot{y}}{\partial m} = \phi(g_m - sy^*), \tag{44}$$

$$j_{22} \equiv \frac{\partial \dot{y}}{\partial y} = \phi(g_y - sm^*), \tag{45}$$

$$j_{32} \equiv \frac{\partial \dot{y}^n}{\partial y} = \delta, \tag{46}$$

$$j_{33} \equiv \frac{\partial \dot{y}^n}{\partial y^n} = -\delta, \tag{47}$$

Where all the elements are evaluated at the steady state.

The baseline model of Flaschel (2008) and Proaño et al. (2011) is that the natural output level is exogenous (i.e. y^n is constant) and the dynamic system consists of the income distribution (equation 9) and the actual output level (equation 16). Then, the local stability conditions are

$$j_{11} + j_{22} = -[(1 - m^*)(\theta_p + \theta_w) + \phi(sm^* - g_y)] < 0, \tag{48}$$

$$j_{11} j_{22} - j_{12} j_{21} = (1 - m^*)\phi[(\theta_p + \theta_w)(sm^* - g_y) + (sy^* - g_m)\theta_m] > 0. \tag{49}$$

Equation (48) is satisfied as far as the Keynesian stability condition is imposed by Assumption 1. Equation (49) is necessarily satisfied for the combinations of the WLD regime and GML distribution regime and of the PLD regime and LML distribution regime because both θ_m and $sy^* - g_m$ take the same sign from Definitions 1 and 2.

However, for the combinations of the PLD regime and GML distribution regime and of the WLD regime and LML distribution regime, whether equation (49) is satisfied *ceteris paribus* depends on the relative strength of the distributional impact of changes in the output level and the output impact of changes in the income distribution. The condition for equation (49) can be rewritten as follows:

$$(\theta_p + \theta_w)(sm^* - g_y) > (g_m - sy^*)\theta_m, \tag{50}$$

where the left-hand side (LHS) is always positive according to the Keynesian stability condition, while the right-hand side (RHS) is also positive because both θ_m and $g_m - sy^*$ take the same sign from Definitions 1 and 2. For the stability condition to be satisfied, given the size of the LHS, when the distributional impact of the change in the output level is strong (i.e. large absolute value of θ_m), the output impact of the change in the income distribution must be weak (i.e. small absolute value of $g_m - sy^*$). Similarly, when the output impact of the change in the income distribution is strong (i.e. large absolute value of $g_m - sy^*$), the distributional impact of the change in the output level must be weak (i.e. small absolute value of θ_m). If this condition is not met, the dynamic path of the economy is a saddle one, which should be regarded as unstable.

Lastly, even when equation (49) is not satisfied, hysteresis can prevent potential instability from arising. We mention this in proving Propositions 3 (2) and 4 (2).

To analyse the local asymptotic stability of the steady state of our original model with hysteresis, let us define the following characteristic equation associated with the Jacobian matrix

$\boldsymbol{J}^*$:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{51}$$

where λ denotes a characteristic root. Coefficients a_1, a_2, and a_3 are given as follows:

$$\begin{aligned} a_1 &= -\text{ trace } \boldsymbol{J}^* \\ &= (1-m^*)(\theta_p+\theta_w) + \phi(sm^* - g_y) + \delta, \end{aligned} \tag{52}$$

$$\begin{aligned} a_2 &= \begin{vmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{vmatrix} + \begin{vmatrix} j_{11} & j_{13} \\ 0 & j_{33} \end{vmatrix} + \begin{vmatrix} j_{22} & 0 \\ j_{32} & j_{33} \end{vmatrix} \\ &= \phi(1-m^*)[(\theta_p+\theta_w)(sm^*-g_y) + (sy^*-g_m)\theta_m] + (1-m^*)(\theta_p+\theta_w)\delta + (sm^*-g_y)\phi\delta, \end{aligned} \tag{53}$$

$$\begin{aligned} a_3 &= -\det \boldsymbol{J}^* \\ &= (1-m^*)(\theta_p+\theta_w)(sm^*-g_y)\phi\delta. \end{aligned} \tag{54}$$

The necessary and sufficient condition for local asymptotic stability is that all the characteristic roots of the Jacobian matrix have negative real parts, which from the Routh–Hurwitz condition, is equivalent to

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1a_2 - a_3 > 0. \tag{55}$$

Equations (52), (53), and (54) imply that the local asymptotic stability of the steady state exclusively depends on the type and combination of the (i) demand regime, (ii) distribution regime, and (iii) speed of the wage γ_w, price γ_p, and quantity ϕ adjustments as well as the magnitude of hysteresis in natural output δ. Therefore, before starting the proof, it is convenient to summarise the stability conditions in the following three systems based on the key variables.

Definition 3 (System θ_m)**.**

$$a_1 = \underbrace{(1-m^*)(\theta_p+\theta_w) + \phi(sm^*-g_y) + \delta}_{\Theta_1>0} = \Theta_1, \tag{56}$$

$$\begin{aligned} a_2 &= \underbrace{(1-m^*)(\theta_p+\theta_w)[\phi(sm^*-g_y)+\delta] + (sm^*-g_y)\phi\delta}_{\Theta_2>0} + \underbrace{(1-m^*)(sy^*-g_m)\phi}_{\Theta_3}\theta_m \\ &= \Theta_2 + \Theta_3\theta_m, \end{aligned} \tag{57}$$

$$a_3 = \underbrace{(1-m^*)(\theta_p+\theta_w)(sm^*-g_y)\phi\delta}_{\Theta_4>0} = \Theta_4, \tag{58}$$

$$a_1a_2 - a_3 \equiv F(\theta_m) = \Theta_1\Theta_2 - \Theta_4 + \Theta_1\Theta_3\theta_m, \tag{59}$$

where $\Theta_1\Theta_2 - \Theta_4 > 0$ regardless of demand and the distribution regimes. In considering system θ_m, we suppose that θ_w and θ_p are given at a constant level and that θ_m varies according to the changes in γ_w and γ_p only.

Definition 4 (System δ)**.**

$$a_1 = \underbrace{(1 - m^*)(\theta_p + \theta_w) + \phi(sm^* - g_y)}_{\Delta_1>0} + \delta = \Delta_1 + \delta, \tag{60}$$

$$\begin{aligned} a_2 &= \underbrace{(1 - m^*)\phi[(\theta_p + \theta_w)(sm^* - g_y) + (sy^* - g_m)\theta_m]}_{\Delta_2} + [\underbrace{(1 - m^*)(\theta_p + \theta_w) + \phi(sm^* - g_y)}_{\Delta_1>0}]\delta \\ &= \Delta_2 + \Delta_1\delta, \end{aligned} \tag{61}$$

$$a_3 = \underbrace{(1 - m^*)(\theta_p + \theta_w)(sm^* - g_y)\phi}_{\Delta_3>0}\,\delta = \Delta_3\delta, \tag{62}$$

$$a_1a_2 - a_3 \equiv G(\delta) = \Delta_1\delta^2 + (\Delta_1^2 + \Delta_2 - \Delta_3)\delta + \Delta_1\Delta_2. \tag{63}$$

Definition 5 (System ϕ)**.**

$$a_1 = \underbrace{(1 - m^*)(\theta_p + \theta_w) + \delta}_{\Phi_1>0} + \underbrace{(sm^* - g_y)}_{\Phi_2>0}\,\phi = \Phi_1 + \Phi_2\phi, \tag{64}$$

$$\begin{aligned} a_2 &= [\{\underbrace{(1 - m^*)(\theta_p + \theta_w) + \delta}_{\Phi_1>0}\}\underbrace{(sm^* - g_y)}_{\Phi_2>0} + \underbrace{(1 - m^*)(sy^* - g_m)}_{\Phi_3}\,\theta_m]\phi + \underbrace{(1 - m^*)(\theta_p + \theta_w)}_{\Phi_4>0}\delta, \quad (65) \\ &= (\Phi_1\Phi_2 + \Phi_3\theta_m)\phi + \Phi_4, \end{aligned}$$

$$a_3 = \underbrace{(sm^* - g_y)}_{\Phi_2>0}\,\underbrace{(1 - m^*)(\theta_p + \theta_w)\delta}_{\Phi_4>0}\,\phi = \Phi_2\Phi_4\phi, \tag{66}$$

$$a_1a_2 - a_3 \equiv H(\phi) = (\Phi_1\Phi_2 + \Phi_3\theta_m)\Phi_2\phi^2 + (\Phi_1\Phi_2 + \Phi_3\theta_m)\Phi_1\phi + \Phi_1\Phi_4. \tag{67}$$

The stability conditions $a_1 > 0$ and $a_3 > 0$ are necessarily satisfied in all the systems. Therefore, we need to prove if $a_2 > 0$ and $a_1a_2 - a_3 > 0$ are also satisfied. First, we provide the proof of Proposition 1.

Proof. From Definitions 1 and 2, $sy^* - g_m < 0$ is established in the PLD regime and $\theta_m < 0$ is satisfied in the LML distribution regime.

- In system θ_m, we have a positive value for $\Theta_3\theta_m$. Consequently, $a_2 > 0$ and $a_1a_2 - a_3 > 0$ are satisfied for any positive value of γ_w and γ_p that make the sign of θ_m negative.

- In system δ, we have a positive value for Δ_2. Then, $a_2 > 0$ is satisfied. In addition, the axis of the downward convex function $G(\delta)$ for the PLD regime and LML distribution regime is

$$\tilde{\delta} = -\frac{\Delta_1^2 + \Delta_2 - \Delta_3}{2\Delta_1} < 0, \tag{68}$$

where the sign of $\Delta_1^2 + \Delta_2 - \Delta_3$ is positive for this combination of regimes, meaning that the axis of $G(\delta)$ is located within $\delta < 0$ in this case. On the contrary, the intercept of $G(\delta)$ in the graph is obviously positive. Therefore, $a_1a_2 - a_3 > 0$ is always satisfied for any positive value of δ.

- In system ϕ, the axis of the downward convex function $H(\phi)$ for the PLD regime and LML distribution regime is

$$\tilde{\phi} = -\frac{\Phi_1}{2\Phi_2} < 0, \tag{69}$$

where the axis of $H(\phi)$ is located within $\phi < 0$. Taking into account that the intercept of $H(\phi)$ is positive, $a_1a_2 - a_3 > 0$ is always satisfied for any positive value of ϕ.

Therefore, the local asymptotic stability conditions of the steady state $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ are satisfied for all the positive adjustment parameters γ_w, γ_p, δ, and ϕ under the PLD regime and LML distribution regime. □

Second, the proof of Proposition 2 is provided in an analogous way to that of Proposition 1. Accordingly, we provide the following sketch for that without going into detail.

Proof. As the proof of Proposition 1 presented, the local asymptotic stability of the steady state depends on the combination of the signs of θ_m and $sy^* - g_m$. The sign of their product for the WLD regime and GML distribution regime is the same as that for the PLD regime and LML distribution regime. Therefore, the local asymptotic stability conditions $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ are satisfied for all the positive adjustment parameters γ_w, γ_p, δ, and ϕ. □

The proof of Proposition 3 is given as follows.

Proof of Proposition 3 (1). In system θ_m, a_2 includes both $sy^* - g_m$ and θ_m. The sign of $sy^* - g_m$ is negative in the PLD regime, while that of θ_m is positive in the GML distribution regime.

First, let us consider the condition for $a_2 > 0$. For this, the values of γ_w and γ_p must ensure the following value of θ_m:

$$\theta_m < \bar{\theta}_{PG} \equiv -\frac{\Theta_2}{\Theta_3}, \tag{70}$$

where the sign of Θ_3 is negative in the PLD regime. As far as $\theta_m < \bar{\theta}_{PG}$ is satisfied, $a_2 > 0$ is ensured.

Second, for $a_1a_2 - a_3$ to also be positive, we need

$$a_1a_2 - a_3 \equiv F(\theta_m) = \Theta_1\Theta_2 - \Theta_4 + \Theta_1\Theta_3\theta_m > 0. \tag{71}$$

Hence,

$$\theta_m < \theta^*_{PG} \equiv \frac{\Theta_4 - \Theta_1\Theta_2}{\Theta_1\Theta_3}, \tag{72}$$

for $\theta_m < \theta^*_{PG}$, and the sign of $a_1a_2 - a_3 > 0$ is established.

Both a_2 and $a_1a_2 - a_3$ are linear functions of θ_m, which takes a positive value under the GML distribution regime. If θ^*_{PG} is smaller than $\bar{\theta}_{PG}$, θ^*_{PG} satisfies $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 = 0$. Hence, Hopf bifurcation occurs for θ_m sufficiently close to θ^*_{PG}. Then, the existence of Hopf bifurcation can be proven as follows. By substituting $\bar{\theta}_{PG}$ into $F(\theta_m)$ and arranging, we obtain

$$F(\bar{\theta}_{PG}) = -\Theta_4 < 0. \tag{73}$$

Since Θ_4 is positive, the value of $F(\bar{\theta}_{PG})$ is obviously negative. This means that the value of θ^*_{PG} is smaller than $\bar{\theta}_{PG}$ and that θ_m satisfies $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 = 0$ at θ^*_{PG}.

To summarise, there exists a positive value θ^*_{PG} such that the unique steady state is locally stable for $\theta_m < \theta^*_{PG}$, locally unstable for $\theta_m < \theta^*_{PG}$, and a limit cycle occurs by Hopf bifurcation for θ_m sufficiently close to θ^*_{PG}. □

Proof of Proposition 3 (2). a_2 includes both $sy^* - g_m$, which is negative for the PLD regime, and θ_m, which is positive for the GML distribution regime in Δ_2. As far as Δ_2 is positive, a_2 is necessarily positive, and all the stability conditions are satisfied, as we have proven for Proposition 1 (δ).

By contrast, Δ_2 is negative when

$$\theta_m > \underline{\theta}_{PG} \equiv -\frac{(\theta_p + \theta_w)(sm^* - g_y)}{sy^* - g_m} > 0. \tag{74}$$

This inequality is the same condition as in the baseline model of Flaschel (2008) and Proaño et al. (2011) to have saddle-path instability, as implied in equation (49). However, we prove that the nature of the transitional dynamics can be stable, unstable, or cycle depending on the degree of hysteresis. Suppose now that equation (74) holds and Δ_2 is negative.

First, for a_2 to be positive even when Δ_2 is negative, the value of δ must be above the following value:

$$\delta > \underline{\delta}_{PG} \equiv -\frac{\Delta_2}{\Delta_1} > 0. \tag{75}$$

Second, $a_1a_2 - a_3$ is a quadratic function of δ, which is

$$a_1a_2 - a_3 = G(\delta) \equiv \Delta_1\delta^2 + (\Delta_1^2 + \Delta_2 - \Delta_3)\delta + \Delta_1\Delta_2. \tag{76}$$

Since $\Delta_1 > 0$, $\Delta_2 < 0$, and $G(0) = \Delta_1\Delta_2 < 0$, $G(\delta) = 0$ has one negative real root and one positive root. Because only the positive root is economically meaningful, we let δ^*_{PG} denote the positive root. If $a_2 > 0$ and $a_1a_2 - a_3 = 0$ are simultaneously established at δ^*_{PG}, Hopf bifurcation arises in the neighbourhood of δ^*_{PG}. Therefore, we compare which of δ^*_{PG} or $\underline{\delta}_{PG}$ is larger.

Substituting $\underline{\delta}_{PG}$, which settles $a_2 = 0$, into $G(\delta)$, we obtain

$$G(\underline{\delta}_{PG}) = \frac{\Delta_2\Delta_3}{\Delta_1} < 0, \tag{77}$$

meaning $\underline{\delta}_{PG} < \delta^*_{PG}$.

Therefore, given $\theta_m > \underline{\theta}_{PG} > 0$, we find that (i) $a_1 > 0$, $a_2 < 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\delta \in (0, \underline{\delta}_{PG})$, (ii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\delta \in (\underline{\delta}_{PG}, \delta^*_{PG})$, whereas (iii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ within the range $\delta > \delta^*_{PG}$. Indeed, at $\delta = \delta^*_{PG}$, we obtain

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad \left.\frac{\partial(a_1a_2 - a_3)}{\partial\delta}\right|_{\delta=\delta^*_{PG}} \neq 0. \tag{78}$$

Consequently, Hopf bifurcation occurs for δ sufficiently close to δ^*_{PG}. □

Proof of Proposition 3 (3). As far as $\Phi_1\Phi_2 + \Phi_3\theta_m$ is positive, a_2 is necessarily positive. Then, the steady state of the system is locally asymptotically stable, as we have proven for Proposition 1 (ϕ).

On the contrary, in the PLD regime and GML distribution regime where Φ_3 is negative but θ_m is positive, the first term in a_2, which is $\Phi_1\Phi_2 + \Phi_3\theta_m$, is negative when

$$\theta_m > \tilde{\theta}_{PG} \equiv -\frac{\Phi_1\Phi_2}{\Phi_3}. \tag{79}$$

Suppose that this is the case. Then, for a_2 to be positive even in this case too, the value of ϕ must be less than the following value:

$$\phi < \overline{\phi}_{PG} \equiv -\frac{\Phi_4}{\Phi_1\Phi_2 + \Phi_3\theta_m}. \tag{80}$$

In addition, $a_1a_2 - a_3$ can be arranged in a quadratic function of ϕ:

$$a_1a_2 - a_3 \equiv H(\phi) = (\Phi_1\Phi_2 + \Phi_3\theta_m)\Phi_2\phi^2 + (\Phi_1\Phi_2 + \Phi_3\theta_m)\Phi_1\phi + \Phi_1\Phi_4. \tag{81}$$

Since $\Phi_1\Phi_2 + \Phi_3\theta_m < 0$, Descartes' rule of signs ensures that the quadratic equation $H(\phi) = 0$ has one negative real root and one positive root. Because only the positive root is economically meaningful, let ϕ^*_{PG} denote the positive root. For the same reason as above, we investigate which is larger, ϕ^*_{PG} or $\overline{\phi}_{PG}$.

Substituting $\overline{\phi}_{PG}$ that ensures $a_2 = 0$ into $H(\phi)$, we obtain

$$H(\overline{\phi}_{PG}) = \frac{\Phi_2\Phi_4^2}{\Phi_1\Phi_2 + \Phi_3\theta_m} < 0, \tag{82}$$

meaning $\phi^*_{PG} < \overline{\delta}_{PG}$.

Therefore, given $\theta_m > \tilde{\theta}_{PG}$, we find that (i) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ within the range $\phi \in (0, \phi^*_{PG})$, (ii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\phi \in (\phi^*_{PG}, \overline{\phi}_{PG})$, whereas (iii) $a_1 > 0$, $a_2 < 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\phi > \overline{\phi}_{PG}$. Indeed, at $\phi = \phi^*_{PG}$, we obtain

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad \left.\frac{\partial(a_1a_2 - a_3)}{\partial\phi}\right|_{\phi=\phi^*_{PG}} \neq 0. \tag{83}$$

Consequently, Hopf bifurcation occurs for ϕ sufficiently close to ϕ^*_{PG}. □

Finally, the proof of Proposition 4 is given as follows.

Proof of Proposition 4 (1). In system θ_m, a_2 includes both $sy^* - g_m$ and θ_m. The sign of $sy^* - g_m$ is positive in the WLD regime, while that of θ_m is negative in the LML distribution regime.

First, we consider the condition for $a_2 > 0$. For this, the values of γ_w and γ_p must ensure the following value of θ_m:

$$\theta_m > \underline{\theta}_{WL} \equiv -\frac{\Theta_2}{\Theta_3}, \tag{84}$$

where the sign of Θ_3 is positive in the WLD regime. As far as $\theta_m < \overline{\theta}_{WL}$ is satisfied, $a_2 > 0$ is ensured.

Second, for $a_1 a_2 - a_3$ to also be positive, we need

$$a_1 a_2 - a_3 \equiv F(\theta_m) = \Theta_1\Theta_2 - \Theta_4 + \Theta_1\Theta_3\theta_m > 0. \tag{85}$$

Hence,

$$0 > \theta_m > \theta^*_{WL} \equiv \frac{\Theta_4 - \Theta_1\Theta_2}{\Theta_1\Theta_3}, \tag{86}$$

for $\theta_m > \theta^*_{WL}$, and the sign of $a_1 a_2 - a_3 > 0$ is established.

Both a_2 and $a_1 a_2 - a_3$ are linear functions of θ_m that now takes a negative value under the LML distribution regime. If $\underline{\theta}_{WL}$ is smaller than θ^*_{WL}, there is such θ^*_{WL} that satisfies $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 = 0$. Hence, Hopf bifurcation occurs for θ_m sufficiently close to θ^*_{WL}. Then, the existence of Hopf bifurcation can be proven as follows. By substituting $\underline{\theta}_{WL}$ into $F(\theta_m)$ and rearranging, we obtain

$$F(\underline{\theta}_{WL}) = -\Theta_4 < 0. \tag{87}$$

Since Θ_4 is positive, the value of $F(\underline{\theta}_{WL})$ is obviously negative. This means that the value of $\underline{\theta}_{WL}$ is smaller than θ^*_{WL} and that θ_m satisfies $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 = 0$ at θ^*_{WL}.

To summarise, there exists one negative value θ^*_{WL} such that the unique steady state is locally stable for $\theta^*_{WL} < \theta_m < 0$, locally unstable for $\theta_m < \theta^*_{WL}$, and a limit cycle occurs by Hopf bifurcation for θ_m sufficiently close to θ^*_{WL}. □

Proof of Proposition 4 (2). a_2 includes both $sy^* - g_m$, which is positive for the WLD regime, and θ_m, which is negative for the LML distribution regime in Δ_2. As far as Δ_2 is positive, a_2 is necessarily positive, and all the conditions are satisfied, as we have proven for Proposition 1.

By contrast, Δ_2 is negative when

$$\theta_m < \overline{\theta}_{WL} \equiv -\frac{(\theta_p + \theta_w)(sm^* - g_y)}{sy^* - g_m} < 0. \tag{88}$$

Again, this inequality is the same condition as in Flaschel (2008)'s and Proaño et al. (2011)'s model to have saddle-path instability, as shown in equation (49). However, the nature of the transitional dynamics can be stable, unstable, and cycle depending on the degree of hysteresis. Suppose now that this is the case and Δ_2 is negative.

First, for a_2 to be positive, even when Δ_2 is negative, the value of δ must be over the following value:

$$\delta > \underline{\delta}_{WL} \equiv -\frac{\Delta_2}{\Delta_1} > 0. \tag{89}$$

Second, $a_1 a_2 - a_3$ is a quadratic function of δ, which is

$$a_1 a_2 - a_3 = G(\delta) \equiv \Delta_1 \delta^2 + (\Delta_1^2 + \Delta_2 - \Delta_3)\delta + \Delta_1 \Delta_2. \tag{90}$$

Since $\Delta_1 > 0$, $\Delta_2 < 0$, and $G(0) = \Delta_1 \Delta_2 < 0$, $G(\delta) = 0$ has one negative real root and one positive root. Because only the positive root is economically meaningful, we let δ^*_{WL} denote the positive root. If $a_2 > 0$ and $a_1 a_2 - a_3 = 0$ are simultaneously established at δ^*_{WL}, Hopf bifurcation arises in the neighbourhood of δ^*_{WL}. Therefore, we compare which of δ^*_{WL} or $\underline{\delta}_{WL}$ is larger.

Substituting $\underline{\delta}_{WL}$, which settles $a_2 = 0$, into $G(\delta)$, we obtain

$$G(\underline{\delta}_{WL}) = \frac{\Delta_2 \Delta_3}{\Delta_1} < 0, \tag{91}$$

meaning $\underline{\delta}_{WL} < \delta^*_{WL}$.

Therefore, given $\theta_m < \overline{\theta}_{WL} < 0$, we find that (i) $a_1 > 0$, $a_2 < 0$, $a_3 > 0$, and $a_1 a_2 - a_3 < 0$ within the range $\delta < \underline{\delta}_{WL}$, (ii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 < 0$ within the range $\delta \in (\underline{\delta}_{WL}, \delta^*_{WL})$, whereas (iii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$ within the range $\delta > \delta^*_{WL}$. Indeed, at $\delta = \delta^*_{WL}$, we obtain

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad \left.\frac{\partial(a_1 a_2 - a_3)}{\partial \delta}\right|_{\delta=\delta^*_{WL}} \neq 0. \tag{92}$$

Consequently, Hopf bifurcation occurs for δ sufficiently close to δ^*_{WL}. □

Proof of Proposition 4 (3). As far as $\Phi_1 \Phi_2 + \Phi_3 \theta_m$ is positive, a_2 is necessarily positive. Then, the steady state of the system is locally asymptotically stable, as we have proven for Proposition 1.

On the contrary, in the WLD regime and LML distribution regime where Φ_3 is positive but θ_m is negative, $\Phi_1 \Phi_2 + \Phi_3 \theta_m$ is negative when

$$\theta_m < \tilde{\theta}_{WL} \equiv -\frac{\Phi_1 \Phi_2}{\Phi_3} < 0. \tag{93}$$

Suppose that this is the case. Then, for a_2 to be positive even in this case too, the value of ϕ must be less than the following value:

$$0 < \phi < \overline{\phi}_{WL} \equiv -\frac{\Phi_4}{\Phi_1 \Phi_2 + \Phi_3 \theta_m}. \tag{94}$$

In addition, $a_1 a_2 - a_3$ can be arranged in a quadratic function of ϕ:

$$a_1 a_2 - a_3 \equiv H(\phi) = (\Phi_1 \Phi_2 + \Phi_3 \theta_m)\Phi_2 \phi^2 + (\Phi_1 \Phi_2 + \Phi_3 \theta_m)\Phi_1 \phi + \Phi_1 \Phi_4. \tag{95}$$

Since $\Phi_1\Phi_2 + \Phi_3\theta_m < 0$, Descartes' rule of signs ensures that the quadratic equation $H(\phi) = 0$ has one negative real root and one positive root. Because only the positive root is economically meaningful, let ϕ^*_{WL} denote the positive root. For the same reason as above, we investigate which is larger, ϕ^*_{WL} or $\overline{\phi}_{WL}$.

Substituting $\overline{\phi}_{WL}$ that ensures $a_2 = 0$ into $H(\phi)$, we obtain

$$H(\overline{\phi}_{WL}) = \frac{\Phi_2\Phi_4^2}{\Phi_1\Phi_2 + \Phi_3\theta_m} < 0, \tag{96}$$

meaning $\phi^*_{WL} < \overline{\delta}_{WL}$.

Therefore, given $\theta_m < \tilde{\theta}_{WL}$, we find that (i) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ within the range $\phi \in (0, \phi^*_{WL})$, (ii) $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\phi \in (\phi^*_{WL}, \overline{\phi}_{WL})$, whereas (iii) $a_1 > 0$, $a_2 < 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\phi > \overline{\phi}_{WL}$. Indeed, at $\phi = \phi^*_{WL}$, we obtain

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad \left.\frac{\partial(a_1a_2 - a_3)}{\partial\phi}\right|_{\phi=\phi^*_{WL}} \neq 0. \tag{97}$$

Consequently, Hopf bifurcation occurs for ϕ sufficiently close to ϕ^*_{WL}. □

References

Arestis, P. and M. Sawyer eds. (2009): *Path Dependency and Macroeconomics*, London: Palgrave Macmillan.

von Arnin, R. and J. Barrales (2015): 'Deamand-driven Goodwin Cycles with Kaldorian and Kaleckian Features', *Reveiew of Keynesian Economics*, **3** (3), pp. 351-373.

Asada, T., P. Chen, C. Chiarella and P. Flaschel (2006): 'Keynesian Dynamics and the Wage-price Spiral: A Baseline Disequilibrium Model', *Journal of Macroeconomics*, **28** (1), pp. 90-130.

Ball, L. (2014): 'Long-term Damage from the Great Recession in OECD Countries', *European Journal of Economics and Economic Policies: Intervention*, **11** (2), pp. 149-160.

Ball, L., G. Mankiw and W. Nordhaus (1999): 'Aggregate Demand and Long-run Unemployment.', *Brookings Papers on Economic Activity*, **1999** (2), pp. 189-251.

Barbosa-Filho, N. and L. Taylor (2006): 'Distributive and Demand Cycles in the US Economy: A Structuralist Goodwin Model', *Metroeconomica*, **57** (3), pp. 389-411.

Bhaduri, A. (2007): ‘On the Dynamics of Profit-led and Wage-led Growth’, *Cambridge Journal of Economics*, **32** (1), pp. 147-160.

Bhaduri, A. and S. Marglin (1990): ‘Unemployment and the Real Wage: the Economic Basis for Contesting Political Ideologies’, *Cambridge Journal of Economics*, **14** (4), pp. 375-393.

Blanchard, O., E. Cerutti and L. Summers (2015): ‘Inflation and Activity: Two Explorations and Their Monetary Policy Implications’, *National Bureau of Economic Research* (w21726), pp. 1-24.

Blanchard, O. (2018): ‘Should We Reject the Natural Rate Hypothesis?’, *Journal of Economic Perspectives*, **32** (1), pp. 97-120.

Carlin, W. and D. Soskice (2015): *Macroeconomics: Institutions, Instability, and the Financial System*, Oxford: Oxford University Press.

——— (2018): ‘Stagnant Productivity and Low Unemployment: Stuck in a Keynesian Equilibrium’, *Oxford Review of Economic Policy*, **34** (1-2), pp. 169-194.

DeLong, B. and L. Summers (2012): ‘Fiscal Policy in a Depressed Economy’, *Brookings Papers on Economic Activity*, **1**, pp. 233-297.

Dutt, A. (2009): ‘Path Dependence, Equilibrium and Economic Growth’, pp. 119-161. in Arestis and Sawyer (2009).

Fazzari, S., P. Ferri. and A. Variato (2018): ‘Demand-led Growth and Accomodating Supply’, *FMM WORKING PAPER* (15).

Flaschel, P. (2008): *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes and Schumpeter*, Berlin: Springer, second revised and enlarged edition edition.

Flaschel, P. and M. Landesmann eds. (2008): *Mathematical Economics and the Dynamics of Capitalism*, London: Routledge.

Fontana, G. and M. Setterfield eds. (2009): *Macroeconomic Theory and Macroeconomic Pedagogy*, New York: Palgrave.

Harvie, D. (2000): ‘Testing Goodwin: Growth Cycles in 10 OECD Countries’, *Cambridge Journal of Economics*, **24** (3), pp. 349-376.

Lavoie, M. (1992): *Foundations of Post-Keynesian Economic Analysis*, Aldershot: Edward Elgar.

——— (2006): ‘A Post-Keynesian Amendment to the New Consensus on Monetary Policy’, *Metroeconomica*,, **57** (2), pp. 165-192.

——— (2009): ‘Taming the New Consensus: Hysteresis and Some Other Post Keynesian Amendment’, pp. 191-213. in Fontana and Setterfield (2009).

——— (2014): *Post-Keynesian Economics: New Foundations*, Cheltenham: Edward Elgar.

Layard, R., S. Nickell and R. Jackman (2005)): *Unemployment: Macroeconomic Performance and the Labour Market*, Oxford: Oxford University Press.

León-Ledesma, M. and P. Thirlwall (2002): ‘The Endogeneity of the Natural Rate of Growth’, *Cambridge Journal of Economics*, **26** (4), pp. 441-459.

Lindbeck, A. and D. Snower (1986): ‘Wage setting, Unemployment, and Insider-outsider Relations’, *American Economic Review*, **76** (2), pp. 235-239.

Michl, T. (2018): ‘Hysteresis in a Three-Equation Model’, *Eastern Economic Journal*, **44** (2), pp. 305-322.

Mohun, S. and R. Veneziani (2008): ‘Goodwin Cycles and the U.S. Economy, 1948-2004’, pp. 107-130. in Flaschel and Landesmann (2008).

Proaño, C., P. Flaschel, H-M. Krolzing and B. Mamadou (2011): ‘Monetary Policy and Macroeconomic Stability under Alternative Demand Regimes’, *Cambridge Journal of Economics*, **35** (3), pp. 569-585.

Rowthorn, R. (1999): ‘Unemployment, Wage Bargaining and Capital-labor Substitutuion’, *Cambridge Journal of Economics*, **23** (4), pp. 413-425.

Sasaki, H. (2013): ‘Cyclical Growth in a Goodwin-Kalecki-Marx Model’, *Journal of Economics*, **108** (2), pp. 145-171.

Setterfield, M. (2009): ‘Path Dependency, Hysteresis and Macrodynamics’, pp. 37-79. in Arestis and Sawyer (2009).

Skott, P. (2005): 'Fairness as a Source of Hysteresis in Employment and Relative Wages', *Journal of Economic Behavior and Organization*, **57** (3), pp. 305-331.

Stockhammer, E. (2004): 'In There an Equilibrium Rate of Unemployment in the Long Run?', *Review of Political Economy*, **16** (1), pp. 59-77.

——— (2008): 'Is the NAIRU Theory a Monetarist, New Keynesian, Post Keynesian or a Marxist Theory', *Metroeconomica*, **59** (3), pp. 479-510.

——— (2011): 'Wage norms, capital accumulation, and unemployment: a post-Keynesian view', *Oxford Review of Economic Policy*, **27** (2), pp. 295-311.

Stockhammer, E. and J. Michell (2016): 'Pseudo-Goodwin Cycles in a Minsky Model', *Cambridge Journal of Economics*, **41** (1), pp. 105-125.

Storm, S. and C. W. M. Naastepad (2017): 'Bhaduri-Marglin Meet Kaldor-Marx: Wages, Productivity, and Investment', *Review of Keynesian Economics*, **5** (1), pp. 4-24.

Figures and Tables

Table 1: Results for comparative static analysis

	WLD						PLD					
	β	κ_p	κ_w	$\bar{m}_C$	$\bar{m}_W$	s	β	κ_p	κ_w	$\bar{m}_C$	$\bar{m}_W$	s
m^*	−	+	−	+	+	0	−	+	−	+	+	0
y^*	+	−	+	−	−	−	−	+	−	+	+	−
y^{n*}	+	−	+	−	−	−	−	+	−	+	+	−

Table 2: Comparison of possible scenarios with Flaschel (2008) and Proaño et al. (2011)

Demand	Distribution	Flaschel (2008) and Proaño et al. (2011)	Our results
PLD	LML	Stable	P.1: Stable
WLD	GML	Stable	P.2: Stable
PLD	GML	Unstable if strong feedback works between demand and distribution dynamics	P.3(1): Unstable (resp. stable, cycle) when the flexibility of change in capitalists' norm to change in the output gap (i.e. absolute value of θ_m) is large (resp. small, intermediate). When capitalists' norm to output gap changes at a certainly flexible rate, P.3(2): Unstable (resp. stable, cycle) if the effect of hysteresis in natural output δ is small (resp. large, intermediate). P.3(3): Unstable (resp. stable, cycle) if the speed of quantity adjustment ϕ is large (resp. small, intermediate).
WLD	LML	Unstable if strong feedback works between demand and distribution dynamics	P.4(1): Unstable (resp. stable, cycle) when the flexibility of change in workers' norm to change in the output gap (i.e. absolute value of θ_m) is large (resp. small, intermediate). When workers' norm to output gap changes at a certainly flexible rate, P.4(2): Unstable (resp. stable, cycle) if the effect of hysteresis in natural output δ is small (resp. large, intermediate). P.4(3): Unstable (resp. stable, cycle) if the speed of quantity adjustment ϕ is large (resp. small, intermediate).

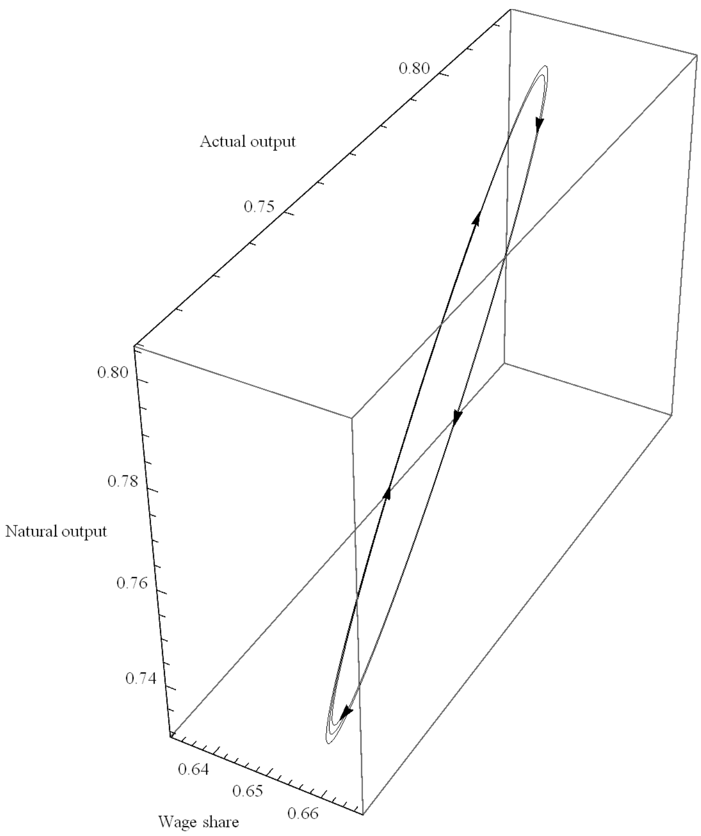

Figure 1: Solution path in the PLD regime with the GML distribution regime

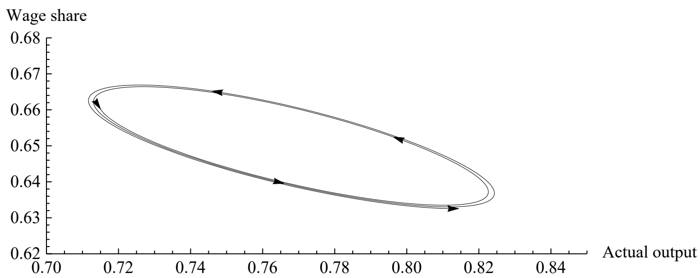

Figure 2: Anticlockwise cycles in the PLD regime with the GML distribution regime

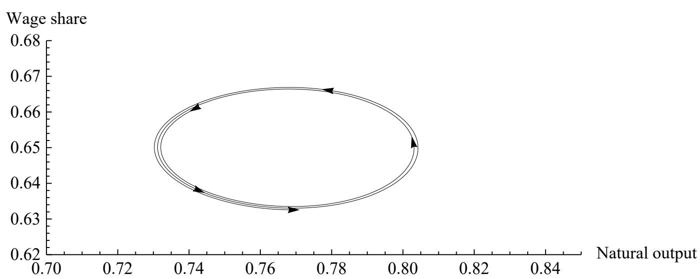

Figure 3: Anticlockwise cycles in the PLD regime with the GML distribution regime

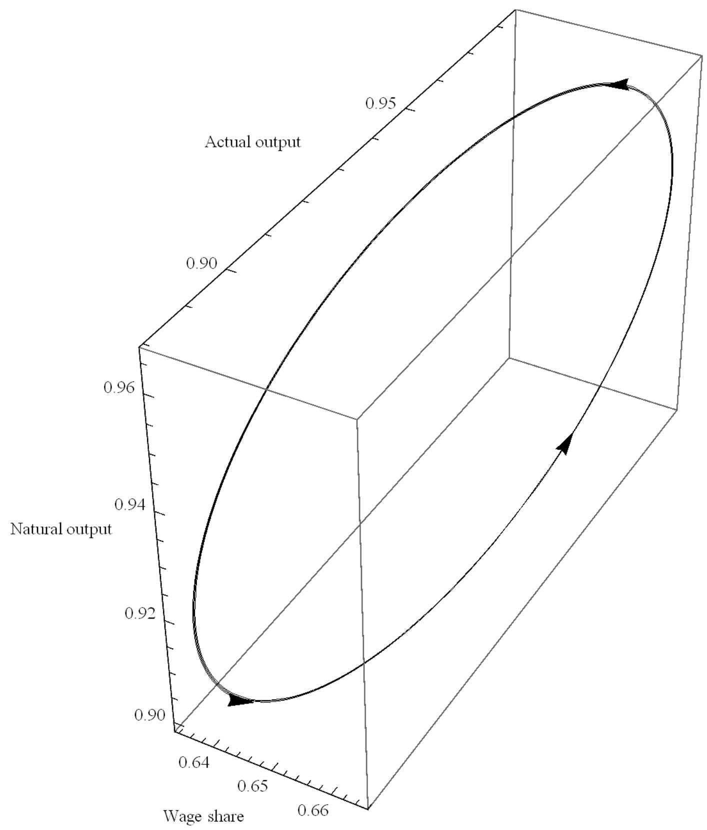

Figure 4: Solution path in the WLD regime with the LML distribution regime

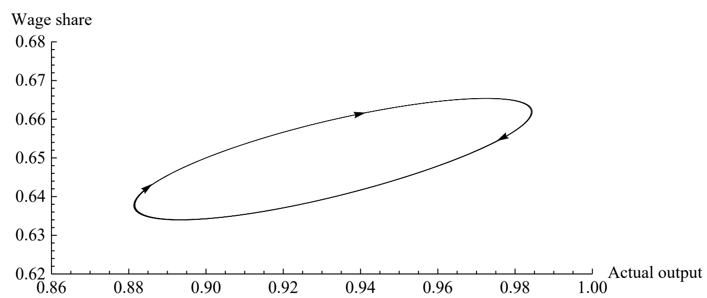

Figure 5: Clockwise cycles in the WLD regime with the LML distribution regime

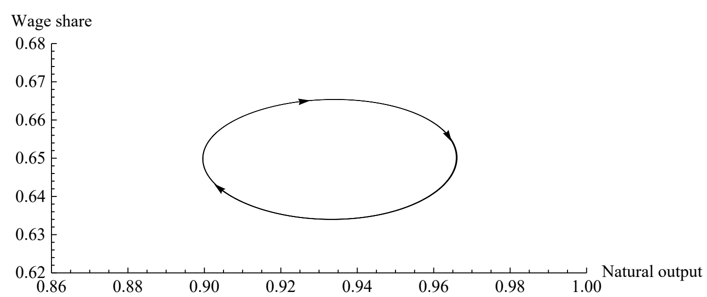

Figure 6: Clockwise cycles in the WLD regime with the LML distribution regime