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Exchange rate dynamics, balance sheet effects, and capital flows. A Minskyan model of emerging market boom-bust cycles

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Abstract

The paper provides a Minskyan open economy model of endogenous boom-bust cycles in emerging market economies, which explains the empirically observed procyclicality of exchange rates and the countercyclicality of the trade balance. It highlights the interaction of exchange rate dynamics and balance sheets. Currency appreciation makes firm balance sheets with foreign currency debt more solid. Throughout the resulting boom phase, the current account position worsens. Pressures on the domestic exchange rate mount until the currency depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their nominal net worth. If capital inflows are driven by exogenous risk appetite, fluctuations can assume the form of shock-independent endogenous cycles. An increase in risk appetite raises the volatility of the cycle. Financial account regulation can reduce macroeconomic volatility, but the larger the risk appetite, the more financial account regulation is required to achieve this.

Keywords: Business cycles, emerging market economies, balance sheet effects, Minsky

JEL Codes: E11, E12, F36, F41

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1 Introduction

Business cycle research has established that macroeconomic fluctuations in output, exchange rates, and the current account are significantly stronger in emerging market economies (EMEs) compared to industrial economies (Agénor et al., 2000; Calderón and Fuentes, 2014; Uribe and Schmitt-Grohé, 2017, chap. 1). The severity of business cycles in EMEs has led some authors to speak of boom-bust cycles (Williamson, 2005; Reinhart and Reinhart, 2009; Herr, 2013). A common pattern is the coincidence of large capital inflows, exchange rate appreciation, and widening current account deficits during the boom period, and capital outflows, currency depreciation, and current account reversals during the bust. The trade balance is thus strongly countercyclical, while nominal exchange rates - unlike in most industrial economies - behave procyclical (Cordella and Gupta, 2015).

A further characteristic of EMEs is the significance of foreign-currency borrowing (Eichengreen et al., 2007). Due to their subordinate position in the international currency hierarchy (Andrade and Prates, 2013; Kaltenbrunner, 2015), EMEs can typically only borrow in foreign currency, which creates currency mismatch on balance sheets. Currency mismatch exposes economic units to exchange rate risk: a depreciation of the domestic currency raises the value of the units foreign currency debt, reducing its net worth. This, in turn, can lead to a drying up of financial sources or outright bankruptcy. Indeed, such balance sheet effects played a significant role in the East Asian crisis in the late 1990s (Kregel, 1998; Arestis and Glickman, 2002). A rich set of macroeconomic models with balance sheet effects has developed ever since (Krugman, 1999; Céspedes et al., 2004; Delli Gatti et al., 2007; Charpe et al., 2011, chap. 2).

Econometric studies confirm that depreciations are more likely to have a negative effect on output and growth in countries with large external debt burdens (Galindo et al., 2003; Bebczuk et al., 2007; Blecker and Razmi, 2008; Kearns and Patel, 2016). The fact that the share of foreign currency-denominated liabilities on the balance sheets of non-financial corporations has increased sharply in many EMEs in the last decade (IMF, 2015, chap. 3; Feyen et al., 2015; Chui et al., 2018) indicates that balance sheet effects will remain an important feature of EM business cycles.

While both phenomena, procyclical exchange rates and balance sheet effects, have been studied in isolation, there is much less research on how they interact. The theoretical literature has not fully acknowledged the fact that foreign-currency debt not only has contractionary effects during depreciations, but also expansionary effects when the currency appreciates. The focus of models with balance sheet effects is typically on currency crises, not business cycles. Business cycle models, in turn, have not yet explained the procyclicality of nominal exchange rates in EMEs in conjunction with foreign currency-denominated business debt.

In this paper, we examine the role of procyclical exchange rates and corporate sector balance sheet effects in EM business cycle dynamics. We argue that a Minskyan framework is ideal for this purpose due to its focus on business investment as the most unstable component of aggregate demand, the role of financial factors in investment decisions, and idiosyncratic investor behaviour (Minsky, 2008 [1975], 2016 [1982]). In this framework, business cycles may arise endogenously through the interaction of the real side of the economy with the financial side. We supplement the Minskyan framework with a structuralist perspective, which highlights the role of the balance-of-payments for the business cycle in open developing economies (Ocampo et al., 2009; Ocampo, 2016).

The contribution of this article is threefold. First, it provides a simple dynamic Minskyan model of EM boom-bust cycles. Thereby, it formalises some mechanisms highlighted in the non-formal structuralist and Minskyan literature on boom-bust cycles in EMEs (Palma, 1998; Taylor, 1998; Arestis and Glickman, 2002; Cruz et al., 2006; Frenkel and Rapetti, 2009; Harvey, 2010; Agosin and Huaita, 2011; Kaltenbrunner and Painceira, 2015). Second, it proposes a business cycle mechanism that not only captures the well-known countercyclicality of the trade balance but also explains the less discussed procyclicality of nominal exchange rates in EMEs. The basic mechanism is as follows: Currency appreciations improve the balance sheets of foreign-currency indebted firms and induce an investment boom. Throughout the boom phase, the exchange rate continues to appreciate, and the current account position worsens. Pressures on the domestic exchange rate mount until it eventually depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their nominal net worth. Third, the model combines the Minskyan notion of endogenous cycles with the structuralist argument that domestic business cycles in EMEs are strongly influenced by external shocks. While the model generates shock-independent endogenous cycles, it shows that the volatility of these endogenous cycles is affected by exogenous capital flow shocks. We find that an exogenous increase in risk appetite increases the amplitude of cycles and that financial account regulation can reduce macroeconomic volatility. Interestingly, the higher the risk appetite, the higher the degree of regulation of the financial account that is needed to reduce volatility.

The article is structured as follows. The second section presents some stylized facts on the cyclical behaviour of exchange rates and real activity in EMEs. It briefly introduces structuralist-Minskyan theories of EM business cycles. The third section develops a simple Minskyan model that explains the stylized facts highlighted in the second section. Its dynamic properties are first examined under the simplifying assumption of a constant external debt ratio, and then under the case of a flexible debt ratio that is driven by exogenous risk appetite. The section derives the conditions under which endogenous cycles emerge and considers financial account regulation as a stabilisation policy. The last section concludes.

2 Stylized facts and the Minskyan approach to business cycles

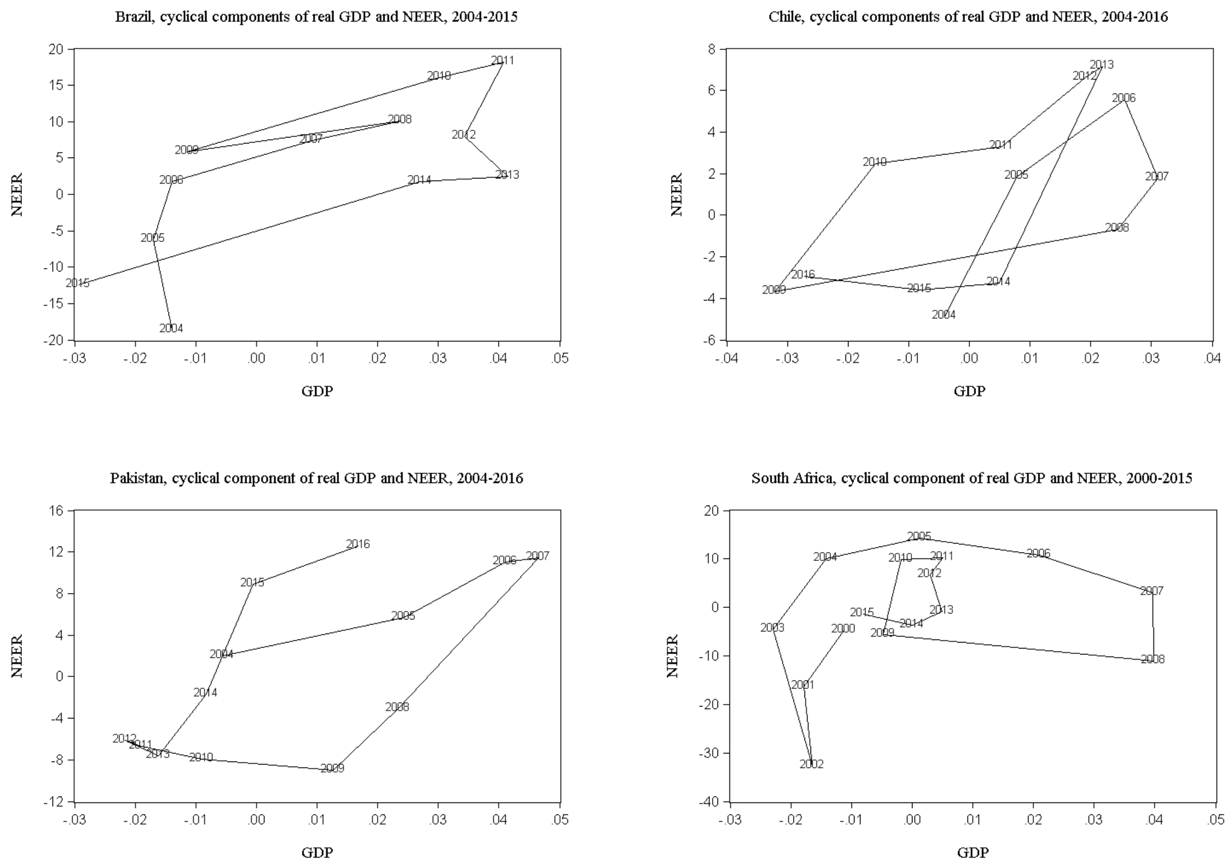
While a lot of the literature on financial crises in EMEs focuses on fixed exchange rate regimes - often under the impression of the 1997-98 Asian financial crisis -, the majority of EMEs today follows some form of floating (Ghosh et al., 2015). The importance of flexible exchange rates for business cycle dynamics has been pointed out in the policy literature, which suggests that exchange rate dynamics and balance sheet effects may increase macroeconomic fluctuations:

> 'In economies that have net external liabilities denominated in foreign currencies, exchange rate fluctuations [...] are pro-cyclical: real appreciation during the boom generates capital gains, whereas depreciation during crises generates capital losses. [...] The exchange rate fluctuations are themselves a result of some of the same forces that give rise to the economic fluctuations: capital inflows can fuel real exchange rate appreciation, at the same time that they lead to a private spending boom, while depreciation may have the opposite effects. In broader terms, in open developing economies the real exchange rate is an essential element in the dynamics of the business cycles' (Stiglitz et al., 2006, p.117).

This link between exchange rate dynamics and balance sheets has also been termed 'the risk-taking' or 'financial channel' of exchange rates (Shin, 2015, 2016; Kearns and Patel, 2016). If the exchange rate of a global funding currency (typically the US-dollar) depreciates against the domestic currency, balance sheets of economic units with currency mismatch improve. This stimulates domestic investment and can lead to a boom. Conversely, an appreciation of the funding currency can depress domestic economic activity. The implied procyclicality of exchange rates in EMEs has been documented by recent empirical studies (Calderón and Fuentes, 2014; Cordella and Gupta, 2015).

To gain further insights into the nature of output and exchange rate cycles, we plot the cyclical component of the nominal effective exchange rate against the cyclical component of log real GDP for four selected EMEs. Figure 1 depicts the most recent one or two complete cycles that could be found in the data.

Figure 1: Phase diagrams for Brazil, Chile, Pakistan, and South Africa



Data sources: IFS (IMF), World Bank.
Notes: NEER: Nominal effective exchange rate (index, 2010 = 100). GDP: Natural logarithm of real gross domestic product. Cyclical components were extracted using the Hodrick-Prescott filter with a smoothing parameter of 100. An increase in the NEER indicates an appreciation of the domestic currency. The cyclical component of log real GDP is the percent deviation from trend.

All four countries exhibit clockwise cycles between output and the nominal effective exchange rate, with boom periods associated with currency appreciation and busts accompanied by depreciation. Converted into the more conventional definition of the exchange rate as domestic currency units per foreign unit, where an increase in the exchange rate indicates a depreciation, we find counter-clockwise cycles in economic activity and the exchange rate. A counter-clockwise direction of cycles indicates that peaks and troughs in economic activity precede peaks and troughs in the exchange rate. This pattern points to predator-prey dynamics where economic activity takes on the role of the prey that is being squeezed by a rising exchange rate (i.e. depreciation), while the currency behaves like a predator that grows together with the prey. Stockhammer et al. (2018) argue that such a real-financial interaction mechanism lies at the heart of theories of financial-real cycles such as Hyman Minsky's Financial Instability Hypothesis (Minsky, 2008 [1975], 2016 [1982]; for a survey see Nikolaidi and Stockhammer, 2017).

A key aspect of the Minskyan approach is the claim that financial fragility increases during economic booms to the point where it spills over to the real economy and turns them into busts. Every boom thus prepares its own bust, and it is finance that plays a decisive role in driving these cycles. In Minskys view (2008 [1975]), corporate investment is the most unstable component of aggregate demand and is thus, together with business debt, at the heart of the cycle. This accords well with the empirical fact that in both emerging and advanced economies investment is typically more than three times as volatile as output and about twice as volatile as consumption (Uribe and Schmitt-Grohé, 2017, chap. 1).

While Minskys approach to financial fragility is tailored to advanced economies, structuralist economists who focus on open developing economies are often sympathetic to Minskyan ideas (Ocampo et al., 2009; Ocampo, 2016). They highlight the importance of the balance-of-payments for the propagation of external financial shocks to domestic business cycle dynamics. A crucial factor in this approach are erratic capital flows that induce private spending booms, which in turn make the economy more vulnerable due to currency mismatch on balance sheets and growing current account deficits. There is a rich non-formal branch of the Minskyan literature that has integrated structuralist ideas into the Minskyan framework (Taylor, 1998; Arestis and Glickman, 2002; Cruz et al., 2006; Frenkel and Rapetti, 2009; Harvey, 2010; Agosin and Huaita, 2011; Kaltenbrunner and Painceira, 2015).

There are only a few attempts to formalise some of the mechanisms discussed in the Minskyan open economy literature (Foley, 2003; Taylor, 2004, chap. 10; Gallardo et al., 2006; Botta 2017). Foley (2003) shows how conservative monetary policy in developing countries can turn positive shocks to exuberance into financial fragility through rising interest rates. Taylor (2004a, chap. 10) presents a model in which risk premia on interest rates are sensitive to the stock of foreign reserves. Boom periods lead to a loss of foreign reserves, which pushes up interest rates to the point where the economy contracts and the current account reveres. Both models, however, completely abstract from nominal exchange rate dynamics. Gallardo et al. (2006) introduce exchange rate dynamics but focus on the interaction between exchange rates and foreign reserve dynamics to assess under which conditions a fixed exchange rate regime is viable. Their model does not provide an account of endogenous cycles. Botta (2017) presents a model in which the nominal exchange rate interacts with external debt in a cyclical manner. Capital inflows lead to nominal exchange rate appreciation. As the stock of external debt successively increases during the boom, foreign investors get anxious and curb the supply of foreign finance. Thus, the nominal exchange rate plays a key role for the dynamics, but the real side of the economy is not explicitly modelled.

The studies that have the largest similarity to ours are the non-Minskyan models in Schneider and Tornell (2004), Korinek (2011) and Müller-Plantenberg (2015), which propose an explanation of the procyclicality of exchange rates in EME business cycles. Schneider and Tornell (2004) offer a microeconomic model in which bailout guarantees encourage currency mismatch in the non-tradables sector, which can generate boom-bust cycles through balance sheet effects. In Korinek (2011), productivity shocks in conjunction with balance sheet effect in the household sector generate procyclical real exchange rates. However, both models disregard capital flows and reduce the exchange rate to a relative price. Thereby, the models fail to capture the role of capital flows in nominal exchange rate determination and their interaction with foreign-currency debt in the corporate sector. Müller-Plantenberg (2015) models the exchange rate as driven by capital and trade flows, but cycles are exogenously imposed, and balance sheet effects are absent.

3 A Minskyan open economy model

We build a Minskyan open economy model with flexible exchange rates and analyse it in two steps. First, under the simplifying assumption of a constant external debt ratio, damped oscillations in the exchange rate and capital accumulation arise. Second, under a flexible debt ratio that is determined by an exogenous debt target, the dynamics turn into shock-independent endogenous cycles.

3.1 The core model

The goods market of the model is kept simple and resembles other Minsky models (Foley, 2003; Charles, 2008). The economy consists of one sector that produces a homogenous good using capital and labour, which can be used for consumption and investment. For simplicity, there is no depreciation of the capital stock and no overhead labour. The technical coefficients of labour and capital are assumed to be constant, so there is no substitution between capital and labour, and no technical progress. Production is demand-driven, so there is Keynesian quantity adjustment to changes in demand. For the sake of simplicity, there is no fiscal policy and no inflation. We the fixed domestic and foreign price level to unity. Furthermore, there is no substitution between the imported good and the domestic good. The economy is small and open, so that all foreign variables are exogenously given. The exchange rate is flexible but adjusts only sluggishly due to restrictions on financial account transactions.

Equilibrium conditions[1]

Aggregate demand (Y^D) in the open economy is composed of consumption (C), investment (I), and net exports ($X - sM$) minus interest payments abroad ($si^f D^f$), where s is the spot exchange rate defined as units of domestic currency per unit of foreign currency so that an increase in s corresponds to a currency depreciation.[2] i^f is the interest rate on foreign-currency denominated bonds (D^f). Equilibrium in the goods market requires that national income (Y) equals aggregate demand:

$$Y = Y^D \equiv C + I + X - sM - si^f D^f. \tag{1}$$

The balance-of-payments (BoP), on the other hand, is given by

$$(X - sM - si^f D^f) + (s\dot{D}^f) = s\dot{Z}, \tag{2}$$

where the first term in brackets represents the current account, i.e. the trade surplus minus interest payments abroad, and the second term is the financial account, i.e. net capital inflows. A surplus in the current (financial) account that is not fully matched by a deficit in the financial (current) account leads to an accumulation of foreign reserves ($s\dot{Z}$). Equilibrium in the balance of payments is given when reserve changes are zero:

$$(X - sM - si^f D^f) + (s\dot{D}^f) = s\dot{Z} = 0. \tag{3}$$

Budget constraints

The economy consists of workers who earn wages (W) and firms who earn profits net of interest payments ($R - si^f D^f$), so that $Y \equiv W + R - si^f D^f$. Workers consume their entire income which exclusively consists of wages. Their budget constraint is thus always satisfied. Firms can finance their investment expenditures through profits net of interest payments and via floating foreign currency-denominated bonds abroad ($s\dot{D}^f$) - a practice that has taken place on a large scale in emerging market economies in the last decade (IMF, 2015, chap. 3; Chui et al., 2018). We furthermore make the simplifying assumption that if the spending and financing decisions of the firm sector do not add up, firms will be provided by an interest-free foreign-currency loan by the monetary authority ($s\dot{L}$). Note that foreign-currency bonds (D) and domestic loans (L) are the only variables in this model that are

[1]The superscript f denotes foreign variables. The time derivative of a variable x is denoted by $\frac{dx}{dt} = \dot{x}$. A list of symbols can be found in Appendix A1.

[2]Notice the difference to the empirically widely used nominal effective exchange rate in Figure 1, where an increase indicates an appreciation.

measured in foreign currency. The aggregate firm budget constraint reads:

$$0 = R - si^f D^f + s\dot{D}^f + s\dot{L} - I. \tag{4}$$

If only workers consume, and they consume their entire wage income, aggregate consumption is given by:

$$C = W = Y - (R - si^f D^f). \tag{5}$$

From (5) and (1), we get:

$$I - R = sM - X. \tag{6}$$

Thus, whenever investment exceeds profits, the economy runs a trade deficit. Substitution of (6) into (2) yields:

$$R - I - si^f D^f + s\dot{D}^f = s\dot{Z}. \tag{7}$$

Eq. (7) in conjunction with the budget constraint of the firm sector (1) implies that $s\dot{Z} = s\dot{L}$. Thus, the BoP and the budget constraint of the firm sector coincide: whenever firms financing needs are not fully covered by the foreign-currency bonds, the monetary authority has to draw on its foreign reserves to fill the financing gap. In BoP equilibrium($s\dot{Z} = 0$), there is no need for such external funding. As we focus on the short- to medium-run, we will assume that the domestic economy commands sufficient foreign reserves for this purpose, and that changes in the stock of reserves have negligible consequences for the domestic macroeconomy.

Aggregate demand and goods market equilibrium

We now scale all variables by the capital stock (K) and use lower case letters henceforth. From the assumption that workers consume all their income, we have the following consumption function:

$$c \equiv \frac{C}{K} = u - (r - si^f \lambda), \qquad u \equiv \frac{Y}{K}; r \equiv \frac{R}{K}; \lambda \equiv \frac{D^f}{K}. \tag{8}$$

We use a simplified investment function with the rate of profit and the external debt ratio measured in domestic currency as the only two arguments.

$$g^d = g_0 + g_r r - g_s s\lambda, \qquad g_r \in (0, 1); g_s > 0. \tag{9}$$

The use of the profit rate as a measure aggregate demand and internal funds in the investment function is conventional in post-Keynesian models.[3] However, inclusion of an external debt in foreign currency ratio is not. The economic rationale for this is the presence of foreign currency-denominated debt on the balance sheets of emerging market firms. Changes in the value of the domestic currency thus exert a strong impact on the net worth of firms, which in turn can affect investment demand. From a Minskyan perspective, the link between net worth and investment is due to borrowers risk, which is the subjective risk of illiquidity and bankruptcy of the entrepreneur due to the possibility of lower than expected cash flows despite fixed payment obligations (Minsky, 2008 [1975], pp.104-110). Note that price-competitiveness effects of the exchange rate affect desired investment expenditures only via their impact on the profit rate which, in equilibrium, is positively related to the exchange rate. Our specification reflects the empirical finding in Kearns and Patel (2016) that in emerging markets, the depreciation of debt-weighted exchange rates depresses investment which indicates that balance-sheet effects outweigh price-competitiveness effects.

Lastly, the net exports ratio is determined by the foreign profit rate (r^f), the domestic profit rate, and the exchange rate:

$$b \equiv \frac{X - sM}{K} = b_{r^f} r^f - b_r r + b_s s, \qquad b_r \in (0,1); b_{r^f} > 0. \tag{10}$$

The foreign rate of profit captures export demand [4], whereas the domestic rate of profit measures import demand. Whether the effect of an increase in the real exchange rate on the trade balance is positive depends on whether the Marshall-Lerner condition (MLC) holds ($b_s > 0$). [5] As the MLC is empirically often not satisfied due to low exchange rate elasticities (Bahmani et al., 2013), we will assume that b_s has a low absolute magnitude and can assume positive or negative values.

Goods market equilibrium is established by quantity adjustment, rendering the profit

[3]The profit rate is, by definition, composed of the rate of capacity utilisation ($u \equiv \frac{Y}{Y^P}$), the profit share ($\pi \equiv \frac{R}{Y}$) and the output-capital ratio ($\frac{1}{v} \equiv \frac{Y^P}{K}$): $r \equiv \frac{R}{K} \equiv \frac{Y}{Y^P}\frac{R}{Y}\frac{Y^P}{K} \equiv \frac{u\pi}{v}$. Since we assume the profit share and the capital-output ratio to be constant, the profit rate is monotonically related to the rate of capacity utilisation, which in turn measures aggregate demand. We abstract from interest payments in the investment function to keep the model simple and to focus on stocks (i.e. balance sheet) effects. For a Minskyan model that examines the role of interest payments in EMEs, see Foley (2003).

[4]Inclusion of the foreign rate of profit requires that the domestic and foreign capital stock grow at the same rate.

[5]The linear specification is in fact a special case that can be considered as an approximation to a more generic net exports function $NX = X(s) - sM(s)$, which is inherently nonlinear in s.

rate endogenous:

$$r^* = \frac{g + b_{r^f} r^f + b_s s}{1 + b_r} = \theta(g + b_{r^f} r^f + b_s s), \qquad \text{where} \quad \theta \equiv \frac{1}{1 + b_r} \in (0, 1). \tag{11}$$

Investment dynamics
As in Charles (2008), we introduce finite adjustment of the actual rate of capital accumulation to the desired one:

$$\dot{g} = \gamma(g^d - g), \qquad \gamma > 0. \tag{12}$$

From (9), (11), and (12) we obtain the law of motion of the investment rate:

$$\dot{g} = \gamma[g(\theta g_r - 1) + s(\theta b_s g_r - g_s \lambda) + g_0 + \theta b_{r^f} r^f g_r]. \tag{13}$$

Balance-of-payments equilibrium and exchange rate dynamics
Eq. (11) in conjunction with the normalised BoP equilibrium condition yields:

$$\underbrace{(\theta b_{r^f} r^f + \theta b_s s + \frac{s\dot{D}^f}{K})}_{\text{FX supply}} - \underbrace{(s i^f \lambda + \theta b_r g)}_{\text{FX demand}} = 0. \tag{14}$$

The BoP can be interpreted as a market-clearing condition for the FX market, since it contains all sources of supply and demand for foreign currency (Federici and Gandolfo, 2012). The positive elements constitute sources of supply (foreign import demand for domestic goods and net capital inflows), whereas the negative elements represent sources of demand for foreign currency (interest payments on foreign debt and domestic import demand). Whenever equality (14) is violated, there is excess demand or supply in the FX market.

Recall that we assume that BoP-disequilibria lead to changes in reserves (see Eq. 7), which ensures that the BoP identity holds and that the firm budget constraint is satisfied. From the perspective of the BoP as an FX market-clearing condition, changes in reserves can be regarded as interventions in the FX market. In a completely unregulated FX market without FX intervention, the exchange rate would instantaneously adjust to clear the market. Interventions take away some of these adjustment pressure on the exchange rate. We assume that BoP-disequilibria lead to gradual changes in the exchange rate, which only sluggishly responds to pressures in the FX market:

$$\dot{s} = \mu[g\theta b_r + s(i^f \lambda - \theta b_s - \frac{s\dot{D}^f}{K}) - \theta b_{r^f} r^f], \qquad \mu > 0. \tag{15}$$

The speed of adjustment of the exchange rate, μ, can be interpreted as the degree of deregulation of the financial account (Bhaduri, 2003).[6] Exchange rate dynamics are thus determined by the rate of capital accumulation (through its effect on the trade balance), as well as by the exchange rate itself (through its influence on net exports and the value of interest payments and capital flows). From the time derivative of the external debt-to-capital ratio $\frac{d\lambda}{dt} \equiv \dot{\lambda} \equiv \frac{\dot{D}^f}{K} - g\lambda$, we get $\frac{\dot{D}^f}{K} \equiv \dot{\lambda} + g\lambda$. Substituting this expression into (15) yields our law of motion for the exchange rate:

$$\dot{s} = \mu[g(\theta b_r + s\lambda) + s(i^f\lambda - \dot{\lambda} - \theta b_s) - \theta b_{r^f} r^f]. \tag{16}$$

3.2 Cyclical dynamics under a constant external debt ratio

To get a thorough understanding of the interaction mechanism between investment and the exchange rate, we first focus on a special case and suppose that the external debt ratio remains constant over time. A constant external debt ratio requires external debt to change proportionally to the rate of investment:

$$\frac{\dot{D}^f}{K} = \lambda g. \tag{17}$$

Under this assumption, we have $\dot{\lambda} = 0$, so the law of motion of the exchange rate (18) reduces to:

$$\dot{s} = \mu[g(\theta b_r + s\lambda) + s(i^f\lambda - \theta b_s) - \theta b_{r^f} r^f]. \tag{18}$$

The laws of motion of capital accumulation (13) and the exchange rate (16) then constitutes a non-linear 2D dynamic system. The Jacobian matrix of the system is:

$$\mathbf{J}(g,s) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s gr - g_s\lambda \\ \mu(\theta b_r - s\lambda) & \mu[(i^f - g)\lambda - \theta b_s] \end{bmatrix}. \tag{19}$$

With respect to the fixed points of the system, we state the following proposition:

Proposition 1 *The dynamic system given by (13) and (16) has at most two fixed points.*

Proof. See Appendix A3.

[6] Montecino (2018) provides econometric evidence that capital controls can slow down the adjustment speed of the exchange rate towards its long-run level.

In the following, we will evaluate the stability and dynamics of the system in the neighbourhood of a fixed point (g^*, s^*), where both g^* and s^* are positive. We are interested in the conditions under which the system gives rise to oscillations.

Oscillations

Element J_{11} constitutes a version of the Keynesian stability condition. We follow the approach of Kaleckian Minsky models[7] and assume that the stability condition is satisfied, so that $J_{11} < 0$. The sign of J_{12} is ambiguous. The first term captures the effect of a depreciation on the trade balance, which is mediated by the exchange rate elasticity of net exports (b_s) and the sensitivity of investment with respect to profitability (g_r). The second term captures the contractionary balance sheet effect of a currency depreciation on investment (g_s). Based on our previous discussion of the empirical evidence (Bahmani et al., 2013; Kearns and Patel, 2016), we suppose that the second effect outweighs the first effect: balance sheet effects are typically strong in EMEs, while exchange rate elasticities are low ($J_{12} < 0$).

The sign of the element J_{21} is positive only if$\theta b_r > s^*\lambda$. The first term captures the fact that an increase in the rate of investment leads to a trade deficit, which in turn creates excess demand in the foreign exchange market. This effect is attenuated by the growth in the capital stock which raises the supply of foreign credit. We assume $J_{21} > 0$, which requires a relatively large propensity to import, which is typically the case in developing countries due to a low share of domestically produced manufactured goods. If the steady state external debt ratio expressed in domestic currency becomes too large, this assumption might be violated. Lastly, J_{22} determines the stability of the BoP. A destabilising element is interest payments on foreign currency debt. Capital accumulation and the exchange rate sensitivity of net exports are stabilising factors. Given that the Marshall-Lerner condition has been frequently rejected in empirical work (Bahmani et al., 2013), and that developing countries often have to cope with high debt service burdens (Frenkel, 2008), we will focus on the case where $J_{22} > 0$.[8]

We then have the following sign structure of the Jacobian matrix:

$$sgn[\mathbf{J}(g,s)] = \begin{bmatrix} - & - \\ + & + \end{bmatrix}.$$

Oscillations arise when the Jacobian matrix of the system exhibits complex eigenvalues. A sufficient condition for complex eigenvalues in a 2D system is $(J_{11} - J_{22})^2 + 4J_{12}J_{21} < 0$ (see

[7]See Ryoo (2013) and Nikolaidi and Stockhammer (2017) on the distinction between Kaleckian Minsky models with stable and Kaldorian Minsky models with unstable goods markets.

[8]Our qualitative findings do not hinge on this assumption.

Appendix A2). This requires

$$\underbrace{\{\gamma(\theta g_r - 1) - \mu[(i^f - g^*)\lambda - \theta b_s]\}^2}_{>0} + 4\mu\gamma \underbrace{(\theta b_r - s^*\lambda)}_{>0} \underbrace{(\theta b_s g_r - g_s\lambda)}_{<0} < 0,$$

which is satisfied when the trade balance responds strongly to domestic demand ($b_r \gg 0$), which we consider to be likely, and when investment is very sensitive towards the external debt in foreign currency ($g_s \gg 0$), which is typical for developing countries where balance sheet effects have been shown to be strong.

Stability

The following proposition states the conditions under which the system is stable:

Proposition 2 *The dynamic system given by (13) and (16) is asymptotically stable if the following conditions are both satisfied:*
(i) $i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{\gamma(1-\theta g_r)}{\mu\lambda}$,
(ii) $i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{(\theta b_r - s^*\lambda)(\theta b_s g_r - g_s\lambda)}{(\theta g_r - 1)\lambda}$.
Depending on whether $\frac{\gamma(1-\theta g_r)}{\mu\lambda} \gtreqless \frac{(\theta b_r - s^*\lambda)(\theta b_s g_r - g_s\lambda)}{(\theta g_r - 1)\lambda}$, *either the first or the second inequality is binding.*

Proof. See Appendix A4.

For given structural parameters of the domestic economy, the system is thus stable if the exogenous foreign interest rate does not exceed a critical threshold. This is more likely to be the case, when the economy exhibits a high steady state growth rate or when the hypothesised interaction mechanism between the exchange rate and the growth rate, i.e. $J_{12} < 0, J_{21} > 0$, is sufficiently strong. In the following, we will assume that the stability condition is satisfied.

Isoclines and dynamic trajectories

To examine the behaviour of the system graphically, we find the isoclines of the system to be:

$$s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{rf} r^f g_r}{g_s\lambda - \theta b_s g_r}; \qquad \frac{\partial s^*_{|\dot{g}=0}}{\partial g} = -\frac{J_{11}}{J_{12}} < 0. \tag{20}$$

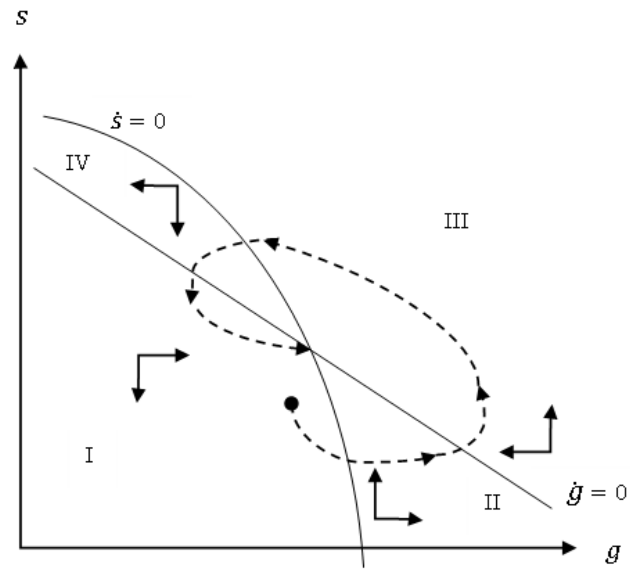
$$s^*_{|\dot{s}=0} = \frac{\theta(b_{rf} r^f - g b_r)}{(i^f - g)\lambda - \theta b_s}; \qquad \frac{\partial s^*_{|\dot{s}=0}}{\partial g} = -\frac{J_{21}}{J_{22}} < 0. \tag{21}$$

While the $\dot{g} = 0$ isocline is linear, the $\dot{s} = 0$ isocline is a rectangular hyperbola with two branches. Its vertical asymptote is $\bar{g} = \frac{i^f\lambda - \theta b_s}{\lambda}$. Given our assumption that $J_{22} = (i^f - g)\lambda -$

$\theta b_s > 0$, we have $g < \bar{g}$. As we impose $|J_{12}J_{21}| > |J_{11}J_{22}|$ for stability, the $\bar{s} = 0$ isocline is steeper than the $\bar{g} = 0$.[9] The corresponding phase diagram is given in Figure 2.

[9] Since $|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}| = |\frac{J_{21}}{J_{22}}|$ and $|\frac{\partial s^*_{|\dot{g}=0}}{\partial g}| = |\frac{J_{11}}{J_{12}}|$, we have $|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}| = |\frac{J_{21}}{J_{22}}| > |\frac{\partial s^*_{|\dot{g}=0}}{\partial g}| = |\frac{J_{11}}{J_{12}}|$.

Figure 2: Adjustment path after an appreciation shock



Note: The isoclines are based on $J_{11} < 0; J_{22} > 22; |J_{11}J_{22}| < |J_{12}J_{21}|$

Starting from an equilibrium position, consider a negative shock to the exchange rate (quadrant I). The currency appreciation feeds into investment demand via an expansionary balance sheet effect and kicks off an investment boom. As a result, profitability and aggregate demand go up. The exchange rate keeps appreciating as it eases the burden of interest payments on foreign debt, which reduces the demand for foreign currency. The currency appreciates until the demand-induced current account deficit becomes large enough to exert pressure on the domestic exchange. There is a short period in which investment keeps accelerating while the currency already depreciates (quadrant II), but this phase is quickly displaced by a sustained contractionary depreciation phase due to balance sheet effects (quadrant III). As a result, the downward trajectory of the current account eventually reverses until the pressure on the exchange rate is removed, and capital accumulation picks up again.

The economy experiences the empirically observed procyclicality of the exchange rate and countercyclicality of the trade balance, where a domestic boom coincides with currency appreciation and trade deficits, while busts are associated with depreciation and current account reversals (Reinhart and Reinhart, 2009; Cordella and Gupta, 2015). This trajectory matches the clockwise direction of the cycles observed in Figure 1 (recall that s is inversely defined compared to the NEER depicted in Figure 1).

3.3 Cyclical dynamics under flexible external debt ratios

3.3.1 Introducing a target external debt ratio

The previous result was established under the simplifying assumption of a constant external debt ratio. This assumption does not correspond well to the experience of large and unstable capital flows to EMEs, highlighted in structuralist literature (Ocampo, 2016; Ocampo et al., 2009). Structuralists argue that capital flows are often driven by external factors rather than domestic fundamentals. This notion is confirmed by a growing empirical literature showing that capital flows to EMEs are primarily determined by global factors such as the liquidity or risk appetite in international financial centres (Nier et al., 2014; Rey, 2015). Recently, Feyen et al. (2015) showed that the surge in external bond issuance by EM firms in the decade after the crisis was predominantly driven by push factors such as expected volatility in the Standard & Poors 500 (the VIX), as well as expansionary US monetary policy. They do not find evidence for a significant role of domestic factors and conclude that 'search-for-yield flows during loose global funding conditions do not strongly discriminate between EMDEs [emerging and developing economies] but are primarily driven by global factors' (Feyen et al. 2015, p.17).

To introduce externally driven capital flows into our model, we employ the notion of a target debt ratio an approach that has recently been used in some closed economy Minsky models (Nikolaidi, 2014; Dafermos, 2018). While this literature explores the dynamics of endogenously changing domestic debt targets, we model the target external debt ratio as exogenous and independent of domestic fundamentals. The target ratio only changes exogenously, for instance in response to changes in risk appetite or liquidity in global financial centres, as found in the empirical literature. Consequently, the actual debt ratio becomes a state variable that varies over time subject to capital inflows and domestic capital accumulation.

Consider a simple adjustment mechanism where capital flows increase whenever the actual debt ratio falls short of the target:

$$\frac{\dot{D}^f}{K} = \delta(\lambda^T - \lambda) \qquad \lambda^T, \delta > 0, \tag{22}$$

where λ^T is the target external debt-to-capital ratio and the adjustment speed of the actual ratio to the desired one. This translates into the following law of motion of the external debt-to-capital ratio:

$$\dot{\lambda} = \delta(\lambda^T - \lambda) - g\lambda. \tag{23}$$

Note that even if the target ratio was met ($\lambda^T = \lambda$), the actual debt ratio would still change, as long as the rate of investment and the actual debt ratio are different from zero. Under normal circumstance, the target is thus never meet and the actual debt ratio will be permanently changing over time.

Using (23) and (16), we obtain:

$$\dot{s} = \mu\{g(\theta b_r + s\lambda) + s[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{r^f} r^f\}. \tag{24}$$

Eqs. (13), (23), and (24) constitute a 3D dynamic system that exhibits an intrinsically nonlinear structure due to valuation effects and normalisation of variables. Notice that this nonlinear structure emerges without having introduced nonlinearities in the behavioural functions. The Jacobian matrix of the system is given by:

$$\mathbf{J}(g, s, \lambda) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s gr - g_s\lambda & -\gamma s g_s \\ \mu(\theta b_r - s\lambda) & \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] & \mu s(i^f + \delta) \\ -\lambda & 0 & -(\delta + g) \end{bmatrix}. \tag{25}$$

With respect to the fixed points of the system, we state following:

Proposition 3 *The dynamic system given by (13), (23), and (24) has at most two fixed points.*

Proof. See Appendix A.5

We will focus on the fixed point where g^*, s^*, and λ^* are all positive.

Dynamic behaviour

We examine if this system can undergo a Hopf bifurcation giving rise to a stable limit cycle (Gandolfo, 1997, chap. 25). A limit cycle is a closed orbit of the variables of a dynamic system around a locally unstable fixed point. In the immediate neighbourhood of the fixed point, the system is unstable and gets pushed away from it. However, rather than exhibiting explosive behaviour, the system eventually reaches a periodic cycle, as it is bounded due to its inherent non-linearity. In contrast to the damped oscillations of the 2D model in the previous section, a limit cycle is periodic and displays persistent shock-independent oscillations.

Given the assumptions made in this paper, the Jacobian matrix evaluated at the positive fixed point has the following sign structure:

$$sgn[\mathbf{J}(g,s,\lambda)] = \begin{bmatrix} - & - & - \\ + & + & + \\ - & 0 & - \end{bmatrix}.$$

We now state the following:

Proposition 4 *If the following conditions simultaneously hold:*
(i) $|J_{11} + J_{33}| > |J_{22}|$
(ii) $|J_{11}J_{33} + J_{12}J_{21}| > |J_{22}(J_{11} + J_{33}) + J_{13}J_{31}|$
(iii) $|J_{33}(J_{11}J_{22} - J_{12}J_{21})| > |J_{31}(J_{12}J_{23} - J_{13}J_{22}|$,
then there exists a critical value of the adjustment speed of the exchange rate, μ_0*, at which the dynamic system in (13), (23), and (24) undergoes a Hopf bifurcation.*

Proof. See Appendix A6.

Economically these conditions require

- that the foreign exchange market is not excessively self-destabilising (i.e. the foreign interest rate must not be too high),
- that the cyclical interaction mechanism between the exchange rate and investment is strong, i.e. strong balance sheet effects coupled with a large propensity to import,
- a moderate steady state external debt ratio,
- the adjustment speed of the exchange rate must exceed a critical value ($\mu_0 > \mu$), i.e. the financial account must be sufficiently open.

These conditions are similar to the conditions needed for damped oscillations in the 2D model. The existence of a self-stabilising debt ratio, however, allows the economy to cope with stronger instability in the foreign exchange market. Moreover, the adjustment speed of the exchange rate now assumes a key role for the dynamics of the model as it induces the Hopf bifurcation.

3.3.2 Endogenous cycles: a numerical illustration

In order to assess whether the limit cycle is stable, we resort to numerical simulation. Consider the following parameterisation:

Table 1: Parameterisation of the 3D model

Parameter	i^f	λ^T	g_0	g_r	g_s	$b_0 r^f$	b_r	b_s	γ	δ
Value	0.08	0.50	0.15	1.3	1.5	0.05	0.7	-0.002	1	1

The foreign interest rate is set at 8%, which roughly corresponds to the time average of the effective yield on the BofA Merrill Lynch Emerging Markets Corporate Plus Index[10] in the decade before the financial crisis (1998-2007). The target debt ratio is set at 50%. Assuming that firms capital stock only consists of debt and equity (i.e. net worth is zero), this implies a target debt-to-equity ratio of 100%.[11] This is in line with foreign debt-to-equity ratios for EME firms in 2007 and 2013 reported in IMF (2015, p.90). The remaining parameters are not directly observable and have to be calibrated. The calibration corresponds to the assumptions made in the text so far. We suppose positive animal spirits (g_0), a moderate sensitivity of investment with respect to demand (g_r),[12] and in line with the evidence in Kearns and Patel (2016) a strong sensitivity of investment with respect to the external debt ratio in foreign currency (g_s). Furthermore, we specify positive external demand for foreign goods ($b_0 r^f$), a relatively large import propensity (b_r), and lastly a negative but low sensitivity of net exports with respect to the exchange rate. This is due to the empirical finding of Bahmani et al. (2013) that the MLC is not satisfied in the majority of empirical studies. The adjustment speeds of investment (γ) and external debt (δ) are set at unity for simplicity.

The system has two fixed points. For the chosen parameterisation, the first fixed point is economically meaningless, as all state variables are negative. We focus on the second equilibrium, which is given by:

$$(g^*, s^*, \lambda^*) = (0.067, 0.245, 0.469).$$

These steady state values imply an average capital stock growth rate of about 6.7%. This

[10]Data were taken from FRED (https://fred.stlouisfed.org/series/BAMLEMCBPIEY). The BofA Merrill Lynch Emerging Markets Corporate Plus Index tracks the performance of US dollar (USD) and Euro denominated emerging markets non-sovereign debt publicly issued within the major domestic and Eurobond markets.

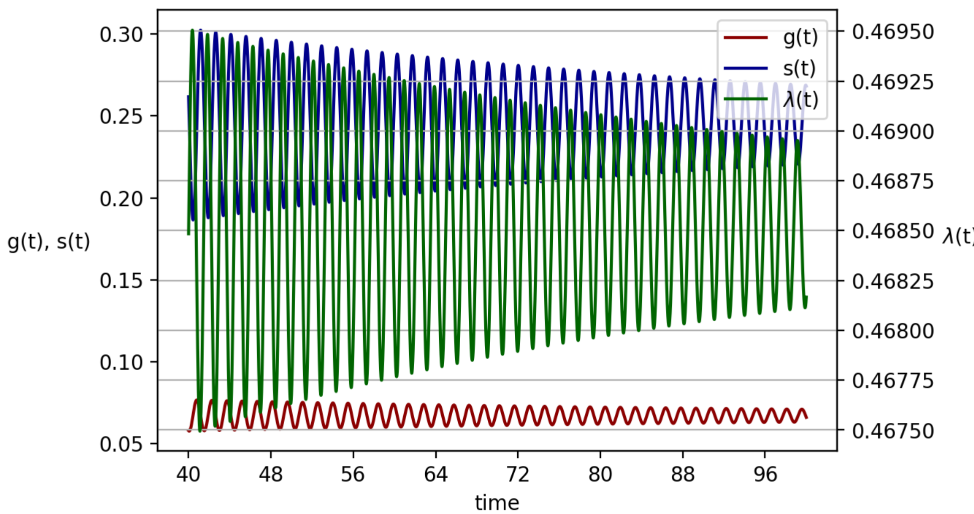
[11]Denote the external debt-to-equity ratio as $d \equiv \frac{D^f}{E}$, where E is equity. If $K = D^f + E$, where K is the capital stock, we can write: $d = \frac{D^f}{K-D^f}$. Dividing the numerator and denominator by K yields $d = \frac{\frac{D^f}{K-D^f}}{1-\frac{D^f}{K}} = \frac{\lambda}{1-\lambda}$. For $\lambda = 0.5$, we thus have $d = 1$.

[12]Recall that the profit share and the capital-to-potential output ratio are fixed. For a profit share (π) of 33% and a capital-to-potential output ratio ($1/v$) of 3, the sensitivity of investment with respect to the utilisation rate (u) would be $\frac{\partial g}{\partial u} = g_r \frac{\pi}{u} = 1.3 \times 0.11 = 0.143$. This is in the order of magnitude used in numerical simulations of Kaleckian models (see Skott and Ryoo, 2008, p. 861).

is well within the range of annual capital stock growth rates of the countries in Figure 1 (Brazil, Chile, Pakistan, South Africa) since the 1960s.[13] Further, we have an equilibrium external debt-to-capital ratio of about 46.9% implying a debt-to-equity ratio of about 88.1% which is within the range reported in IMF (2015, p.90).

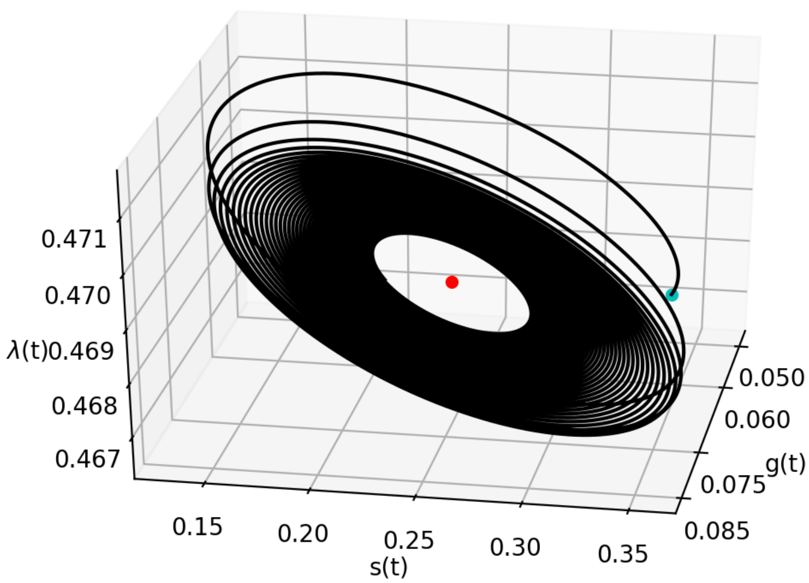
For this parameterisation, the conditions stated in Proposition 4 are satisfied. A positive critical value for which the Hopf bifurcation arises is given by $\mu_0 = 71.65$. Consider first an adjustment speed of the exchange below the critical value, $\mu = 65 < \mu_0$. The fixed point is locally stable, and we observe damped oscillations that converge towards the fixed point (Figures 3 and 4).

Figure 3: Damped oscillations of capital accumulation, the exchange rate, and the external debt-to-capital ratio



[13]Own calculations based on capital stock data at current purchasing power parities from Penn World Tables 9.

Figure 4: 3D phase plot of damped oscillations



Notes: The simulations are based on the parameters provided in Table 1, with $\mu = 65$, and the following initial values: $[g(0), s(0), \lambda(0)] = [0.067, 0.35, 0.469]$. The red dot demarks the steady state towards which the trajectory of the system is converging. The cyan blue dot demarks the initial condition.

Now, consider a second parameterisation, where we leave all other parameters unchanged but set a value of above the critical value: $\mu = 71.8 > \mu_0$. Now the fixed point loses its local stability and a limit cycle around the unstable equilibrium arises (Figures 5 and 6). The simulations suggest that the Hopf bifurcation is supercritical and gives rise to a stable limit cycle in capital accumulation, the exchange rate, and the external debt-to-capital ratio. The middle panel in Figure 5 reveals that peaks and troughs in the investment rate precede those in the exchange rate, which implies a counter-clockwise cycle similar to the empirical ones depicted in Figure 1. This also corresponds to the results derived analytically in the simplified 2D model. However, in contrast to the 2D model, the external debt ratio now moves over time, as can be seen in the last panel.

Figure 5: Limit cycle dynamics of capital accumulation, the exchange rate, and the external debt-to-capital ratio

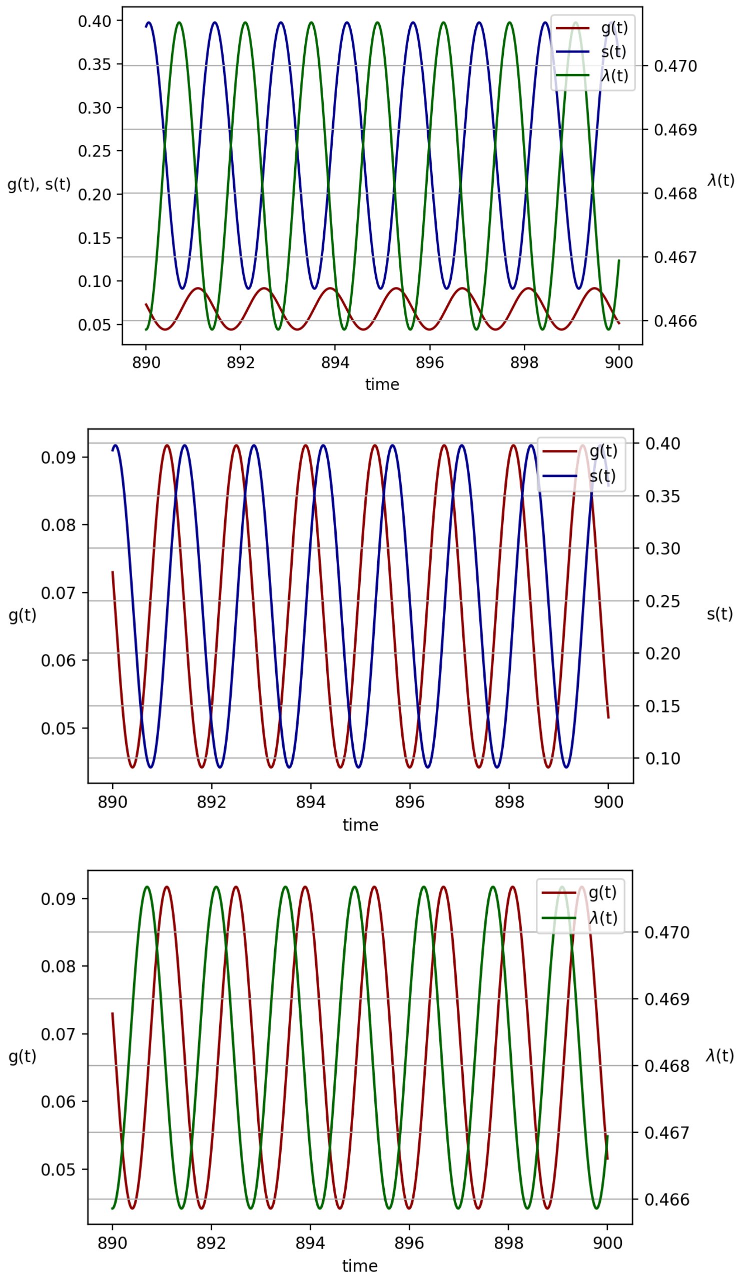
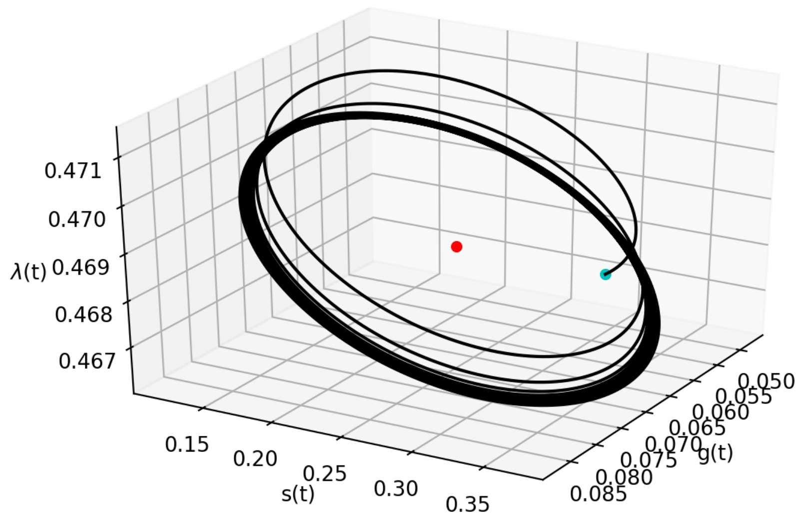


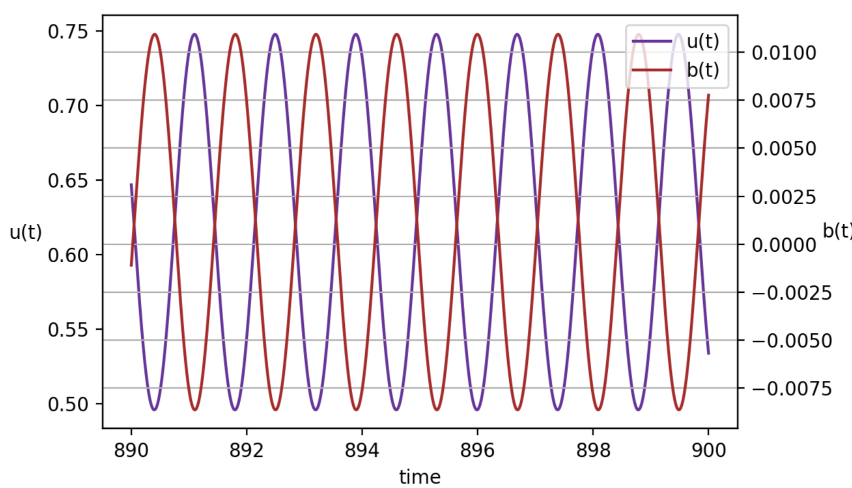
Figure 7 further plots the time paths of the rate of capacity utilisation and the trade balance, which reveals a strong negative correlation. As the economy moves from the peak of the boom to the trough, the trade balance reverses from a deficit to a surplus. Thus, the model captures the strong countercyclicality of the trade balance, as well as the phenomenon of current account reversals observed during EME cycles (Reinhart and Reinhart, 2009).

Figure 6: Phase plot of 3D limit cycle dynamics



Notes: The simulations are based on the parameters provided in Table 1, with $\mu = 71.8$, and the following initial values: $[g(0), s(0), \lambda(0)] = [0.067, 0.35, 0.469]$. The red dot demarks the steady state towards which the trajectory of the system is converging. The cyan blue dot demarks the initial condition.

Figure 7: Dynamics of the rate of capacity utilisation and trade balance



Notes: The simulations are based on the parameters provided in Table 1, with $\mu = 71.8$, and the following initial values: $[g(0), s(0), \lambda(0)] = [0.067, 0.35, 0.469]$. The rate of capacity utilisation is defined as $u = \frac{rv}{\pi}$ and is constructed based on the equilibrium profit rate (equation 11), $\pi = \frac{1}{3}$, and $v = 3$. The trade balance is given by equation (11) in conjunction the equilibrium profit rate.

To understand what happens over the cycle, consider again an appreciation shock to the exchange rate that pushes the economy off the locally unstable equilibrium. As we have $\lambda^T > \lambda^*$, this situation is accompanied by net capital inflows and the currency keeps appreciating. Due to the improvement of balance sheets, the appreciation induces a boost to capital accumulation. As a result, the actual external debt ratio starts declining. This process induces further capital inflows. However, as the current account deficit grows faster, there is downward pressure on the domestic currency. At some point, the depreciation becomes contractionary due to its effects on balance sheets. The initial phase of the contractionary depreciation is accompanied by rising external debt ratios, as the capital stock shrinks. Once the pressure on the current account is relieved, the exchange rate starts to appreciate again. The cycle then repeats itself.

3.3.3 Shocks to risk appetite and policy intervention

What happens if there is a shock to the external debt target, for instance because of an increase in risk appetite? Table 2 presents the comparative dynamic effects of a change in the target debt ratio based on the parameterisation reported in Table 1. [14] An increase in the target debt ratio raises the steady state investment rate and leads to a more appreciated exchange rate. This corresponds to the empirical finding that surges in capital flows to EMEs are typically associated with higher domestic growth rates and exchange rate appreciation (Reinhart and Reinhart, 2009). An increase in the target also leads to a higher steady state debt ratio, as one would expect.

Table 2: Comparative dynamics of a change in the target debt ratio

	$\frac{\partial g^*}{\partial \lambda^T}$	$\frac{\partial s^*}{\partial \lambda^T}$	$\frac{\partial \lambda^*}{\partial \lambda^T}$	$\frac{\partial \mu_0}{\partial \lambda^T}$
Sign	$+$	$-$	$+$	$-$

Notes: The partial effects are based on the parameterisation in Table 1.

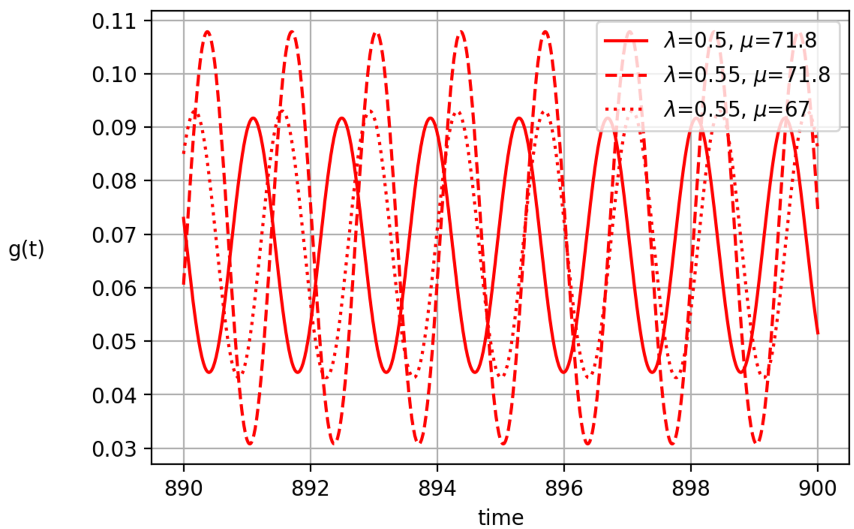
Interestingly, a rise in the target external debt ratio lowers the adjustment speed of the exchange rate for which the limit cycle occurs. This implies that if there is a positive shock to risk appetite, stricter regulations on capital flows are needed to prevent limit cycle dynamics. On the other hand, for a given target leverage ratio, a sufficiently strong regulation of the financial account may prevent the occurrence of volatile limit cycle dynamics altogether.

Figure 8 explores the effects of an increase in the target ratio on business cycle dynamics. An increase in the debt target ratio by 10% (dashed line) significantly increase the amplitude

[14] Analytical solutions for the comparative statics of the steady state solutions do not allow for a clear-cut interpretation. We thus consider a numerical evaluation more helpful.

of the cycle compared to the baseline (solid line). Thus, although the cycle is driven by an endogenous mechanism, exogenous capital flow surges increase the volatility of the domestic macroeconomy. The model thereby combines the notion of endogenous cycles with the phenomenon of exogenous capital flow shocks that make the domestic economy more volatile.

Figure 8: Amplitude of limit cycle for different target debt ratios and policy intervention



Notes: The simulations are based on the parameters provided in Table 1 and on the parameters stated in the legend. The following initial values were used: $[g(0), s(0), \lambda(0)] = [0.067, 0.35, 0.469]$. The solid line is the baseline parameterisation. The dashed line represents an increase in the target debt ratio compared to the baseline. The dotted line depicts a reduction in the adjustment speed of the exchange rate for the scenario with the increased debt target.

Lastly, Figure 8 also investigates how policy intervention can reduce the volatility of the cycle. The dotted line displays the case where the target debt ratio is still increased compared to the baseline, but where the adjustment speed of the exchange rate is reduced. Empirical research has shown that such a reduction in the adjustment speed of the exchange rate can be achieved by restrictions on financial account transactions (Montecino, 2018). Our model shows that such capital account regulation can also have stabilising on the macroeconomy a view that has been taken by structuralists (e.g. in Ocampo and Palma, 2008) and has recently also gained cautious support from mainstream institutions such as the IMF (Ostry et al., 2011).

4 Conclusion

This paper has been concerned with boom-bust cycles in emerging market economies. It has provided a Minskyan approach to business cycles in EMEs with structuralist features, incorporating the stylized facts of procyclical exchange rates, a countercyclical trade balance, and significant levels of foreign currency-denominated corporate debt. The model proposes a causal mechanism that not only explains the observed procyclicality of exchange rates, but also shows how they interact with foreign-currency debt to generate business cycle dynamics. Although business cycles are thus generated endogenously, they are affected by exogenous capital flows, which increase the amplitude of domestic fluctuations. The model shows that by imposing stricter regulations on the financial account, the volatility of the business cycle can be reduced. The higher the risk appetite that drives capital flows, the more financial account regulation is needed to reduce volatility.

The model captures the key role of exchange rate and balance-of-payments dynamics in emerging market business cycles that has been highlighted in the structuralist and Minskyan literature (Ocampo et al., 2009; Frenkel and Rapetti, 2009; Harvey, 2010; Ocampo, 2016). However, unlike previous formal models (Foley, 2003; Taylor, 2004, chap. 10; Gallardo et al., 2006) the approach presented in this paper shifts the focus away from interest rate issues and currency crises towards exchange rate dynamics and balance sheet effects. We consider this an important step forward, given that the majority of emerging markets economies presently follow some form of exchange rate floating. Moreover, our approach combines the Minskyan notion of endogenous cycles with the structuralist emphasis on external shocks. Capital flow surges amplify fluctuations, but the business cycle ultimately emanates endogenously from the interaction of foreign currency debt on domestic balance sheets with a procyclical exchange rate.

This combined approach allows us to identify three areas for policy interventions: first, on the external front, capital controls may curb macroeconomic fluctuations that stem from capital inflow shocks (Ocampo and Palma, 2008; Ostry et al., 2011). Second, a more active exchange rate policy can smoothen exchange rate fluctuations and try to reduce its procyclical effects. Such managed floating has gained growing theoretical support among structuralist and post-Keynesian authors (Frenkel, 2006; Ocampo, 2016; de Paula et al., 2017). Third, at the domestic front, strengthening the banking system through prudential regulation and development banks may reduce foreign-currency indebtedness (Herr and Priewe, 2005). This would remove one of the key domestic driving forces of the boom-bust cycle identified in this paper.

Although we place a strong emphasis on the interaction of exchange rate dynamics and balance sheet effects, we do no claim that this is the only channel that drives business cycles in EMEs. Firstly, interest rates and their effect on capital flows can play an important role too, especially in inflation-targeting regimes. Monetary policy that raises interest rates during a boom may attract more capital inflows which can further fuel the boom. Similarly, market interest rates may be endogenous to economic activity due to risk premia on external debt (Kohler, 2017). Secondly, exchange rate dynamics are presently modelled in a highly simplified way. The presence of heterogenous agents in the foreign exchange market may have important ramifications for macroeconomic stability (Proaño, 2011). Future research could introduce these aspects into the present framework.

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Appendix

A1 List of symbols

Roman Letters

$b \equiv \frac{X-sM}{K}$	Net exports rate
b_r	Sensitivity of net exports w.r.t. domestic rate of profit
b_{r^f}	Sensitivity of net exports w.r.t. foreign rate of profit
b_s	Sensitivity of net exports w.r.t. exchange rate
C	Consumption
$c \equiv \frac{C}{K}$	Consumption rate
D^f	Foreign currency-denominated corporate bonds
$g \equiv \frac{I}{K}$	Investment rate
g^d	Desired investment rate
g_0	Animal spirits
g_r	Sensitivity of capital accumulation w.r.t. profit rate
I	Investment
i^f	Interest rate on foreign currency-denominated corporate bonds
L	Domestic loan (in foreign currency)
M	Imports (denominated in foreign currency)
R	Profits
$r \equiv \frac{R}{K}$	Profit rate
r^f	Foreign profit rate
s	Spot exchange rate (units of domestic currency per unit of foreign currency)
t	Time
u	Rate of capacity utilisation
W	Wage bill
Y	National income
Y^D	Aggregate demand
Z	Foreign reserves

Greek Letters

γ	Adjustment speed of investment rate
δ	Adjustment speed of external debt-to-capital ratio
$\theta \equiv \frac{1}{1+b_r}$	Composite parameter
$\lambda \equiv \frac{D^f}{K}$	External debt-to-capital ratio
λ^T	Target external debt-to-capital ratio
$\upsilon \equiv \frac{K}{Y^P}$	Capital-output ratio
$\pi \equiv \frac{R}{Y}$	Profit share
μ	Adjustment speed of exchange rate (Hopf bifurcation parameter)
μ_0	Critical value of Hopf bifurcation parameter

A2 Mathematical condition for oscillations in 2D systems of differential equations

Consider the characteristic equation of a 2D Jacobian matrix:

$$\lambda^2 - \lambda Tr(J) + Det(J) = 0.$$

The characteristic roots of this equation are given by:

$$\lambda_\pm = \frac{Tr(J) \pm \sqrt{Tr(J)^2 - 4Det(J)}}{2}.$$

Oscillatory behaviour occurs when the characteristic roots are a pair of complex conjugates, which requires the discriminant $\Delta = Tr(J)^2 - 4Det(J)$ to become negative. This condition can be simplified as follows:

$$\begin{aligned}\Delta &= Tr(J)^2 - 4Det(J) \\ &= (J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21}) < 0 \\ &= (J_{11} - J_{22})^2 + 4J_{12}J_{21} < 0.\end{aligned}$$

A3 Proof of Proposition 1: Fixed points of the 2D system

First, we determine the fixed points of s by setting equations (13) and (18) equal to zero and solving for g:

$$g^*_{|\dot{g}=0} = \frac{s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{r^f} r^f g_r}{1 - \theta g_r},$$

$$g^*_{|\dot{s}=0} = \frac{s(i^f\lambda - \theta b_s) - \theta b_{r^f} r^f}{s\lambda - \theta b_r}.$$

Setting these two equations equal and solving for s yields:

$$0 = s^2\lambda(\theta b_s g_r - g_s\lambda) + s[(g_0 + \theta b_{r^f} r^f g_r)\lambda - (\theta b_s g_r - g_s\lambda)\theta b_r + (i^f\lambda - \theta b_s)(\theta g_r - 1)]$$
$$+ (1 - \theta g_r)\theta b_{r^f} r^f - (g_0 + \theta b_{r^f} r^f g_r)\theta b_r.$$

This is an inverted U-shaped parabola. Let us define:

$$\alpha_s = (\theta b_s g_r - g_s\lambda) < 0,$$
$$\beta_s = \underbrace{(g_0 + \theta b_{r^f} r^f g_r)\lambda}_{>0} - \underbrace{(\theta b_s g_r - g_s\lambda)\theta b_r}_{<0} + \underbrace{(i^f\lambda - \theta b_s)(\theta g_r - 1)}_{<0},$$
$$\varsigma_s = \underbrace{(1 - \theta g_r)\theta b_{r^f} r^f}_{>0} - \underbrace{(g_0 + \theta b_{r^f} r^f g_r)\theta b_r}_{>0}.$$

Its roots, which are the two fixed points of s, are given by:

$$s^*_{1,2} = \frac{-\beta_s \pm \sqrt{\beta_s^2 - 4\alpha_s\varsigma_s}}{2\alpha_s}. \tag{A.1}$$

Second, we obtain the two fixed points of g by taking (22) and (23),

$$s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{r^f} r^f g_r}{g_s\lambda - \theta b_s g_r}, \tag{20}$$

$$s^*_{|\dot{s}=0} = \frac{\theta(b_{r^f} r^f - g b_r)}{(i^f - g)\lambda - \theta b_s}, \tag{21}$$

setting them equal and solving for g:

$$0 = g^2\lambda(1 - \theta g_r) + g[(\theta g_r - 1)(i^f\lambda - \theta b_s) - \lambda(g_0 + \theta b_{r^f} r^f g_r) + \theta b_r(g_s\lambda - \theta b_s g_r)] +$$
$$(g_0 + \theta b_{r^f} r^f g_r)(i^f\lambda - \theta b_s) - \theta b_{r^f} r^f(g_s\lambda - \theta b_s g_r).$$

This is a U-shaped parabola. We define

$$\begin{aligned}
\alpha_g &= \lambda(1-\theta g_r) > 0,\\
\beta_g &= \underbrace{(\theta g_r - 1)(i^f\lambda - \theta b_s)}_{<0} - \underbrace{\lambda(g_0 + \theta b_{rf} r^f g_r)}_{>0} + \underbrace{\theta b_r(g_s\lambda - \theta b_s g_r)}_{>0},\\
\varsigma_g &= \underbrace{(g_0 + \theta b_{rf} r^f g_r)(i^f\lambda - \theta b_s)}_{>0} - \underbrace{\theta b_{rf} r^f (g_s\lambda - \theta b_s g_r)}_{>0}.
\end{aligned}$$

The fixed points of g are then given by:

$$g^*_{1,2} = \frac{-\beta_g \pm \sqrt{\beta_g^2 - 4\alpha_g\varsigma_g}}{2\alpha_g} \tag{A.2}$$

where $\beta_g^2 - 4\alpha_g\varsigma_g = \beta_s^2 - 4\alpha_s\varsigma_s$, i.e. the discriminants of (A.1) and (A.2) are identical. If the discriminant is positive, there will be two fixed points. ■

A4 Proof of Proposition 2: Asymptotic stability of the 2D system

The trace and determinant of the Jacobian (19) evaluated at the positive fixed point are given by:

$$\begin{aligned}
Tr(J) &= \gamma(\theta g_r - 1) + \mu[(i^f - g^*)\lambda - \theta b_s],\\
Det(J) &= \gamma\mu\{(\theta g_r - 1)[(i^f - g^*)\lambda - \theta b_s] - (\theta b_s g_r - g_s\lambda)(\theta b_r - s^*\lambda)\}.
\end{aligned}$$

Stability requires $Tr(J) < 0$ and $Det(J) > 0$. Some algebra shows that

$$\begin{aligned}
Tr(J) < 0 &\Leftrightarrow i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{\gamma\overbrace{(1-\theta g_r)}^{>0}}{\mu\lambda},\\
Det(J) > 0 &\Leftrightarrow i^f < g^* + \frac{\overbrace{(\theta b_s g_r - g_s\lambda)(\theta b_r - s^*\lambda)}^{<0}}{\underbrace{(\theta g_r - 1)\lambda}_{<0}}.
\end{aligned}$$

■

A5 Proof of Proposition 3: Number of fixed points of the 3D system

The 3D system is reproduced here for convenience:

$$\dot{g} = \gamma[g(\theta g_r - 1) + s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{rf} r^f g_r]$$
$$\dot{\lambda} = \delta(\lambda^T - \lambda) - g\lambda$$
$$\dot{s} = \mu\{g(\theta b_r + s\lambda) + s[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{rf} r^f\}.$$

First, we set (13) equal to zero and solve for g:

$$g = \frac{s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{rf} r^f g_r}{1 - \theta g_r}. \tag{A.5}$$

In order to reduce clutter, we introduce the following composite parameters:

$$\Phi_0 = \frac{g_0 + \theta b_{rf} r^f g_r}{1 - \theta g_r} > 0,$$
$$\Phi_1 = \frac{\theta b_s g_r}{1 - \theta g_r} > 0,$$
$$\Phi_2 = \frac{g_s}{1 - \theta g_r} > 0,$$

which allows us to re-write (A.5) as:

$$g = \Phi_0 + \Phi_1 s - \Phi_2 s\lambda. \tag{A.6}$$

Substituting (A.6) into (23), setting it equal to zero and solving for s yields:

$$s = \frac{\delta\lambda^T - \lambda(\delta + \Phi_0)}{\lambda(\Phi_1 + \Phi_2\lambda)}. \tag{A.7}$$

Likewise, substituting (A.6) into (24), setting it equal to zero and solving for s yields:

$$s = \frac{\theta(\Phi_0 b_r - b_{rf} r^f)}{\lambda(\Phi_2\theta b_r + \delta - i^f) - \Phi_1\theta b_r - \delta\lambda^T - \theta b_s}. \tag{A.8}$$

Let us introduce further composite parameters:

$$\rho_0 = \theta(\Phi_0 b_r - b_{rf} r^f),$$
$$\rho_1 = \Phi_2\theta b_r + \delta - i^f,$$
$$\rho_2 = -(\Phi_1\theta b_r + \delta\lambda^T + \theta b_s) < 0.$$

We can then rewrite (A.8) as:

$$s = \frac{\rho_0}{\lambda\rho_1 + \rho_2}. \tag{A.9}$$

Setting (A.7) and (A.9) equal and solving for λ yields the following second-order polynomial:

$$\lambda^2[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)] + \lambda[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T] - \rho_2\delta\lambda^T = 0, \tag{A.10}$$

with roots:

$$\lambda^*_{1,2} = \frac{-[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T] \pm \sqrt{[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T]^2 + 4[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)]\rho_2\delta\lambda^T}}{2[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)]}. \tag{26}$$

If the discriminant is positive there are exactly two real roots, which constitute the fixed points of λ. Thus, there are at most two fixed points in the 3D system. ■

A6 Proof of Proposition 4: Hopf bifurcation in the 3D system

The Jacobian matrix of the 3D system (25) evaluated at the positive fixed points is given by:

$$\mathbf{J}(g, s, \lambda) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s gr - g_s\lambda^*) & -\gamma s^* g_s \\ \mu(\theta b_r - s\lambda^*) & \mu[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] & \mu s^*(i^f + \delta) \\ -\lambda & 0 & -(\delta + g^*) \end{bmatrix}. \tag{27}$$

The Jacobian has the characteristic equation

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{28}$$

where

$$\begin{aligned} a_1 &= -Tr(J), \\ a_2 &= Det(J_1) + Det(J_2) + Det(J_3), \\ a_3 &= -Det(J), \end{aligned}$$

and where J_i is the 2x2 minor obtained by deleting row and column i from the Jacobian. The Hopf bifurcation emerges for $a_1, a_2, a_3 > 0$ and $a_1 a_2 - a_3 = 0$, where the Jacobian matrix exhibits a non-zero real root and a pair of pure imaginary eigenvalues (Gandolfo, 1997, pp. 475-479). We thus have the following four conditions:

$$Tr(J) = -a_1 < 0, \tag{HBF.1}$$

$$Det(J_1) + Det(J_2) + Det(J_3) = a_2 > 0, \tag{HBF.2}$$

$$Det(J) = -a_3 < 0, \tag{HBF.3}$$

$$-Tr(J)[\sum_{i=1}^{3} Det(J_i)] + Det(J) = a_1 a_2 - a_3 = 0. \tag{HBF.4}$$

(HBF.1). The first condition is:

$$-a_1 = J_{11} + J_{22} + J_{33} = \underbrace{\gamma(\theta g_r - 1)}_{<0} + \underbrace{\mu[i^f \lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]}_{>0} - \underbrace{(\delta + g^*)}_{>0},$$

which becomes negative if $|J_{11} + J_{33}| > |J_{22}|$.

(HBF.2). The second condition is:

$$\begin{aligned} a_2 &= \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} \\ &= \underbrace{\begin{vmatrix} \mu[i^f \lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] & \mu s^*(i^f + \delta) \\ 0 & -(\delta + g^*) \end{vmatrix}}_{<0} + \begin{vmatrix} \gamma(\theta g_r - 1) & -\gamma s^* g_s \\ -\lambda^* & -(\delta + g^*) \end{vmatrix} \\ &+ \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s \lambda^*) \\ \mu \theta b_r & \mu[i^f \lambda - \delta(\lambda^T - \lambda) - \theta b_s] \end{vmatrix}}_{>0} \\ &= \underbrace{J_{22} J_{33}}_{<0} + \underbrace{J_{11} J_{33}}_{>0} - \underbrace{J_{13} J_{31}}_{>0} + \underbrace{J_{11} J_{22}}_{<0} - \underbrace{J_{12} J_{21}}_{<0}, \end{aligned}$$

which becomes positive if $|J_{11} J_{33} + J_{12} J_{21}| > |J_{22}(J_{11} + J_{33}) + J_{13} J_{31}|$.

(HBF.3). The third condition is given by:

$$
\begin{aligned}
-a_3 &= J_{31} \begin{vmatrix} J_{12} & J_{13} \\ J_{22} & J_{23} \end{vmatrix} + J_{33} \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_2 \end{vmatrix} \\
&= -\lambda^* \underbrace{\begin{vmatrix} \gamma(\theta b_s g_r - g_s \lambda^*) & -\gamma s^* g_s \\ \mu[i^f \lambda - \delta(\lambda^T - \lambda) - \theta b_s] & \mu s^*(i^f + \delta) \end{vmatrix}}_{<0} - (\delta + g^*) \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s \lambda^*) \\ \mu \theta b_r & \mu[i^f \lambda - \delta(\lambda^T - \lambda) - \theta b_s] \end{vmatrix}}_{>0} \\
&= \underbrace{J_{31}}_{<0} \underbrace{(J_{12} J_{23} - J_{13} J_{22})}_{<0} + \underbrace{J_{33}}_{<0} \underbrace{(J_{11} J_{22} - J_{12} J_{21})}_{>0},
\end{aligned}
$$

which becomes negative if $|J_{33}(J_{11}J_{22} - J_{12}J_{21})| > |J_{31}(J_{12}J_{23} - J_{13}J_{22})|$.

(HBF.4). Lastly, we have $a_1 a_2 - a_3$, which must become zero for a Hopf bifurcation to take place. We will use μ as the Hopf bifurcation parameter as it has the convenient property that it does not affect the steady state values. Let us write the three parameters of the characteristic equation as functions of μ:

$$
\begin{aligned}
a_1 &= \mu \kappa_1 + \kappa_2, \\
a_2 &= \mu \kappa_3 + \kappa_4, \\
a_3 &= \mu \kappa_5.
\end{aligned}
$$

We can then rewrite (HBF.4) as:

$$
a_1 a_2 - a_3 = f(\mu) = \kappa_1 \kappa_3 \mu^2 + (\kappa_1 \kappa_4 + \kappa_2 \kappa_3 - \kappa_5)\mu + \kappa_2 \kappa_4 = 0.
$$

This expression must become zero for a Hopf bifurcation to occur. The coefficients κ_1 to κ_5

are given by:

$$\kappa_1 = -[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] < 0,$$
$$\kappa_2 = \gamma(1-\theta g_r) + (\delta + g^*) > 0,$$
$$\kappa_3 = \underbrace{[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]}_{>0}\underbrace{[\gamma(\theta g_r - 1) - (\delta + g^*)]}_{<0} - \underbrace{\theta b_r\gamma(\theta b_s g_r - g_s\lambda^*)}_{<0},$$
$$\kappa_4 = \gamma(1-\theta g_r)(\delta + g^*) - \lambda^*\gamma s^* g_s,$$
$$\kappa_5 = \lambda^*\underbrace{\{\gamma(\theta b_s g_r - g_s\lambda^*)s^*(i^f + \delta) + [i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]\gamma s^* g_s\}}_{<0}$$
$$+ (\delta + g^*)\underbrace{\{\gamma(\theta g_r - 1)[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] - \theta b_r\gamma(\theta b_s g_r - g_s\lambda^*)\}}_{>0}.$$

κ_3 is likely to be positive given that we assume strong interaction effects and weak independent dynamics. κ_4 and κ_5 can be positive or negative. We then have

$$f(\mu) = \underbrace{\kappa_1\kappa_3}_{<0}\mu^2 + (\kappa_1\kappa_4 + \underbrace{\kappa_2\kappa_3}_{<0} - \kappa_5)\mu + \kappa_2\kappa_4 = 0. \tag{A.11}$$

This parabola is opened downward. If its discriminant is positive, the equation has two real roots. This requires the expression $\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5$ to be sufficiently large in absolute value. At these two roots, a Hopf bifurcation occurs:

$$\mu_\pm = \frac{-(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5) \pm \sqrt{(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)^2 - 4\kappa_1\kappa_2\kappa_3\kappa_4}}{2\kappa_1\kappa_3}.$$

■