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Engelbert Stockhammer[1], Giorgos Gouzoulis[2], and Rob Calvert Jump[3]

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Abstract

Minsky (1975) proposed a theory of endogenous cycles that results from the interaction of real and financial variables. Minsky's work has inspired a growing body of literature on theoretical business cycle models, but relatively little work has been done in the empirical field. In particular, while interest in financial cycles has risen significantly after the 2007-8 financial crash, and recent empirical studies have explored the impact of debt on aggregate demand or its effect on the probability of financial crises, the literature does not test for endogenous cycle mechanisms. In contrast, the present paper investigates econometrically whether or not business cycles are driven by corporate debt and/or by mortgage debt. We estimate simple vector autoregressive moving average (VARMA) models, using historical macroeconomic data for the USA (1889-2015) and the UK (1882-2010). We find robust evidence of endogenous corporate debt-driven cycles for the USA, weak evidence of mortgage debt-driven cycles in the USA and no evidence of corporate or mortgage debt-driven cycles for the UK.

Keywords: Minsky cycles; corporate debt; mortgage debt; business cycles; historical data.

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[1] King's College London, Department of European and International Studies, Virginia Woolf Building, 22 Kingsway, London WC2B 6LE; E-mail address: engelbert.stockhammer@kcl.ac.uk

[2] King's College London, Department of European and International Studies, Virginia Woolf Building, 22 Kingsway, London WC2B 6LE; E-mail address: giorgos.gouzoulis@kcl.ac.uk

[3] University of the West of England, Department of Accounting, Economics and Finance, Bristol BS16 1QY; Email address: rob.calvertjump@uwe.ac.uk

1. Introduction

Interest in financial cycles has grown significantly since the 2007-8 crisis, which has highlighted the destabilizing role of liberalized financial sectors. Recent theoretical studies such as Eggertsson and Krugman (2012), Farmer (2013), and Bhattacharya *et al.* (2015) have analyzed financial instability by enriching the standard New Keynesian (NK) model with insights from the pioneering works of Hyman Minsky (1975, 1986, 1992). Minsky's financial instability hypothesis has also been a pillar for a wide variety of post-Keynesian (PK) endogenous business cycle models (see Nikolaidi and Stockhammer, 2017), which emphasize the inherent instability that arises due to the interaction between real and financial variables.

The simplest version of Minsky's argument suggests that endogenous fluctuations are generated by the procyclicality of the corporate leverage ratio and the negative effect of debt service payments on investment. Specifically, investors' attitudes to risk relax during the euphoria of a boom, which leads to rising debt ratios and increased interest payments, which eventually hamper investment and growth. While the theoretical Minsky literature keeps expanding, empirical works are quite limited, centering on the effects of consumer and household debt on GDP growth in the US economy (Palley 1994; Kim 2013, 2016). However, there is a growing empirical literature that explores the different lengths of real and financial cycles through univariate filtering techniques (see e.g. Drehmann *et al.* 2012; Borio 2014) or estimates the impact of financial variables on real variables and the probability of financial crises, using historical macroeconomic data (see e.g. Schularick and Taylor, 2012; Jordá *et al.* 2013, 2015, 2016; Aikman *et al.* 2015).

The aim of the present paper is to contribute to the growing empirical literature on debt-driven business cycles by testing simple Minsky models with historical macroeconomic data. These models generate endogenous cycles through the interaction of a pro-cyclical leverage ratio and negative effects of debt on growth. We estimate two-equation models for corporate and mortgage debt and real GDP growth using historical macroeconomic data for the USA (1890-2015) and the UK (1882-2010). We also report results for real investment growth. Several empirically oriented macroeconomic history papers (e.g. Schularick and Taylor, 2012; Jordá et al. 2013, 2016) find negative growth effects of debt in a panel context. In contrast, our approach focuses on the interaction of debt and growth, i.e. we check for the negative investment and GDP growth effects of debt as well as the procyclical nature of debt ratios, both of which are necessary to generate endogenous oscillations.

Since we are estimating pairs of growth and debt equations, rather than using binary variables for financial crises, we can take advantage of the length of the historical time series and apply country-specific time series analysis. Compared to the empirical PK Minskyan literature (Palley, 1994; Kim, 2013, 2016), first, our study covers the interaction between corporate (non-mortgage) debt and investment growth following Minsky's original writings, as well as household (mortgage) debt, and, second, it covers a significantly longer historical period. Compared to the literature that examines the effects of private credit on output and investment (Bezemer et al. 2015; Mian et al. 2016), our paper tests for endogenous cycles, which require both a procyclical leverage ratio and a negative effect of debt on investment and GDP growth. The main finding of our study is that the US economy has indeed experienced corporate debt-driven Minsky business cycles over the sample period. For the UK economy, we find insufficient evidence for corporate debt-driven cycles. Our estimations using mortgage debt yield weak evidence for mortgage debt-driven Minsky cycles for the USA, but negative effect of debt on growth is below conventional levels of statistical significance. We fail to find evidence for mortgage debt-riven cycles for the UK. These results are comparable to the results found in a sister paper to the present paper, Stockhammer *et al.* (2018), which covers the post-1970 period.

The rest of the paper is structured as follows. Section 2 discusses the Minsky-inspired business cycle literature. Section 3 reviews the existing empirical literature on financially and debt-driven business cycles, highlighting their methodological differences and shortcomings. Section 4 presents data sources and our econometric modeling approach, underlining its theoretical relevance. Section 5 discusses the estimation results. Finally, the concluding section recapitulates the main findings and suggests future research paths.

2. Minsky debt-driven cycle models

Minsky's (1975, 1986, 1991) analysis is rooted in the PK and Old Institutionalist traditions. His writings offer rich insights on financial dynamics, but no canonical model. As a result, economists inspired by Minsky have taken his approach in different directions and developed different elements of his analysis into formal models. Nikolaidi and Stockhammer (2017) survey the literature with a focus on models that generate endogenous cycle and distinguish between debt cycle models and asset price models. There are further variations within each of these. In the debt cycle models the cycle arises from the interaction between financial and real variables. The key financial variable is usually the debt-to-income ratio and interest rate movement often plays a key role. The asset price models are based on the nature of expectation formation and often feature the interaction of different valuation strategies. Differences exist on whether interest rates are set by the central bank in response to changes in inflation (Fazzari et al. 2008) or by commercial banks in response to change in their costumers' leverage (e.g. Keen, 1995; Lima and Meirelles, 2007); on whether they assume stable goods markets (Charles, 2008) or Harrodian instability (Ryoo, 2013). While most models deal with corporate debt, some focus on household debt (Kapeller and Schutz, 2014; Ryoo, 2016). While most models use small scale macroeconomic models there are also full specified stock flow-consistent models (Nikolaidi, 2014; Dafermos, 2018) and heterogeneous agent models (Jump et al. 2017).

Some scholars have emphasized the role of institutional change in Minsky's analysis and developed this into a theory of long waves. Palley (2011) discuss Minsky's contribution by distinguishing between shorter finance-driven cycles and long waves of financial expansion. His short waves is close to what we discussed above. As regarding long waves, Palley highlights that firms and policy makers tend to forget that financial liberalisations in the past have led to major systemic crises and recessions. Policy makers eventually allow the deregulation of institutions and markets, while firms (and households) take on more risk (and debt) indebtedness taking advantage of this deregulation process. Ultimately, this results in a secular increase in financial fragility, leading to major financial crises like the 2007-8 crash. While the short cycles operate in a given regulatory environment, the long cycles (or: super-cycles) are about endogenous regulatory change. Similarly, Wray (2009) discusses long Minsky cycles and underlines that a substantial difference between the 1929 financial crisis and the 2007-8 crash in the USA is that in the latter case real estate prices and the engagement of households in finance played a much more important role, which is consistent with recent econometric studies.

This paper is testing for endogenous cycles arising from the interaction between debt and real expenditures. In Palley's terminology we analyse the basic Minsky cycle. We will analyse versions with corporate debt and with mortgage debt. The distinguishing characteristic of Minskyan corporate debt-driven cycle models is that the hypothesized residual source of finance for investment is business debt (Asada, 2001; Lima and Meirelles, 2007; Charles, 2008; Fazzari et al. 2008). Thus, the theory is consistent with a "pecking order" theory of finance, in which firms only resort to debt financing after retained earnings have been exhausted. The central postulate of the theory is that the desired investment rate rises rapidly during the euphoria of a boom, exceeding retained earnings. The gap between desired investment and actual internal funding resources is then covered by corporate debt, and the debt-to-income ratio rises as a result. As the debt-to-income ratio increases, relevant interest payments rise and a rising share of retained profits will be devoted to debt service. This makes the balance sheet of the firm increasingly fragile, which eventually leads to a slowdown in investment growth, thus on GDP growth. A typical reduced-form Minsky corporate debt-driven cycle model can be expressed in the following system of difference equations,

$$\begin{bmatrix} g_t \\ d_t \end{bmatrix} = \begin{bmatrix} \pm & - \\ + & \pm \end{bmatrix} \begin{bmatrix} g_{t-1} \\ d_{t-1} \end{bmatrix}, \tag{1}$$

where g is the growth rate of investment or GDP, and d is some measure of the debt-to-income ratio. Following Stockhammer *et al.* (2018), necessary conditions for oscillations in (1) are debt-burdened investment or GDP growth ($J_{12} < 0$) and a pro-cyclical corporate leverage ratio ($J_{21} > 0$). While our model in principle is closer to Palley's (2011) short cycles within a given institutional structure, there is nothing in our model that restricts the cycle length.

An important simplifying assumption of most Minskyan debt cycles models, implicit in the explanation given above, is that they do not explicitly account for the role of equity markets. There are a few models in which asset prices are procyclical, hence permit the increase of debt ratios by relaxing firms' collateral constraint, such as Kiyotaki and Moore (1997). However, it is worth noting that in that model asset prices are endogenous (but not state variables), therefore system (1) is fully consistent with its reduced form. Ryoo (2010) also presents a model where asset price expectations along with liquidity preferences generate endogenous instability building on non-linear higher order systems. The incorporation of behavioural variables makes the estimation of such a system significantly less straightforward than a typical linearised Minsky debt cycle model, thus it is beyond the scope of this paper to test this model. We leave the empirical estimation of fully specified systems with non-linear asset price functions to future research.

While Minsky's original emphasis was on business debt-driven cycles, several authors attempt to formalize consumer debt and real estate prices in the context of Minskyan models. Palley (1994) presents a Minskyan model that includes procyclical consumer debt accumulation. Modifying a simple multiplier-accelerator model, Palley shows that initially debt flows increase aggregate demand through consumption, and thus output, but eventually rising debt accumulation decreases aggregate demand. Ryoo (2016) develops a real estate price Minsky model, in which momentum traders expect further price increases when house prices grow. Ultimately, households' demand for houses will slow down, curbing house prices, and thus the housing cycle. Here, the key variable is expected capital gains, which are not observable. Based on Palley (1994) and Ryoo (2016), we propose a reduced form Minsky mortgage debt-driven models similar to the 2D corporate debt-driven model above, i.e. households' confidence during the boom period makes them increase their debt ratio in order to purchase a house. Eventually, increasing debt payments decrease growth, hence endogenous fluctuations are generated. Such a Minsky household debt model can be depicted in the following system of difference equations,

$$\begin{bmatrix} g_t \\ m_t \end{bmatrix} = \begin{bmatrix} \pm & - \\ + & \pm \end{bmatrix} \begin{bmatrix} g_{t-1} \\ m_{t-1} \end{bmatrix}, \tag{2}$$

where g is the growth rate of GDP, as above, and m is the mortgage or household debt-to-income ratio. As in the reduced-form corporate debt-driven model, necessary conditions for oscillations in (2) are $J_{12} < 0$ and $J_{21} > 0$.

3. Debt-driven Business Cycles: A Review of the Empirical Literature

In contrast to the theoretical literature on Minskyan cycles, the related empirical literature is quite limited and has started growing only recently. An overview is given in Table 1. Palley (1994) tests a Minsky-inspired household debt cycle model using quarterly data for the USA (1975-1991) to evaluate the effects of consumer debt on real GNP per capita. Results from a single-equation distributed lag model and a 3-dimensional vector autoregressive (VAR) model indicate that increases in consumer debt produce damped oscillations.[4] Kim (2013) follows Palley's single-equation approach, using quarterly US data (1951-2009) with

[4] The paper does not report the coefficient values from which the oscillations generated, so we cannot draw any conclusion about the signs of the implied Jacobian matrix elements, i.e. whether the oscillations are endogenous as in Minsky

household net worth and consumer debt as financial variables. He finds a positive effect of a change in household debt, but negative level effects, implying an underlying financial accelerator mechanism. Kim (2016) reports Johansen cointegration tests of vector error correction models of GDP, net worth, consumption, and either household, mortgage, or consumer debt, and finds that shocks in the debt variables decrease output, while the leverage ratio is procyclical. These results are indeed consistent with our approach on Minsky cycles as the necessary conditions are fulfilled: private indebtedness decreases output, while the leverage ratio is procyclical. However, the lag structure of the specifications is not similar to a typical Minsky debt-capital stock difference equation system, thus it is not possible to evaluate the sufficient conditions.

Greenwood-Nimmo and Tarassow (2016) offer a more policy-oriented Minsky model which focuses on the effects of monetary aggregates and macroprudential policy shocks on the aggregate credit-to-GDP and the corporate credit-to-internal funds ratios, using quarterly data for the US economy (1960-2007). Using a sign-restricted VAR model, they report that contractionary monetary policy shocks raise the credit-to-GDP ratio and the corporate credit ratio. Contrariwise, a credit-constraining macroprudential policy shock reduces the total credit ratio but does not have significant effects on the corporate financial ratio.

Next to the few empirical business cycle studies, there is a larger group of studies that focus on particular aspects of Minsky's analysis. In particular Fazzari and Mott (1986), Fazzari et al. (1988), Ndikumana (1999), and Arza and Espanol (2008) analyse the effect of corporate debt on business investment. There are also various studies that analyse the financial fragility non-financial sectors' (Isenberg, 1989; Wolfson, 1990; Mulligan, 2013; Nishi, 2016). All of these studies are consistent with our approach, but cover only one of the two mechanisms that are necessary for endogenous cycles.

In recent years, the wider financial cycles literature has grown, including the field of quantitative macroeconomic history. This research is empirically driven and refers to New Keynesian theories of credit rationing and the financial accelerator models and, to some extent, to Minsky as motivation. Drehman *et al.* (2012) and Borio (2014) utilize descriptive analysis using band-bass (Christiano and Fitzgerald 1999) and Hodrick-Prescott filtering (Whittaker 1922; Hodrick and Prescott 1997), and turning-point analysis (Burns and Mitchell 1946), on quarterly and annual data for several countries (1960 to the present). They conclude that financial cycles tend to be longer than real business cycles, while the length and the amplitude of the former tends to increase after the mid-1980s. Claessens *et al.* (2011, 2012) analyze the duration of recessions and recoveries, and their amplitude. Panel estimations based on quarterly datasets (1960Q1 – 2010Q4) of 21 and 44 countries, respectively, demonstrate that house price movements amplify recessions and recoveries, while slowdowns deteriorate when downturns in credit and asset prices synchronize. Bezemer *et al.* (2015) examine the effects of financial variables on growth, using a 46 country panel (1970-2011) and an industry-level dataset. Controlling for government spending, trade, inflation, and education, they find that credit booms decrease growth. Mian *et al.* (2016) explore the impact of household debt on growth (30 countries, 1960-2012) and show that the household debt-to-GDP ratio is related to future growth. Household debt has positive short-term, but negative long-term effect. As a further step, they focus on the global level by taking the sample averages of the variables. The resulting model suggests that the growth effect of household debt remains negative.

In their seminal paper, Schularick and Taylor (2012) find negative cumulative output and investment effects of credit, and that the impact of credit becomes stronger in the post-WWII era. Using binary financial crises variables in logit and probit models, they report strong effects of credit expansions and stock market booms, if the financial sector, measured by credit volumes, is large. Jordá *et al.* (2013) draw similar conclusions about impact of total credit on the growth rate of real GDP per capita, controlling for excess credit, i.e. the percentage change of the loans-to-output ratio compared to the last expansion period. Aikman *et al*. (2015) utilize band-bass filtering and spectral density analysis, using historical macroeconomic data on total credit (14 countries, 1870-2008). They find that financial cycles are longer than real cycles, while their logit model estimations (full sample and sub-sample, excluding the war years) suggest that credit expansions increase the probability of financial crises. Lastly, Jordá *et al.* (2016) disaggregate debt into mortgage debt and non-mortgage debt and estimate probit and logit models to explain banking and financial crises. They find that both categories of debt increase the probability of financial crises and note that the results for mortgage debt are due to the post-WWII period. This result is consistent with Wray's (2009) argument that the 2007-8 crisis was driven by the real estate boom. Five-year cumulated impulse responses for real per capita GDP, real investment per capita and real lending per capita show that when a crisis coincides with a credit boom, recessions tend to be longer and recoveries slower. In particular, after WWII mortgage booms led to deeper recession projections.

The vast majority of these studies support Minsky's debt-burdened growth hypothesis, as they find that either total, mortgage, and/or non-mortgage debt affect growth negatively (Bezemer et al. 2015; Mian et al. 2016; Schularick and Taylor, 2012; Jordá et al. 2013, 2016). However, that finding alone provides information only about one aspect of Minsky's mechanism of endogenous debt-driven cycles. The other, i.e. the procyclicality of the leverage ratio, remains unexplored in this literature. In addition, as summarized in Table 1, the empirical literature on finance-driven cycles that uses historical macroeconomic data uses panel data models, often with a binary dependent variable. In contrast, the present study is the first to test for endogenous cycles arising from the interaction of debt and growth using historical macro data.

Table 1: Overview of related empirical studies.

Authors	Dependent variables	Financial variables	Data
Palley (1994)	Per capita GNP	Consumer debt	USA, quarterly (1975-1991, 1951- 2009)
Kim (2013, 2016)	GDP	Consumer, household, mortgage debt	USA, quarterly (1951-2009)
Greenwood-Nimmo and Tarassow (2016)	Financial fragility	Monetary and macroprudential shocks	USA, quarterly (1960-2007)
Claessens et al. *(2011, 2012)*	Recessions' and recoveries' durations	Credit, asset prices	Panel, quarterly (21 countries, 1960-2007; 44 countries, 1960-2007)
Bezemer et al. *(2015)*	Per capita GDP	Credit stocks and flows	Panel (46 countries, 1990-2011)
Mian et al. *(2016)*	GDP	Corporate and household debt	Panel (30 countries, 1960-2012)
Schularick and Taylor (2012), Jordá et al. (2013), Aikman et al. *(2015)*	Financial crises (binary), per capita GDP	Total credit, money, bank assets, stock prices, financial crises (binary)	Panel (14 countries, 1870-2008)
Jordá et al. *(2016)*	Financial crises (binary), per capita GDP	Mortgage and corporate (non-mortgage) credit	Panel (17 countries, 1870-2012)

4. Data and Econometric Approach

The historical macroeconomic dataset of the present study covers the period from the mid-to-late 19th century to date for the USA (1890-2015) and the UK (1882-2010). We obtain data from various sources. The four main variables of interest are real GDP, real investment, the business debt-to-income ratio, and the mortgage debt-to-income ratio. Debt refers to bank loans, i.e. does not include debt to other institutions. Since business debt is not directly available for the USA before 1960, we approximate it by subtracting mortgage debt from total private credit to the non-financial private sector. The series for the UK come from Hills *et al.* (2015) and Jordà *et al.* (2017), whereas the US data were derived from US national accounts and Jordà *et al.* (2017). The investment series for the USA covers a longer period than GDP, since real investment data are available from 1890 in Kuznets and Jenks (1961). A summary of variable definitions, periods covered, data sources, and the results of augmented Dickey-Fuller (ADF) unit root tests can be found in Appendix A1. Summary statistics of the relevant variables are reported in Table 2, where $BDEBT$ is the business (non-mortgage) debt-to-GDP ratio, $MDEBT$ is the mortgage debt-to-GDP ratio, I is the logarithm of real investment, and GDP is the logarithm of real gross domestic product.

As outlined in Section 2, a simple debt-driven business cycle model can be specified by a pair of difference equations which give us necessary conditions for interaction-driven oscillations derived from the Jacobian matrix. However, as the lag structure is minimalistic, the reduced form in (1) and (2) may well be a poor approximation to the joint distribution of real GDP and corporate (or mortgage) debt. As a result, we estimate the following VARMA models on Investment or GDP growth (g) and the change in corporate (or mortgage) debt-to-income ratio (Δd),

$$\begin{bmatrix} g_t \\ \Delta d_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} g_{t-1} \\ \Delta d_{t-1} \end{bmatrix} + \begin{bmatrix} \theta(L) & 0 \\ 0 & \varphi(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}, \tag{3}$$

where $\theta(L)$ and $\varphi(L)$ are lag polynomials. As the lag length for the MA terms will be restricted to be 1, given that data are annual, and the resulting models do not suffer from serial correlation, a necessary condition for oscillations in (3) is $\beta_{12}\beta_{21} < 0$. For the purpose of examining the existence of Minskyan cycles, we can restrict this necessary condition to the pair of conditions $\beta_{12} < 0$ (debt burdened growth) and $\beta_{21} > 0$ (procyclical leverage ratio). The sufficient condition for oscillations in (3) is the existence of complex conjugate eigenvalues (see Stockhammer *et al.* (2018) and Appendix A2). So, when the necessary conditions are met, we can also calculate the discriminant to evaluate the sufficient conditions.

Table 2: Summary statistics

	USA				UK			
	BDEBT	*Δ(GDP)*	*Δ(I)*	*MDEBT*	*BDEBT*	*Δ(GDP)*	*Δ(I)*	*MDEBT*
Mean	0.205	0.030	0.007	0.214	0.183	0.019	0.022	0.160
Median	0.205	0.032	0.048	0.187	0.191	0.022	0.032	0.101
Max	0.320	0.158	0.591	0.432	0.354	0.101	0.604	0.718
Min	0.059	-0.147	-2.053	0.061	0.063	-0.112	-0.454	0.013
First ob.	1889	1930	1889	1889	1880	1851	1851	1880
Last ob.	2013	2015	2015	2013	2009	2015	2015	2009
# obs.	125	86	126	125	130	165	165	130

Two points need to be noted concerning (3) before we continue. First, the matrix of MA terms is restricted to be diagonal. As is well known, VARMA models are not identified in the general case and require some form of restriction prior to estimation. Our approach follows Dufour and Pelletier (2011), who point out that the standard approach to imposing identifying restrictions in VARMA models, known as the "echelon form", is considerably more complicated than simply choosing lag orders (as in VAR models), and is a major reason why VARMA models are infrequently used in practice. The "diagonal MA equation form" of (3), on the other hand, is extremely simple to specify and can be seen as a straightforward extension of a VAR model. Second, well behaved estimates require that the roots of $\theta(L)$

and $\varphi(L)$ lie within the unit circle (Brooks 2014, pp. 267-281). Again, as the MA terms will be of order 1 in practice, the inverted roots are identical to the absolute values of the estimated MA coefficients, which accordingly must be less than one.

The model in (3) can be estimated either by maximum likelihood (ML) using the Kalman filter, or generalized least squares (GLS) on an equation-by-equation basis. There are advantages and disadvantages to both methods. As ML using the Kalman filter is a system estimation procedure, in principle it should be more efficient. On the other hand, any misspecification in one equation will affect the estimates of all other equations when using system estimation, and therefore, in principle, equation-by-equation estimation should be more robust to misspecification. In addition, simulation exercises on ARMA models (Koreisha and Pukkila 1990; Koreisha and Fang 2001) suggest that the GLS approach is less sensitive to initial values than the ML approach for small sample sizes (50-200 observations). As a result, we initially report GLS estimates as our baseline results below, and follow these with ML estimates as robustness checks.

We note that an alternative to MA errors would be to use autoregressive (AR) errors, which is common in the DSGE literature. However, this approach yields models that are considerably prone to non-identification - even more so than VARMA models - and experimentation with models with AR errors using our data resulted in badly identified models. As a result, we confine our analysis to VARMA models. Finally, in addition to real GDP growth, we also use real investment growth as an alternative dependent variable.

5. Results

5.1. Corporate debt–growth cycles

Table 3 summarizes our baseline results for the USA, using the change in the corporate debt-income ratio as the financial variable. These are the results for the VARMA model estimated on an equation-by-equation basis using GLS, where, in addition, we add dummy variables for each of the world war years. In both the models using real GDP growth and real investment growth, the partial effect of the real variable on the debt variable is positive, and the partial effect of the debt variable on the real variable is negative. In addition, the Jacobian discriminant is negative in both models, thus the sufficient condition for oscillations is met. The Breusch-Godfrey LM test does not reject the null hypothesis of no serial correlation in all four equations, while White tests point towards the existence of heteroskedasticity. The inverted MA roots, i.e. the absolute values of the estimated coefficients of the MA(1) terms, are below 1 in each equation, and therefore the MA processes are invertible. Overall, the estimates in table 3 support the existence of endogenous corporate debt-driven fluctuations in the US economy from the late 19th / early 20th century to date. Calculating the length of the interaction cycle[5] we find that it is 16 years for the GDP growth-corporate debt system and 11.3 years for the investment growth-corporate debt system.

[5] We calculate the interaction cycle length of the 2D system following Stockhammer *et al.* (2018, p. 6).

Table 3: USA (1889-2013), Corporate debt cycles – main results

real GDP and corporate debt		
Dependent variable:	**Δ(GDP)$_t$**	**Δ (BDEBT)$_t$**
Δ(GDP)$_{t-1}$	0.624***	0.113***
Δ(BDEBT)$_{t-1}$	-0.528***	0.548***
MA(1)	-0.534***	-0.105
R^2	*0.562*	*0.432*
B-G LM test	*0.235*	*0.510*
White test	*0.000*	*0.000*
Discriminant:	*-0.233*	
Cycle length:	*16.086*	

real investment and corporate debt		
Dependent variable:	**Δ(I)$_t$**	**Δ(BDEBT)$_t$**
Δ(I)$_{t-1}$	0.523***	0.023***
Δ(BDEBT)$_{t-1}$	-3.990***	0.446***
MA(1)	-0.521***	-0.113***
R^2	*0.343*	*0.416*
B-G LM test	*0.673*	*0.165*
White Test	*0.001*	*0.000*
Discriminant:	*-0.361*	
Cycle length:	*11.318*	

Notes: *, **, and *** denotes statistical significance at the 10%, 5%, and 1% levels respectively. Values for specification tests are p-values corresponding to nR^2. B-G LM test at first lag only. Constant terms and dummy variables for the World War years are included but not reported.

For the UK economy we cover the period 1882 to 2010. Table 4 summarizes our baseline results, using the change in the corporate debt-income ratio as the financial variable, which are again the results for the VARMA model estimated on an equation-by-equation basis using GLS. In both the models using real GDP growth and real investment growth, the partial effect of the real variable on the debt variable is positive and statistically significant. The partial effect of the debt-to-income ratio on GDP and investment growth are negative, but not statistically significant in both cases. However, the discriminant of the system is positive, i.e. the sufficient condition for cycles is not met. The Breusch-Godfrey LM test fails to reject the null hypothesis of no serial correlation in all equations. This is remedied to some extent by increasing the MA lag order by one, but we still get no evidence for cycles (see Appendix A.3.). However, as we do not find evidence of interaction cycles in this model, we do not pursue this sign of (possible) misspecification further. As in the models estimated on US data, the inverted MA roots are below 1 in each equation, and therefore the MA processes are invertible. The estimates in Table 4 do not support the existence of endogenous corporate debt-driven fluctuations in the UK economy from the late 19th / early 20th century to date.

Table 4: UK (1882-2010), Corporate debt cycles – main results

real GDP and corporate debt			*real investment and corporate debt*		
Dependent variable:	**Δ(GDP)$_t$**	**Δ (BDEBT)$_t$**	*Dependent variable*:	**Δ(I)$_t$**	**Δ(BDEBT)$_t$**
Δ(GDP)$_{t-1}$	0.734***	0.100**	**Δ(I)$_{t-1}$**	0.836***	0.023*
Δ(BDEBT)$_{t-1}$	-0.114	-0.431***	**Δ(BDEBT)$_{t-1}$**	-0.397	-0.593***
MA(1)	-0.811***	0.611***	**MA(1)**	-1.000	0.706***
R^2	*0.327*	*0.344*	R^2	*0.572*	*0.345*
B-G LM test	*0.016*	*0.005*	*B-G LM test*	*0.001*	*0.004*
White test	*0.002*	*0.004*	*White Test*	*0.000*	*0.293*
Discriminant:	*1.311*		*Discriminant:*	*2.005*	

Notes: *, **, and *** denotes statistical significance at the 10%, 5%, and 1% levels respectively. Values for specification tests are p-values corresponding to nR^2. B-G LM test at first lag only. Constant terms and dummy variables for the World War years are included but not reported.

To evaluate the robustness of our findings for the USA, we estimate the model as a system with maximum likelihood using the Kalman filter. The results confirm the robustness of the findings, as the off-diagonal elements keep the expected signs and remain statistically significant. Table 5 reports the results for the VARMA models estimated with maximum likelihood using the Kalman filter. All of the coefficients are now statistically significant, at least at the 10% level, and thus confirm that the necessary conditions for Minskyan interaction cycles are satisfied for the USA. Moreover, the discriminants of the relevant Jacobian matrices for each VARMA model is negative, and therefore the sufficient conditions for oscillations are met. The GDP growth-corporate debt system exhibits a cycle length of 15.6 years, whilst the investment growth-corporate debt system generates a cycle of 10.7 years. Our results provide strong evidence for the existence of Minskyan corporate debt and output cycles in the USA, while there is insufficient evidence for such cycles for the UK.

Table 5: USA (1889-2013), Corporate debt cycles – robustness checks (VARMA maximum likelihood)

real GDP and corporate debt			*real investment and corporate debt*		
Dependent variable:	**Δ(GDP)$_t$**	**Δ (BDEBT)$_t$**	*Dependent variable*:	**Δ(I)$_t$**	**Δ(BDEBT)$_t$**
Δ(GDP)$_{t-1}$	0.481***	0.058***	**Δ(I)$_{t-1}$**	0.409*	0.022***
Δ(BDEBT)$_{t-1}$	-0.718*	0.475**	**Δ(BDEBT)$_{t-1}$**	-3.481***	0.419**
MA(1)	0.062	0.024	**MA(1)**	-0.344	-0.095
R^2	*0.335*	*0.265*	R^2	*0.078*	*0.232*
White test	*0.000*	*0.000*	*White Test*	*0.000*	*0.000*
Discriminant:	*-0.167*		*Discriminant:*	*-0.306*	
Cycle length:	*15.573*		*Cycle length:*	*10.665*	

Notes: *, **, and *** denotes statistical significance at the 10%, 5%, and 1% levels respectively. Values corresponding to specification tests are p-values. The R^2 values are computed from the one-step ahead predictions of the state space model, so differ slightly from those presented in tables 3 and 5.

As our dataset covers a long time period, we estimate recursive estimates to evaluate parameter stability of the main results for the USA an additional test for the robustness (Figures 1 and 2). We begin the recursive regressions with the initial sample set between the start date of the series and 1950, and then we increase by one year at a time. Both figures suggest that all four elements of the Jacobians for both systems are indeed stable over time.

Figure 1: Recursive coefficients GDP growth system

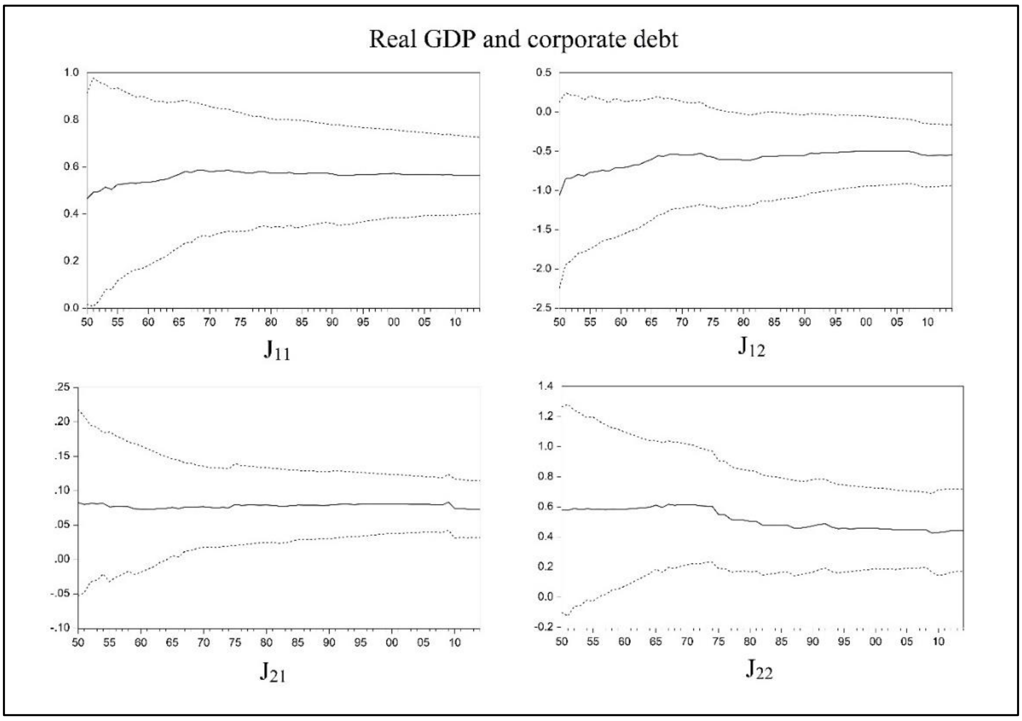
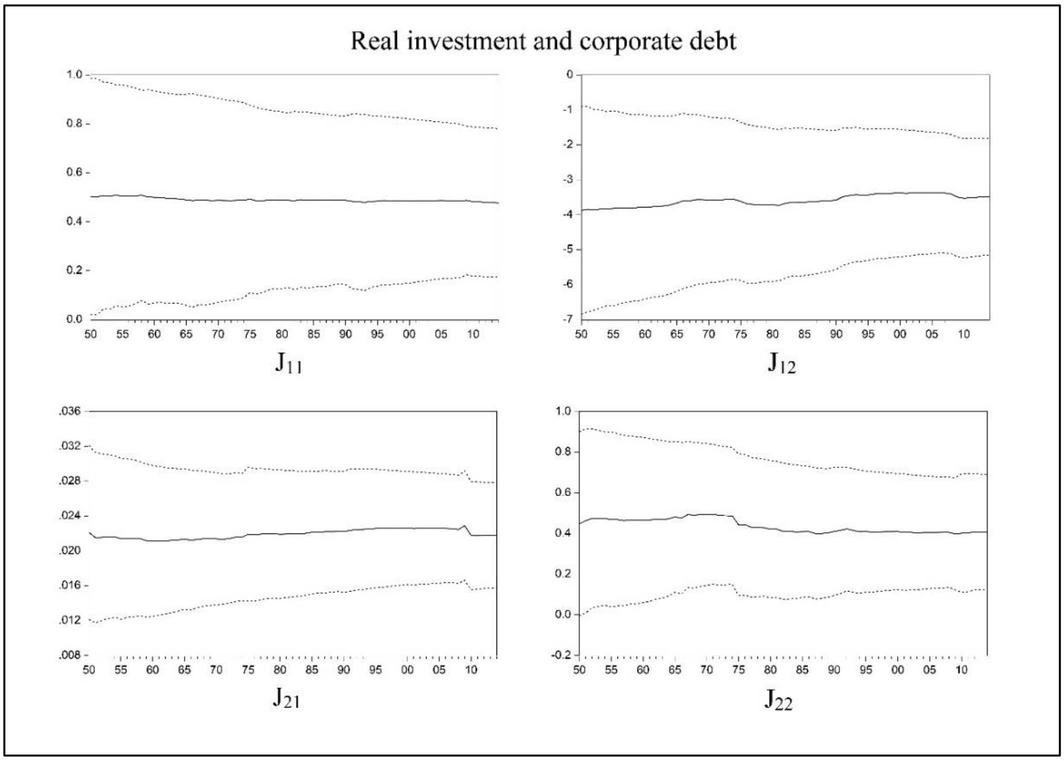


Figure 2: Recursive coefficients Investment growth system



5.2. Mortgage debt – growth cycles

Finally, to evaluate the possibility of oscillations driven by mortgage debt, we estimate the system of real GDP growth and the change in the mortgage debt-income ratio using the baseline ARMA-GLS approach. Table 6 reports the results. For the USA, we find that the mortgage leverage ratio is procyclical as expected, and statistically significant at the 1% level. We also find that increases in the mortgage debt decrease growth, but the coefficient is not statistically significant. In addition, the discriminant is negative, thus necessary and sufficient conditions for cycles are met and the implied cycle length for this system is 40.2 years. Thus for the USA there is weak evidence for endogenous mortgage-driven cycles, but below standard levels of statistical significance. For the UK, we find that the partial effect of real GDP growth on the mortgage debt ratio is negative and not statistically significant. With respect to the effects of the mortgage debt ratio on real GDP growth, we find a positive sign, and the coefficient is again not statistically significant. Overall, we do not find any evidence for Minskyan household debt-driven oscillations for the UK.

Table 6: Mortgage debt cycles - USA and UK

USA			*UK*		
Dependent variable:	**Δ(GDP)$_t$**	**Δ (MDEBT)$_t$**	*Dependent variable*:	**Δ(GDP)$_t$**	**Δ(MDEBT)$_t$**
Δ(GDP)$_{t-1}$	0.606***	0.100**	**Δ(GDP)$_{t-1}$**	0.313	-0.010
Δ(MDEBT)$_{t-1}$	-0.134	0.444***	**Δ(MDEBT)$_{t-1}$**	0.307	0.901***
MA(1)	-0.604***	0.618***	**MA(1)**	0.010	-0.062
R^2	*0.515*	*0.562*	*R^2*	*0.290*	*0.767*
B-G LM test	*0.020*	*0.413*	*B-G LM test*	*0.318*	*0.392*
White test	*0.000*	*0.000*	*White Test*	*0.001*	*0.096*
Discriminant:	*-0.027*		*Discriminant:*	*0.333*	
Cycle length:	*40.215*		*Cycle length:*		

Notes: *, **, and *** denotes statistical significance at the 10%, 5%, and 1% levels respectively. Values for specification tests are p-values corresponding to nR^2. B-G LM test at first lag only. Constant terms and dummy variables for the World War years are included but not reported.

6. Conclusions

While most contributions to the literature on financial cycles either focus on univariate cycles in financial variables or on the negative growth effects of indebtedness, this paper provides an empirical examination of Minsky's theory of endogenous cycles resulting from the interaction of pro-cyclical debt-to-income ratios and the negative effect of debt on growth. We estimate simple VARMA models in debt and output, in which the conditions for oscillations driven by the interaction between debt and GDP growth can be easily evaluated. We use historical macroeconomic data for the USA and the UK that covers a period between 1880 and 2015. This is the longest time horizon examined in any econometric study of Minskyan cycles (see Palley, 1994; Kim 2013, 2016; Stockhammer et al. 2018), and ours is the first to use VARMA models.

Our results provide robust evidence for Minskyan cycles driven by the interaction of corporate debt and real investment growth or real GDP growth for the USA. We find that increases in the corporate debt-income ratio decrease investment and GDP growth, while increases in real investment or GDP growth lead to increases in the debt-income ratio. The implied cycle length is 16 and 11 years (for GDP and investment respectively), thus longer than the regular business cycle. For the UK, while the necessary conditions hold in terms of signs, the partial effect of debt on real GDP or investment growth is not statistically significant and the discriminant of the system is positive. Thus, there is no evidence for corporate cycles in the UK.

Our results for mortgage debt-driven cycles offer weak evidence for the existence of a Minskyan cycle driven by the interaction of mortgage debt and real GDP growth for the USA, where necessary and sufficient conditions hold, but the effect of debt on growth is not statistically significant. The implied cycle length is 40 years, substantially beyond business cycle frequency. For the UK there is no evidence of mortgage debt driven cycles.

Our approach sheds light on the endogeneity of a debt-driven cycle mechanism which is consistent with Hyman Minsky's theory of endogenous crises and New Keynesian theories of the financial accelerator. It suggests that corporate debt plays the key role in the business cycle. Perhaps surprisingly, given the prominence that household debt has gained debates on financial instability since the Global Financial Crisis, our results for mortgage debt are weaker than for corporate debt. Our results are to some extent comparable to those of Stockhammer *et al.* (2018), who apply a similar methodology for a larger group of countries but for a much shorter time span. Across seven countries they report some evidence for business debt-driven cycles, which are longer than regular business cycles, but no support for mortgages debt-driven cycles. However, their sample is too short to reliably test for cycles of 40 years. There are differences in the details: They find only weak evidence for corporate debt cycles in the USA, but stronger results corporate debt-driven cycles for the UK. Next to the different time spans, a possible explanation for the differences in statistical significance is the use of different data sources.[6]

The fact that we find only weak evidence for mortgage debt-driven business cycles does not necessarily imply that mortgage debt is not important. In particular, our results are not inconsistent with findings that household debt deepens recessions. A possible way to reconcile our findings with the Minskyan literature on household debt is to interpret changes in mortgage debt as driven by speculative real estate prices (e.g. Ryoo 2016). This suggests that integration of asset prices (and possibly monetary policy) and the estimation of higher order, fully specified finance-driven models would be a useful next step. Finally, it would be interesting to explore effects operating through second moments - for example, whether or not raised debt levels affect the volatility of GDP - given that the majority of our models display some form of heteroskedasticity. We leave this to future work.

[6] Stockhammer *et al.* (2018) use BIS data, which has broader definition of debt that includes debt to non-banks.

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Appendices

A.1. Historical Macroeconomic Data sources and unit root tests

Table A1: Data Sources

Country	Variable	Period	Source
UK	GDP (real)	1850-2015	Hills et al. (2015)
	Investment (real)	1850-2015	Hills et al. (2015)
	Business Debt (nominal)	1880-2013	Jordà et al. (2017)
	Mortgage Debt (nominal)	1880-2013	Jordà et al. (2017)
	GDP (nominal)	1870-2013	Hills et al. (2015)
USA	GDP (real)	1929-2015	BEA NIPAs
	Investment (real)	1889-1929	Kuznets and Jenks (1961)
		1929-2015	BEA NIPAs
	Total Credit (nominal)	1889-2013	Jordà *et al.* (2017)
	Mortgage Credit (nominal)	1889-2013	Jordà et al. (2017)
	GDP (nominal)	1889-2013	Jordà *et al.* (2017)

Table A2: ADF Unit Root Tests

Country	Variable	ADF test		
		Levels	1st Differences	Conclusion
UK	*GDP*	(1)	(0)	I(1)
	BDEBT	(0.95)	(0)	I(1)
	MDEBT	(0.99)	(0.1)	I(1)
	I	(0.14)	(0)	I(1)
US	*GDP*	(0.92)	(0)	I(1)
	BDEBT	(0.53)	(0)	I(1)
	MDEBT	(0.03)	(0)	I(1)
	I	(0.954)	(0)	I(1)

Notes: Values corresponding to ADF tests are p-values.

A.2. Endogenous oscillations in 2D dynamic systems

A system of difference equations exhibits endogenous oscillations if the eigenvalues of the relevant Jacobian matrix are complex conjugates (see Chiang 1984, pp. 633-45). In a two dimensional system, the eigenvalues are those λ which satisfy,

$$\lambda^2 - \lambda Tr(J) + \det(J) = 0 \;\Rightarrow\; \lambda_{1,2} = \pm \frac{Tr(J) \pm \sqrt{Tr(J)^2 - 4\det(J)}}{2}.$$

The sufficient condition for oscillations is therefore that the discriminant $\Delta = Tr(J)^2 - 4\det(J)$ is negative. The discriminant of a 2D Jacobian matrix can be calculated as a function of its trace and determinant as follows:

$$\Delta = Tr(J)^2 - 4\det(J) < 0 \;\Leftrightarrow (J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{21}J_{12}) < 0$$
$$\Leftrightarrow (J_{11} - J_{22})^2 + 4J_{21}J_{12} < 0.$$

As $(J_{11} - J_{22})^2$ is positive, a necessary conditions for oscillations is therefore that the product of the off-diagonal elements of the Jacobian matrix, $J_{21}J_{12}$, is negative. See e.g. Stockhammer *et al.* (2018) for further discussion.

Figure A2: USA series graphs

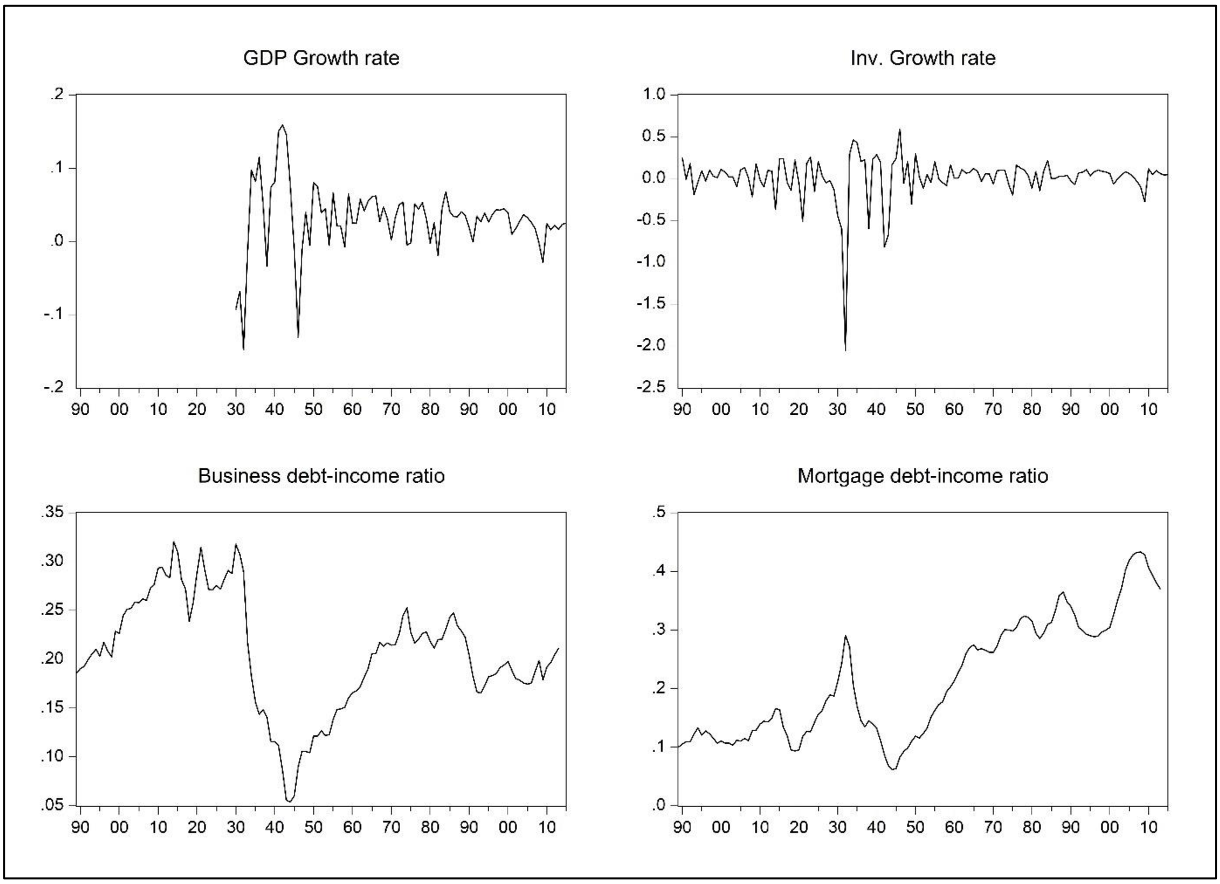
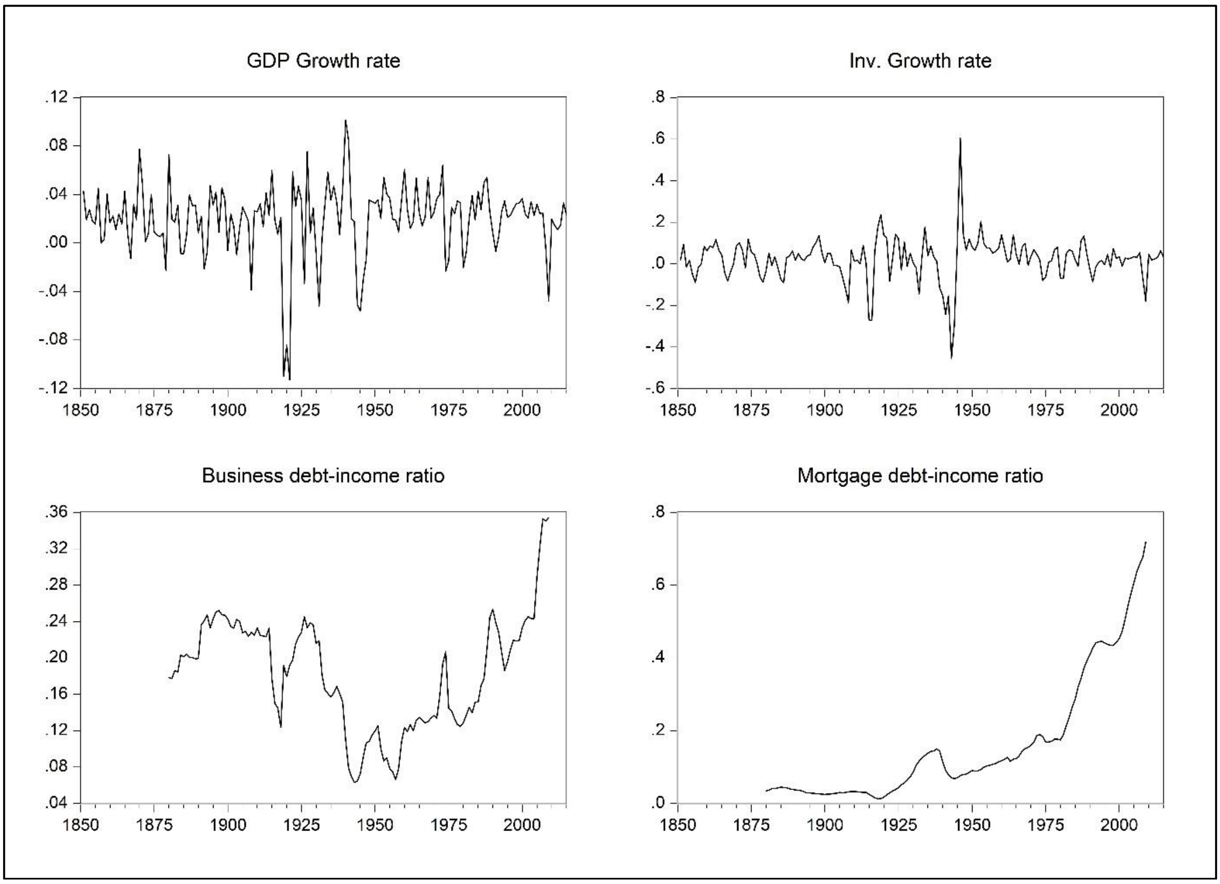


Figure A3: UK series graphs



A.3. UK (1882-2010), Corporate debt cycles – MA(2) errors

Table A3: UK (1882-2010), Corporate debt cycles – main results

real GDP and corporate debt		
Dependent variable:	**Δ(GDP)$_t$**	**Δ (BDEBT)$_t$**
Δ(GDP)$_{t-1}$	-0.105	0.102**
Δ(BDEBT)$_{t-1}$	-0.088	-0.468***
MA(1)	0.483**	0.776***
MA(2)	0.428***	0.303**
R^2	*0.315*	*0.388*
B-G LM test	*0.288*	*0.330*
White test	*0.001*	*0.025*
Discriminant:	*0.096*	

real investment and corporate debt		
Dependent variable:	**Δ(I)$_t$**	**Δ(BDEBT)$_t$**
Δ(I)$_{t-1}$	0.837***	0.019*
Δ(BDEBT)$_{t-1}$	-0.393	-0.647***
MA(1)	1.029	0.928***
MA(2)	0.029	0.262***
R^2	*0.572*	*0.387*
B-G LM test	*0.001*	*0.400*
White Test	*0.003*	*0.060*
Discriminant:	*2.172*	

Notes: *, **, and *** denotes statistical significance at the 10%, 5%, and 1% levels respectively. Values for specification tests are p-values corresponding to nR^2. B-G LM test at first lag only. Constant terms included but not reported.