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Convergence of actual, warranted, and natural growth rates in a Kaleckian-Harrodian-classical model

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Abstract

This paper describes a dynamic one-sector macroeconomic model that draws on both post-Keynesian and classical/neo-Marxian themes. The model features an equilibrium in which Harrod's actual, warranted, and natural growth rates coincide. Dynamic processes unfolding over both short and long time scales lead the economy to exhibit both business cycles and long waves. The Keynesian stability condition is assumed not to hold, so the model features short-run instability, which is bounded from above by a utilization ceiling. Labor constraints affect distribution through conflict pricing. In contrast to other Kaleckian-Harrodian models, we do not assume an exogenous source of demand. Instead, short-run instability is bounded from below by firms' expectations that the downturn will eventually reverse.

Keywords: Kaleckian; Harrodian; classical; neo-Marxian; cycles; long waves; technological change

JEL codes: B50, E32, O40

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1 Introduction

Post-Keynesian theory has a rich tradition (Hein, 2014; Kurz and Salvadori, 2010). Besides Keynes, important contributors include Kalecki, Kaldor, Harrod, and Robinson (Lavoie, 2006, Fig. 1.1). While all post-Keynesian theory retains Keynes' argument that output is determined by the level of effective demand, that rich diversity offers sometimes conflicting explanations of economic phenomena. Kalecki explained resource underutilization when prices are set in oligopolistic markets (Kalecki, 1969; Sawyer, 1999), with capital utilization as the accommodating variable; Kaldor (1955), elaborating on themes developed by Robinson and others (*op. cit.* fn. 3), explained how the functional income distribution is determined under an assumption of fixed saving propensities, with the profit (or wage) share as the accommodating variable; Harrod (1939) identified potential instabilities along a long-run growth path.

Theoretical divergence has been a source of both inspiration and consternation to contemporary post-Keynesian theorists. In the search for a common core, the neo-Kaleckian family of models, first proposed by Del Monte (1975), has performed well. It explains underutilization of capital and cleanly illustrates the paradoxes of thrift and of costs (Lavoie, 2014). Work by Rowthorn (1981), Dutt (1984, 1987), Taylor (1985), Blecker (1989), and Bhaduri and Marglin (1990), among others, considerably expanded Kalecki's original ideas to cover stagnation, "exhilaration", and open economies. Many of these results follow from an assumption that "Keynesian stability" holds; that is, that savings responds more strongly than does investment to a change in capacity utilization.

With greater prominence, the neo-Kaleckian model drew closer scrutiny, and – perhaps unavoidably – criticism (Lavoie, 1995; Hein et al., 2011). A major point of contention is whether capacity utilization can deviate persistently from the level desired by firms. As Skott (2012) points out, firms in fact have a substantial measure of control over the factors that determine long-run utilization, so there is no practical reason they should be unable to meet (or at least continually move towards) their target. And firms are aware of their potential utilization; Corrado and Mattey (1997) argue that "those who discuss production capability with plant managers quickly discover that managers generally are quite precise about how much their facilities can produce without extraordinary efforts." It is an open question how this accommodation might be accomplished in the neo-Keynesian model.

Kurz (1986) argues that firms ought to target the least-cost level of utilization as part of their choice of technique, a proposition that appears consistent with observed micro-level firm behavior (Mattey and Strongin, 1997). Building on this idea, Nikiforos (2013) created a model in which utilization is determined by a cost-minimizing firm that was subsequently applied to a Kaleckian analysis by Nikiforos (2016) and Dávila-Fernández et al. (2019).

If target utilization is exogenous, then a natural parameter to adjust endogenously in order to meet it is the "animal spirits" term in an investment function or, in the terminology of Hicks (1950), "autonomous investment", which reflects firms' medium-run to long-run demand expectations. Yet, this gives rise to Harrodian instability (see, e.g., Hein, 2014, p. 32). The sources of the instability and some proposed solutions are reviewed and critiqued by Hein et al. (2011) and Girardi and Pariboni (2019); from these papers, and the response by Lavoie (2019) to Dávila-Fernández et al. (2019), it is reasonable to conclude that neither Kaleckians nor Harrodians feel convinced by the mechanisms proposed by their counterparts in the debate.

One possibility that has been explored by a small number of authors is to abandon the Keynesian stability condition in order to tame long-run Harrodian instability. Skott (2012) proposes such a "Kaleckian-Harrodian" model, in which the Keynesian stability condition holds in the short run but not the long run. Fazzari et al. (2013) propose a model in which the Keynesian stability condition does not hold even in the short run. We follow the latter approach. As this stabilizes adaptive expectations of autonomous investment, the result, somewhat paradoxically, is long-run *stability*, as the actual growth rate tends towards the warranted rate (Fazzari et al., 2013, 2018). Thus, abandoning the Keynesian stability condition resolves Harrod's unstable dynamics in the long run. The price that must be paid for that result is unstable dynamics in the short run, which are contained from above by one or more inputs to production. In Fazzari et al. (2013), labor is the limiting input, while in this paper the capital stock sets a hard upper bound. Instability must also be contained from below. In a novel contribution to the Kaleckian-Harrodian literature, we implement such a bound by assuming that firms anticipate that downturns will eventually reverse. Thus, in contrast to some recent Kaleckian-Harrodian models (e.g., Fazzari et al., 2018; Serrano et al., 2019; Fiebiger and Lavoie, 2019), we need not assume an autonomous source of demand.

But first we address a possible objection. On its face, the evidence appears to support the Keynesian stability condition, contrary to what we assume in this paper. The mid-20th century saw an empirically-grounded debate over the relative merits of the impulse-response business cycle models of Frisch (1933), on one hand, and multiplier-accelerator models (Clark, 1917; Kahn, 1931; Samuelson, 1939; Hicks, 1950), on the other. While multiplier-accelerator models predominated for about half a century, by the early 1970s the consensus view among econometricians was that endogenous saving and expenditure dynamics are stable, so cycles must be the result of external shocks from which the economy subsequently recovers (Hymans, 1972). This result can be interpreted as justifying the long-standing post-Keynesian assumption that utilization dynamics are stable. In the mainstream literature, theoretical developments followed empirical developments with a lag, emerging in the form of real business cycle (RBC) models (Lucas, 1975; Long and Plosser, 1983; King and Rebelo, 1999).

However, the evidence is not as strong as it first appeared. Blatt (1978) showed that the econometric tests in use in the early 1970s, which assumed a linear model, would falsely suggest stable dynamics even when the underlying process was unstable and nonlinear. Blatt showed that linear models produce symmetric cycles, so the well-known asymmetry of business cycles, with short contractions and long expansions, is *prima facie* evidence of nonlinearity. Subsequent econometric tests have provided additional evidence of nonlinearity for business cycles in the US and Europe (Brock, 1991; Teräsvirta and Anderson, 1992; Pesaran and Potter, 1997; Clements and Krolzig, 1998; Razzak, 2001; Belaire-Franch and Contreras, 2003). These findings are complementary to those of Skott and Zipperer (2012), who found, in contrast to Lavoie et al. (2004), that empirical estimates of parameters for a Kaleckian model specification imply unstable rather than stable dynamics.

In this paper we use a Kaleckian-Harrodian mechanism to bring Harrod's actual and warranted rates into alignment. At the peak of the business cycle, the goods market is closed through forced saving. We combine those behaviors with classical (or neo-Marxian) mechanisms to endogenously bring the natural and actual rates into alignment (Foley, 2003; Julius, 2005; Shaikh, 2016, p.652, and citations in the Appendix), to create a Kaleckian-Harrodian-classical model. The classical mechanisms are cost share-induced technological change combined with conflict-based price and wage setting. With these assumptions, productivity growth depends on the functional income distribution, which is in turn influenced by productivity. The result is a stabilizing mechanism that aligns Harrod's actual and natural growth rates. A recurring finding in the literature is that this mechanism damps Goodwin (1951) cycles (Foley, 2003; Julius, 2005). In this paper, a Goodwin-type mechanism drives changes in the functional income distribution over long-period cycles, while utilization varies over short-period cycles arising from the Harrodian instability.

The main novelty of this paper is to demonstrate how the combination of the post-Keynesian and classical/neo-Marxian mechanisms leads to convergence of the actual, warranted, and natural growth rates as a long-run tendency while generating both business cycles and Kondratieff (1979) type long waves. The convergence takes place within a disequilibrium framework. Thus, persistent cycles and bounded dynamics are derived, rather than assumed. This stands in sharp contrast to computable general equilibrium (CGE) or dynamic stochastic general equilibrium (DSGE) models, which implicitly assume that stable dynamics drive the economy rapidly towards equilibrium. As noted above, a further novelty is that firms' expectations that business cycle downturns will eventually reverse place a floor under a contraction.

2 Two linear Kaleckian-Harrodian models

We build towards a model with nonlinear utilization dynamics by examining two models with linear investment functions: a standard neo-Kaleckian model and a neo-Kaleckian model with linear forecasts of the expected utilization rate.

In all of the models in this paper, we work in discrete time, and the goods market is closed by adjusting capacity utilization. The models are demand-led, in that orders for investment goods in the current period are issued in the previous period based on anticipated output, while saving adjusts to accommodate investment. For each model, the saving function is

$$g^s = s\kappa u, \tag{1}$$

where s is the saving rate, κ is capital productivity at full utilization, and u is capacity utilization. In the two linear models, s and κ are exogenously specified. The investment function differs between the models.

2.1 Standard neo-Kaleckian model

In a variation on the standard neo-Kaleckian model, we propose an investment function that depends on previous-period utilization,

$$g^i = \gamma + \alpha\left(u_{-1} - u_d\right). \tag{2}$$

In this equation, u_d is the capacity utilization desired by firms. If they indeed operate in the subsequent period with that level of utilization, then their investment decision will have been justified. In Harrod's terms, it leaves them "satisfied that they have produced neither more nor less than the right amount" (Harrod, 1939, p.16). The parameter α is the response of investment to a change in utilization, while γ is the investment rate that firms believe they should finance when utilization is at its desired level.

We close the goods market by setting $g^s = g^i$, which yields

$$u = \frac{\alpha}{s\kappa}u_{-1} + \frac{\gamma - \alpha u_d}{s\kappa}. \tag{3}$$

This recurrence relation is stable (it generates a stationary time series) if the coefficient on lagged utilization, u_{-1}, is less than one. It therefore generates stable dynamics if $\alpha < s\kappa$, which means that the investment response to a change in utilization is smaller than the saving response – the Keynesian stability condition. The equilibrium utilization, u^* is, however, not equal to the utilization rate desired by firms. Instead, setting $u = u_{-1} = u^*$ and solving for u^*, it is equal to

$$u^* = u_d + \frac{\gamma - s\kappa u_d}{s\kappa - \alpha}. \tag{4}$$

The numerator in second term on the right-hand side of this expression includes Harrod's warranted growth rate $g_w = s\kappa u_d$. It is the rate of growth at which desired saving at the desired level of utilization is equal to actual saving. If $\gamma = g_w$, then $u^* = u_d$. Thus, if growth expectations are equal to the warranted growth rate, then utilization will be at its desired level. However, unless γ adjusts to satisfy that condition, it will only happen by accident.

Following Hein (2014, p. 32), we introduce a Harrodian adjustment mechanism, allowing γ to adjust through adaptive expectations,

$$\gamma_{+1} = \gamma + \beta\left(g^i - \gamma\right) = \gamma + \alpha\beta\left(u_{-1} - u_d\right). \tag{5}$$

Together with Eqn. (3) this gives a two-variable system in u and γ. It is convenient to rewrite this system, as captured in Eqns (3) and (5), in terms of new variables. We define $x \equiv u_{-1} - u_d$ and $y \equiv \gamma - s\kappa u_d$ and rearrange to find

$$x_{+1} = \tfrac{\alpha}{s\kappa}x + \tfrac{1}{s\kappa}y, \tag{6a}$$

$$y_{+1} = y + \alpha\beta x. \tag{6b}$$

This two-variable, two-equation system of difference equations has two independent solutions in which the variables advance (or, possibly, oscillate) at a common rate. A general solution is a linear combination of the two independent solutions. Denoting the common growth rate for either of the independent solutions by r, we have $x_{+1} - x = rx$, $y_{+1} - y = ry$, so

$$r\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{s\kappa} - 1 & \frac{1}{s\kappa} \\ \alpha\beta & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}. \tag{7}$$

This system is stable if the trace of the matrix of coefficients $\mathrm{Tr} = \alpha/s\kappa - 1$ is negative and its determinant $\mathrm{Det} = -\alpha\beta/s\kappa$ is positive (these are the Routh-Hurwitz stability criteria for a two-variable linear dynamic system). The Keynesian stability condition ensures that the trace is negative, but, because α, β, s, and κ are all positive, the determinant is also negative. Thus, this system has an unstable mode – the Harrodian instability.

In this model, long-run instability is driven by firms observing and reacting to short-run changes in utilization. That is unsatisfying because it assumes that firms extrapolate their moment-by-moment observations over the course of a business cycle into long-run expectations. More plausibly, they will build their past experience of business cycles into their expectations. We remedy this shortcoming in the next section, where we separate short-run from long-run behavior.[1]

[1] Hein et al. (2011) also separate the short and long run, but unlike in this paper, they assume short-run Keynesian stability.

2.2 The neo-Kaleckian model with linear expectations

As an intermediate step towards a model with nonlinear utilization dynamics, we modify the standard neo-Kaleckian model introduced above by adding linear expectations. In this model, firms determine their investment plans based on their expectations for utilization in the next period rather than observed utilization. The saving function is the same as in Eqn. (1), while the investment function becomes

$$g^i = \gamma + \alpha \left(u_{-1} + \Delta u^e - u_d \right). \tag{8}$$

We assume that firms have accumulated experience of the business cycle, and they expect, as utilization exceeds or falls below its long-period goods market clearing equilibrium value u^*, as given by Eqn. (4), that the trend is increasingly likely to reverse. Specifically, we assume

$$\Delta u^e = -\phi \left(u_{-1} - u^* \right) = -\phi \left(u_{-1} - u_d \right) + \phi \left(u^* - u_d \right). \tag{9}$$

Note that while u^* is expected to equal u_d in the long run, we do not impose it. Rather, we allow it to emerge from the model dynamics.

In specifying Eqn. (9) we assume that firms have become aware of the realized equilibrium utilization u^* and anchor their expectations to it. While we could explicitly introduce adaptive expectations for firms' estimate of the equilibrium utilization, this model is merely a stepping-stone on the way to a nonlinear model, where we do include adaptive expectations. The reason to introduce this linear model as a preliminary step is that linear specifications are easier to conceptualize and analyze than are nonlinear specifications.

Closing the (instantaneous, out-of-equilibrium) goods market, and defining, as before, $x \equiv u_{-1} - u_d$, the evolution of x is given by

$$x_{+1} = \frac{(1-\phi)\alpha}{s\kappa} x + \frac{\alpha\phi}{s\kappa} \left(u^* - u_d \right) + \frac{\gamma - s\kappa u_d}{s\kappa}. \tag{10}$$

In this case, the dynamics are stable as long as the coefficient on x_{-1} is less than one; that is, if $(1-\phi)\alpha < s\kappa$. Depending on the magnitudes of the parameters, that condition can be satisfied even if $\alpha > s\kappa$. Requiring ϕ to lie between zero and one, these conditions can be written

$$1 > \frac{s\kappa}{\alpha} > 1 - \phi > 0. \tag{11}$$

This outcome is plausible – at least, within the motivating story for this model. Firms will, over time, find the value of ϕ that allows them to anticipate reversals in utilization over the business cycle (the second inequality). As with the firms' beliefs about u^*, we could make ϕ an explicitly adaptive parameter, but as with u^*, we avoid additional complications in a model we are using only as a way-point towards our proposed nonlinear model.

By its definition in Eqn. (9), $\Delta u^e = 0$ when u is at its equilibrium value u^*, which is the same as in Eqn. (4). Substituting that expression into Eqn. (10) and defining, as before, $y \equiv \gamma - s\kappa u_d$, we have

$$x_{+1} = \frac{(1-\phi)\alpha}{s\kappa} x + \frac{s\kappa - (1-\phi)\alpha}{s\kappa - \alpha} \frac{1}{s\kappa} y. \tag{12}$$

Defining γ_{+1} as in Eqn. (5), but using Eqn. (8) for g^i, and substituting using x and y, we find

$$y_{+1} = \left(1 + \frac{\phi\alpha\beta}{s\kappa - \alpha} \right) y + \alpha\beta(1-\phi)x. \tag{13}$$

Following the previous development, this system of difference equations, in matrix form, gives the following equation for the growth rates of the independent solutions:

$$r\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{(1-\phi)\alpha}{s\kappa} - 1 & \frac{s\kappa-(1-\phi)\alpha}{s\kappa-\alpha}\frac{1}{s\kappa} \\ \alpha\beta(1-\phi) & \frac{\phi\alpha\beta}{s\kappa-\alpha} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}. \tag{14}$$

We assume the inequalities in (11), in which case the signs of the terms in the matrix are

$$\operatorname{sgn}\begin{pmatrix} \frac{(1-\phi)\alpha}{s\kappa} - 1 & \frac{s\kappa-(1-\phi)\alpha}{s\kappa-\alpha}\frac{1}{s\kappa} \\ \alpha\beta(1-\phi) & \frac{\phi\alpha\beta}{s\kappa-\alpha} \end{pmatrix} = \begin{pmatrix} - & - \\ + & - \end{pmatrix}. \tag{15}$$

From this it can be seen that the trace will be negative and the determinant positive. This system is therefore stable.

2.3 Comments

In the standard neo-Kaleckian model with adaptive expectations for "animal spirits", the accelerator is purely from the γ dynamics, while utilization is damped. The result is Harrodian instability, because there is nothing in the standard model to stop γ from growing without bound.[2] Effectively, there is no business cycle; there is a business explosion.

In the model with expectations, while short-run expectations are driven by observations of recent changes in utilization, long-run growth expectations, which inform beliefs about the parameter γ, are formed with reference to the realized equilibrium utilization u^*. On short time scales, firms expect utilization to revert to its equilibrium value and they build that expectation into their investment plans; on long time scales, they realize that potential growth rates and utilization may drift, and they adjust their expectations accordingly. The result is that the long-run and short-run dynamics are separated, with the combined system exhibiting a stable equilibrium at which long-run realized utilization is equal to firms' desired utilization: $u^* = u_d$.

This is close to what we want, but it goes too far. There are no cycles in this model, which can be traced back to an unsatisfying assumption that firms immediately start anticipating a turnaround at the slightest deviation from long-run realized utilization. That is unrealistic. It is more plausible to assume that firms will take advantage of the opportunities provided by an expansion, and only anticipate a reversal close to the upper capacity limit.[3] This leads us to a nonlinear model, which is the subject of the next section.

3 A nonlinear Kaleckian-Harrodian model

For the nonlinear model, we retain the investment function with expectations from the previous section as given in Eqn. (8). However, we assume a different specification for

[2] Harrod (1939) is vague about whether his instability refers to long-run expectations (captured in this model by γ) or short-run responses (that is, Keynesian instability). Long and short run dynamics are conflated in passages such as, "A departure from equilibrium, instead of being self-righting, will be self-aggravating. [The warranted growth rate] represents a moving equilibrium, but a highly unstable one. Of interest this for trade-cycle analysis!" (Harrod, 1939, p. 22). The analysis in this paper does not depend on what precisely Harrod meant. Purely for purposes of presentation, we identify Harrodian instability with long-run expectations.

[3] This approach can be contrasted with that of Setterfield (2019), who suggests, as we do, that firms change their behavior near a utilization boundary. However, for Setterfield the boundary is a dynamic parameter, whereas here we assume that it is determined by firms' production capacity.

expected utilization. Below, we will argue that there is likely to be a maximum economy-wide utilization $u_{\max}$ that is reached below full utilization of all capacity in the economy. For now, we assume such a value exists and that firms have a reasonable understanding of what it is. Away from that limit, we assume that during an expansion, firms expect utilization to increase. During a contraction, firms do not take the previous-period change in utilization as a reliable indicator of the next period. They may believe that it will be larger, or at least wish to prepare for that case, if they are more concerned about potential losses than they are hopeful of future gains (Tversky and Kahneman, 1991). Alternatively, they may anticipate a reversal, believing that the economy is fundamentally viable but has suffered a correction at the top of the cycle; the correction may be deep or shallow, but it will eventually end, and they want to ensure they have sufficient capacity to participate in the recovery. These considerations lead us to the following specification,

$$\Delta u^e = \begin{cases} \Delta u_{-1}, & \Delta u_{-1} > 0 \text{ and } u_{-1} + \Delta u_{-1} < u_{\max}, \\ u_{\max} - u_{-1}, & \Delta u_{-1} > 0 \text{ and } u_{-1} + \Delta u_{-1} > u_{\max}, \\ \theta \Delta u_{-1}, & \Delta u_{-1} < 0, \text{ where } \theta > 0. \end{cases} \tag{16}$$

This is nonlinear because behavior changes at thresholds, whether at the peak ($u = u_{\max}$) or between an expansion and a contraction.

During an expansion, when $u_{-1} + \Delta u_{-1} < u_{\max}$, the specification above gives the following recurrence relation for utilization,

$$u = \frac{\alpha}{s\kappa}\left(u_{-1} + \Delta u_{-1}\right) + \frac{\gamma - \alpha u_d}{s\kappa} = \frac{\alpha}{s\kappa}\left(2u_{-1} - u_{-2}\right) + \frac{\gamma - \alpha u_d}{s\kappa}. \tag{17}$$

To study utilization dynamics in isolation, we shift utilization by a constant factor and define

$$z \equiv u - \frac{\gamma - \alpha u_d}{s\kappa - \alpha}. \tag{18}$$

That eliminates the constant term in Eqn. (17), and we find

$$z = \frac{\alpha}{s\kappa}\left(2z_{-1} - z_{-2}\right). \tag{19}$$

We insert a growth factor $R = 1 + r$, so that $z_{-1} = Rz_{-2}$ and $z = R^2 z_{-2}$, and find the following quadratic equation for R,

$$R^2 = \frac{\alpha}{s\kappa}\left(2R - 1\right). \tag{20}$$

The solutions are

$$R_{\pm} = \frac{\alpha}{s\kappa}\left(1 \pm \sqrt{1 - \frac{s\kappa}{\alpha}}\right). \tag{21}$$

If $\alpha < s\kappa$ – that is, if the Keynesian stability condition holds – then this gives rise to stable behavior for utilization, as in the standard Kaleckian model. The magnitudes of R_+ and R_- are both less than one, and they have an imaginary component that indicates a damped cycle. Of interest for the present paper is the case in which the Keynesian stability condition fails to hold, so that $\alpha > s\kappa$. In that case, both R_+ and R_- are real, and the magnitude of R_+ is greater than one. That is, the system exhibits an unstable mode.

The instability may drive either expansion or contraction. In this paper we focus on the case of an unstable expansion. (An unstable contraction could lead to a severe recession, requiring a degree of government intervention that is not captured in the model.) The unstable expansion is contained by the upper limit on utilization, as in Eqn. (16). Firms are aware, from experience, that the economy cannot produce more than a maximum level (below full capacity), so their expectations for next-period utilization do not exceed that level. When the limit is reached, the next-period utilization consistent with desired saving, which we denote u_s, is given by

$$u_s = \frac{\alpha}{s\kappa} u_{\max} + \frac{\gamma - \alpha u_d}{s\kappa}. \tag{22}$$

We now explore the implications of this equation.

3.1 Utilization in an economy with multiple sectors

In the models presented above, capacity utilization normally clears the goods market. However, utilization is also bounded by a ceiling. In implementing the ceiling, the limitations of a one-sector model become apparent, because a one-sector economy has no coordination problems. In an economy with many sectors, some sectors (for example, construction) will reach capacity constraints before others at the peak of a business cycle, so the economy in the aggregate hits its peak before average utilization reaches 100%. This is the reason to expect a maximum level below full capacity utilization, $u_{\max} < 1$.

In this section we provide a simple mechanism for determining utilization in the presence of capacity constraints, assuming that the economy in fact contains many sectors. In principle, any constraint might limit an expansion, whether on capital utilization, employment, critical raw materials (such as energy or, in an agricultural economy, the harvest), foreign exchange to buy intermediate and investment goods, or any other necessary input to production. In this paper we show how capacity utilization can provide a constraint in a Kaleckian-Harrodian model. In the next section, we will introduce the labor constraint through a Marxian-inspired wage conflict model.

As noted above, closing the goods market by setting $g^s = g^i$ gives the level of utilization u_s compatible with desired saving behavior. When that level is below the maximum, there is no impediment to meeting it, and $u = u_s$. Otherwise, if $u_s > u_{\max}$, realized utilization will be at a lower level u'. The saving propensity must therefore be at a higher level s', such that actual saving is sufficient to fund investment,

$$s'\kappa u' = s\kappa u_s = \frac{S}{K}\frac{I}{K} \qquad \Rightarrow \qquad s' = s\frac{u_s}{u'}. \tag{23}$$

In Kaldorian models, saving adjustment occurs through a change in the functional income distribution at fixed saving propensity. In this paper, we assume that orders for investment goods, which were placed in the previous period, are always filled. As a consequence, when the economy as a whole is operating at full capacity, such that $u_s > u_{\max}$, the model exhibits forced saving. That crowds out household final consumption, so that sectors filling orders for final household goods can fall below full capacity. If that happens, then it can lower average utilization across the whole economy.

We explore this question with a 3-sector model containing a consumption goods sector with output C, an investment goods sector with output I, and an intermediate goods sector with output N that supplies each of the others. The consumption and investment goods sectors are assumed to only provide finished goods, so the input-output system for this economy is

$$\begin{pmatrix} C \\ I \\ N \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{NC} & a_{NN} & a_{NI} \end{pmatrix} \begin{pmatrix} C \\ I \\ N \end{pmatrix} + \begin{pmatrix} C \\ I \\ 0 \end{pmatrix}. \tag{24}$$

The equations for output from the consumption and investment goods sectors cancel out, leaving one independent equation. We solve it for C to find

$$C = \frac{1}{a_{NC}} \left[(1 - a_{NN}) N - a_{NI} I \right]. \tag{25}$$

We assume that the production of investment goods is more intermediate goods-intensive than the consumption goods sector, which includes labor-intensive retail and other services. That assumption means that $a_{NI} > a_{NC}$.

GDP, Y, is equal to total final supply, $I + C$. Adding I to the expression for C in Eqn. (25) gives an expression for GDP,

$$Y = \frac{1 - a_{NN}}{a_{NC}} N - \left(\frac{a_{NI}}{a_{NC}} - 1 \right) I. \tag{26}$$

We find the economy-wide average utilization u' by dividing this expression by κK,

$$u' = \frac{1 - a_{NN}}{a_{NC}} \frac{N}{\kappa K} - \left(\frac{a_{NI}}{a_{NC}} - 1 \right) \frac{I}{\kappa K}. \tag{27}$$

Because orders for investment goods are assumed to always be filled, we can use Eqn. (23) to replace I/K by $s\kappa u_s$ in Eqn. (27). This gives

$$u' = \frac{1 - a_{NN}}{a_{NC}} \frac{N}{\kappa K} - \left(\frac{a_{NI}}{a_{NC}} - 1 \right) s u_s. \tag{28}$$

If there are no constraints on production from sector N, then we can replace u_s with u and solve this equation for N in terms of utilization. However, if N is at it maximum potential, which we denote by $N_{\max}$, then utilization is constrained.

Suppose that the intermediate sector is at its potential, so that $N = N_{\max}$, but we have just reached the potential, so that utilization corresponding to desired saving is equal to realized utilization: $u' = u_s$. They are both equal to the maximum level of utilization before a correction becomes necessary, so we can replace both u' and u_s with $u_{\max}$ in Eqn. (28). That gives the following relationship,

$$\left[1 + \left(\frac{a_{NI}}{a_{NC}} - 1 \right) s \right] u_{\max} = \frac{1 - a_{NN}}{a_{NC}} \frac{N_{\max}}{\kappa K}. \tag{29}$$

Substituting this expression into Eqn. (28), we find

$$u' = u_{\max} - \left(\frac{a_{NI}}{a_{NC}} - 1 \right) s \left(u_s - u_{\max} \right). \tag{30}$$

Earlier, we argued that a_{NI} is expected to be greater than a_{NC}, so this equation is declining in u_s as it exceeds $u_{\max}$.

The form for utilization under capacity constraints in Eqn. (30) is specific to the three-sector model in Eqn. (24), which is very special indeed. More generally, we retain the feature of Eqn. (30) that realized utilization is declining in investment (captured by $su_s = I/\kappa K$) once the economy has reached its maximum utilization rate $u_{\max}$. We generalize the expression by replacing the composite technical coefficient $(a_{NI}/a_{NC} - 1)$ with a generic coefficient $\tau > 0$, so that

$$u' = u_{\max} - \tau s\,(u_s - u_{\max})\,. \tag{31}$$

3.2 Updating growth expectations and closing the model

During an expansion, changes in utilization are dominated by an unstable mode with growth factor R_+ as given in Eqn. (21). As noted earlier, that instability can drive either an expansion or a contraction, but we assume in this paper that it drives an expansion. Because we have not put a hard floor under the dynamics, an unstable movement towards negative values would lead to collapse in the model. More realistically, it would threaten a deep recession, which might be contained by government intervention. Below, we choose parameters that are plausible, but that also allow for recurring cycles without collapse.

In nonlinear models, equilibrium values generally differ from long-run averages (Blatt, 1983b). Rather than anchoring to the (unstable) equilibrium utilization, we assume that firms anchor their expectations to observed long-run utilization by smoothing over a time period T_u that encompasses at least one business cycle. Denoting firms' assessment of long-run utilization by $\bar{u}$, they update it using

$$\bar{u}_{+1} = \frac{T_u - 1}{T_u}\bar{u} + \frac{1}{T_u}u. \tag{32}$$

Firms then adjust their expectation for growth based on how far the long-run average utilization is from their desired level, smoothing over a time period T_γ,

$$\gamma_{+1} = \gamma + \frac{1}{T_\gamma}\left(\bar{u} - u_d\right). \tag{33}$$

Note that in this equation $1/T_\gamma$ replaces the product $\alpha\beta$ that we used before.

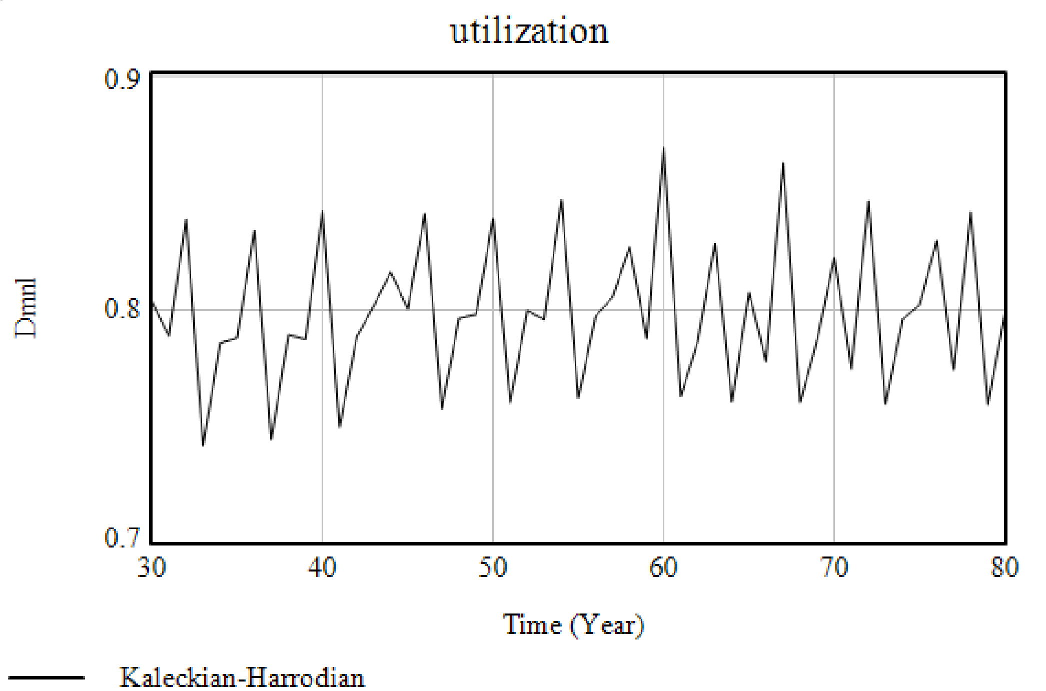
With this specification for updating the parameter γ, the model is closed. It can exhibit a wide range of behaviors, including collapse, as we noted above. We are particularly interested in persistent cycles, and avoid parameter ranges that exhibit collapse. We set desired utilization u_d at 80% and $u_{\max}$ to 87%, which are typical for total industry in the US prior to the onset of the "Great Moderation" in the mid-1980s. We set the saving rate to 23.5%, which is the average of an assumed rate of saving out of wages of 10% and out of profits of 40%, with a profit share of 45%. Capital productivity at desired utilization[4] is set to 0.30/year, which is close to the historical average for the US.[5] Other parameters are provided in the caption to Fig. 1, which shows an example of a business cycle regime in this "Kaleckian-Harrodian" model. The figure starts at year 30 to allow for transients to die out during a "burn-in" period, and extends for 50 years to year 80 of the simulation. The cycles vary in length, from three to seven years (36-84 months), with an average of 4.9 years (58.5 months), and are visibly asymmetric, with short contractions (one year, or 12 months) and long expansions. Cycles in the model are comparable to, although slightly

[4] That is, it is a typical value for $Y/K = \kappa u$, which can be calculated from historical data. Setting $u = u_d$ gives the value of κ used in the model.

[5] Using data from the Penn World Tables ver. 9.1 (Feenstra et al., 2015), the mean capital productivity between 1950 and 2017 is 0.28/year, and between 2000 and 2017 was 0.31/year.

shorter than, post-war US business cycles,[6] which had an average contraction of 11 months and cycle length of 69 months.

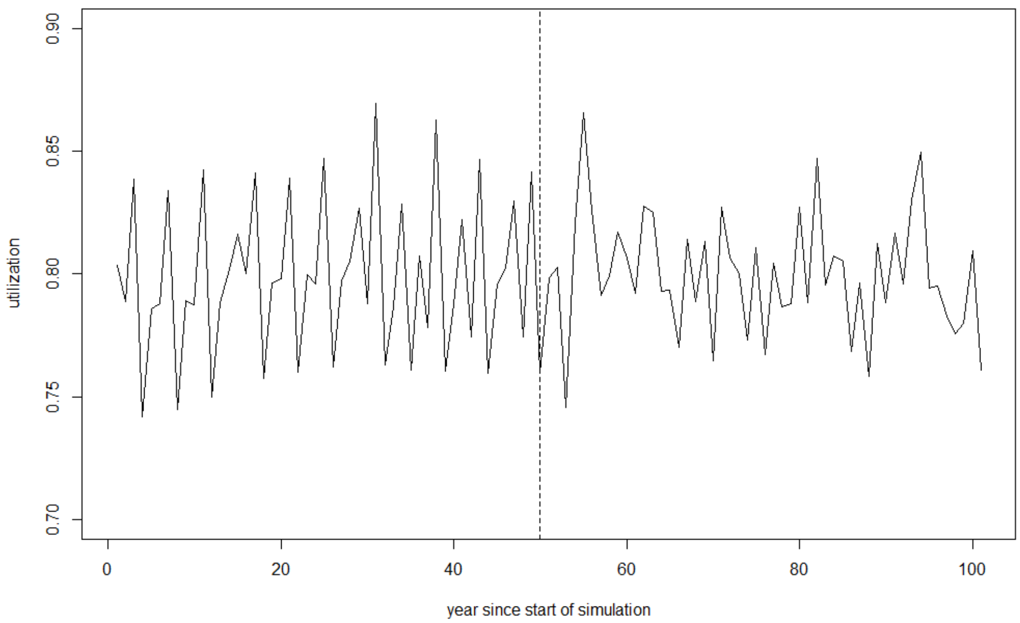
Figure 1: Utilization with $u_d = 0.80$, $u_{\max} = 0.87$, $\theta = 0.30$, $\tau = 2.5$, $s = 23.5\%$, $\kappa u_d = 0.30/\text{year}$, $\alpha = 0.17$, $T_u = 10$ years, $T_\gamma = 30$ years.



Keynesian stability holds if $\alpha/s\kappa$ is less than one. For the parameters used to generate Fig. 1, $\alpha/s\kappa = 1.93$, so the Keynesian stability condition does not hold. The instability is contained by both the utilization ceiling and expectations for a recovery in the downturn. An autoregression performed on the utilization time series finds an optimal order of $n = 3$. The roots of the characteristic polynomial lie outside the unit circle, indicating stability, contrary to the model construction. This is an example of the phenomenon identified by Blatt (1978), in which tests with a linear model of a time series generated by a nonlinear process that is locally unstable but bounded can incorrectly yield parameter estimates consistent with a stable (stationary) process. The difference between the simulation model and the autoregressive model can be seen in Fig. 2. While broad features of the utilization series produced by the Kaleckian-Harrodian model are reproduced in the autoregressive model, the quasi-regular cycles are absent.

[6]As recorded by the US National Bureau of Economic Resesarch (NBER): see https://www.nber.org/cycles.html

Figure 2: Utilization time series produced by the Kaleckian-Harrodian model (to year 50) and by an autoregressive model with stochastic innovations (after year 50)



3.3 Comments

In the previous section, we showed how short-run expectations about utilization can be separated from long-run expectations about growth. In this section, we applied that insight in a nonlinear, multiplier-accelerator, model of business cycles along the lines of Samuelson (1939) and Hicks (1950). As pointed out by Blatt (1983a,b), such models are quite plausible as a description of actual economies. The combined behavior leads to the convergence of Harrod's actual and warranted growth rates as in Fazzari et al. (2013, 2018). We have accomplished this in a demand-led model – firms only invest if they believe that what they produce will be bought – but without an exogenous source of autonomously increasing demand, as proposed by Serrano (1995). The mechanism is an upward instability that repeatedly drives the economy to expand, thereby repeatedly pushing against capacity constraints. This is reminiscent of Minsky's claim that "the fundamental instability of capitalism is upward" (Minsky, 1980, p. 512), although he was writing about finance.

Setterfield (2019) calls models such as the one presented above "neo-Keynesian" (or classical), in contrast to post-Keynesian (Kaleckian) models. There may be some value in making such a distinction. But while the model above does allow firms to target a desired level of capacity utilization, it also encompasses the possibility of collapse, corresponding to a deep recession. At that point, classical (or Robinsonian/Kaldorian) analysis fails and Keynesian remedies are called for. Moreover, unstable and nonlinear dynamics dominate economic life in the model. In nonlinear models, averages are not equal to equilibrium values, so the identification of long-run "centers of gravitation" cannot be separated from short-run behavior. The model presented in this paper thus touches on neo-Kaleckian themes. We return to these points in the Discussion.

4 A Kaleckian-Harrodian-classical model

Income distribution is one of the most important explanatory factors in post-Keynesian theory, yet we have not yet introduced it in the model. Neither have we introduced technological change, which is an important feature in any long-run growth theory. We now introduce both factors in a linked distribution and growth model and show that they lead to a convergence of Harrod's natural and actual growth rates.

First, we introduce distribution into the saving assumption by setting the desired saving rate as a function of the profit share, π. With desired saving out of wages at a rate s_w and desired saving out of profits at a rate s_p, the desired saving rate $s(\pi)$ is given by

$$s(\pi) = s_w + (s_p - s_w)\,\pi. \tag{34}$$

We emphasize that this is desired saving; when planned investment exceeds desired saving at any viable utilization rate, the saving rate must rise above the desired level. In a Kaldorian model that would happen by changing π, as firms raise the funds they need for investment by increasing their prices at fixed saving propensities. As noted above, in this model we assume a forced-saving mechanism, in which saving propensities change at fixed profit share. In reality, both are likely to occur to some degree.

When saving accommodates investment at a fixed profit share, as assumed in our model, it may indeed be "forced", but may also be voluntary or induced through price changes. As an example of truly forced saving, households may wish to make home improvements, but they find that all of the construction firms are busy meeting business investment needs. Postponing their planned expenditure would drive up s_w (or possibly s_p if the purchase was to be made out of dividend income). Saving would be voluntary if, e.g., firms retain more of their profits (raising s_p) to cover needed investment. Saving could be induced by, for example, higher deposit interest rate offers from commercial banks or rising stock prices, or by rising construction costs. For the model developed in this paper, the precise channel is not important. The behavioral rule is that at the peak of the business cycle the saving rate rises, while the profit share remains steady.

We further link distribution and productivity growth through cost share-induced technological change. Specifically, we apply a classical-evolutionary theory of cost share-induced technological change developed in Kemp-Benedict (2019) that generalizes a model proposed by Duménil and Lévy (1995, 2010). A simplified derivation of the core result is presented in the Appendix, along with a brief discussion of theories of cost share-induced technological change. Two important implications of the theory are, first, that both labor and capital productivity must respond simultaneously when cost shares change, and, second, that the functional relationship between distribution and productivity growth must satisfy certain conditions. As shown by Kemp-Benedict (2019), the cost-share induced mechanism is compatible with the Kaldor-Verdoorn mechanism. Using a "hat" to indicate a growth rate and writing labor productivity as λ and capital productivity as κ, we assume the following functional forms,

$$\hat{\lambda} = a + b\left(g^i - \delta\right) + c\left(\ln\frac{1-\pi}{\pi} - \ln d\right), \tag{35a}$$

$$\hat{\kappa} = c\left(d - \frac{1-\pi}{\pi}\right). \tag{35b}$$

These expressions satisfy the conditions listed in the Appendix. The parameter d is a "neutral" distribution in the sense that when $(1-\pi)/\pi = d$, labor productivity growth is given solely by the Kaldor-Verdoorn mechanism, while capital productivity is constant. Under those conditions or, equivalently, setting $c = 0$, these expressions reduce to the conventional post-Keynesian productivity assumptions of constant capital productivity and labor productivity tied to the growth rate of the economy (through the Kaldor-Verdoorn law).

It remains to say how the profit share changes, and we propose two mechanisms. Both rely on conflict theories of pricing (e.g., as in Goodwin, 1967; Rowthorn, 1977). We assume that firms try to defend their profit rates, aiming for a minimal target level r^*, while workers demand raises when they have leverage.

For the firms' goal, their desired profit share, π^* is given by

$$\pi^* = \frac{r^*}{u_d \kappa}. \tag{36}$$

When the actual profit share is below the desired level, firms would like to close the gap. However, they will not all act at once, and firms at least face oligopolistic competition, so we assume that firms' target profit share for the next period partially closes the (positive) gap between their target and the current level,

$$\pi_{+1}^{\text{firm}} = \pi + \psi^{\text{firm}} \max\left(0, \pi^* - \pi\right), \qquad 0 < \psi < 1. \tag{37}$$

Firms may not be able to set their profit share according to their target if workers are able to effectively negotiate for a higher wage. We assume that workers have more negotiating power when the growth in labor demand exceeds growth in labor supply. Unlike Fazzari et al. (2013, 2018), we do not implement a hard limit on output due to labor constraints. The ultimate limit is the working-age population, rather than the currently active workforce, and participation rates are often well below 100%. When labor demand is increasing faster than the working-age population and wages are rising faster than labor productivity, it tends to draw new workers into the workforce; when labor demand is comparatively weak, people leave the workforce, often involuntarily as they become discouraged.[7]

The growth in labor demand, $\hat{L}$, is given by the difference between GDP growth, $\hat{Y}$, and labor productivity growth,

$$\hat{L} = \hat{Y} - \hat{\lambda}. \tag{38}$$

We assume a constant growth rate of the working-age population, n. When $\hat{L}$ exceeds n, workers have leverage, and seek to increase the wage share at the expense of the profit share,

$$\pi_{+1}^{\text{worker}} = \pi - \psi^{\text{worker}} \max\left(0, \hat{L} - n\right). \tag{39}$$

[7] Fazzari et al. (2018) allow for this behavior to gradually alter the labor supply, but retain a hard upper bound at the current labor supply.

For the purposes of this paper, we assume a simple rule for wage competition, that workers prevail whenever $\hat{L} > n$, and firms prevail otherwise, so that

$$\pi_{+1} = \begin{cases} \pi_{+1}^{\text{worker}}, & \hat{L} > n, \\ \pi_{+1}^{\text{firm}}, & \hat{L} \leq n. \end{cases} \tag{40}$$

With this dynamic included, we ran the model using the same parameters as those used for Fig. 1, with additional values for the new parameters, such as a, b, and c in Eqns. (35). Values for each of the parameters are reported in the caption to Fig. 3, which shows the utilization time series for both the "Kaleckian-Harrodian" model of the previous section and the "Kaleckian-Harrodian-classical" model introduced in this section. Running the model for a longer time than shown in the figure reveals that the business cycle is overlaid by a long cycle with a period of about 60 years, corresponding in length to Kondratieff (1979) cycles, or "long waves". In the model, the long waves are driven by the classical/neo-Marxian dynamics: cost share-induced technological change and conflict wage-setting. Those dynamics link labor constraints to the functional income distribution and technological change. Together, they drive the growth in labor demand towards the growth rate of the working-age population, with the result

$$\hat{L} \to n \qquad \Rightarrow \qquad \hat{Y} \to \hat{\lambda} + n. \tag{41}$$

In these expressions the arrows indicate a tendency rather than a strict equality. This result means that the growth rate of the economy approaches the natural growth rate, so the cost share-induced technological change mechanism, combined with conflict-based pricing, leads the actual growth rate to converge towards the natural growth rate through accommodation of labor productivity and GDP growth.

5 Discussion

The Kaleckian-Harrodian-classical model features a combination of post-Keynesian and classical mechanisms that bring Harrod's actual, warranted, and natural growth rates into alignment. The equilibrium tendency (denoted by a subscript "eq"), is therefore

$$s(\pi_{\text{eq}})\kappa_{\text{eq}}u_d = \hat{Y}_{\text{eq}} + \delta = \hat{\lambda}_{\text{eq}} + n + \delta. \tag{42}$$

Due to a combination of cost share-induced technological change (Eqns. [35]) and target-return pricing (Eqn. [36]), capital productivity is constant at the equilibrium (Kemp-Benedict, 2019). That constant level occurs at a specific value of the profit share. Thus, π_{eq} is determined by

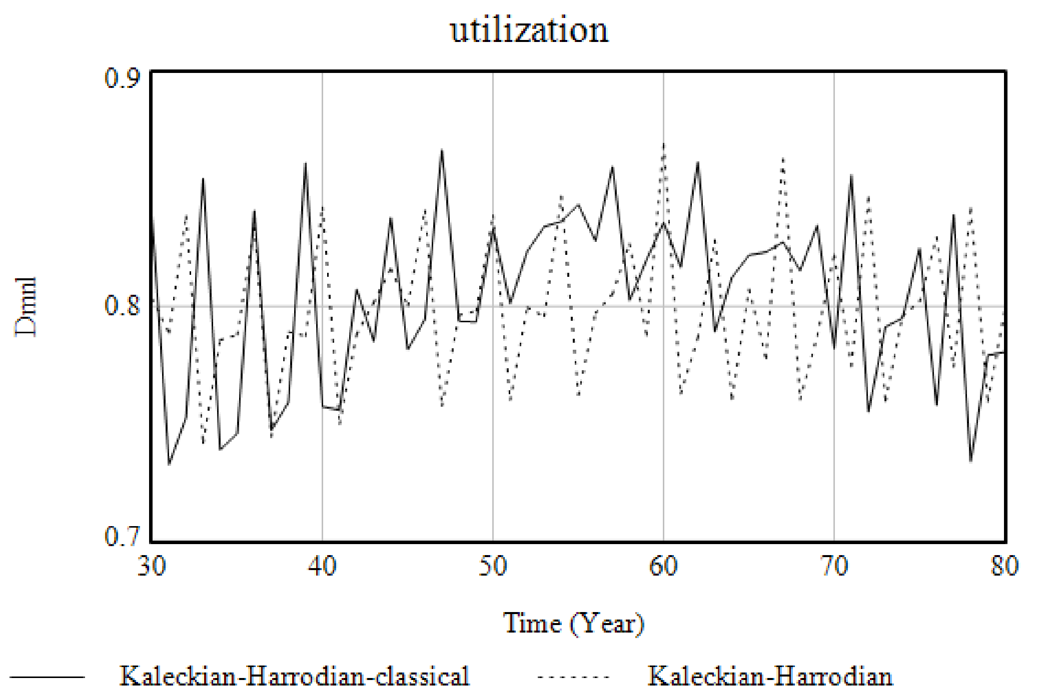
$$\hat{\kappa}(\pi_{\text{eq}}) = 0. \tag{43}$$

From the expression for the capital productivity growth rate, Eqn. (35b), we see that this equation is satisfied when π_{eq} is at the neutral distribution, $\pi_{\text{eq}} = 1/(1+d)$.

5.1 A Cambridge equation

The equilibrium growth rate of the capital stock, g_{eq}, can be set equal to the expression for any of Harrod's growth rates, whether actual, warranted, or natural. Setting it equal to

Figure 3: Utilization with distribution and technological change, using the same parameters as in Fig. 1, and $a = 0.002$/year, $b = 0.40$, $c = 0.10$/year, $d = 1.22$, $\delta = 3.0\%$/year, $s_w = 10\%$, $s_p = 40\%$, $n = 1.75\%$/year, $\psi^{\text{firm}} = 0.10$/year, $\psi^{\text{worker}} = 0.02$.



$\hat{Y}_{\text{eq}} + \delta$, substituting into the equation for labor productivity growth, Eqn. (35a), and using the second equality in Eqn. (42), we find

$$g_{\text{eq}} = a + b\,(g_{\text{eq}} - \delta) + n + \delta \quad \Rightarrow \quad g_{\text{eq}} = \frac{a+n}{1-b} + \delta. \tag{44}$$

Next, setting g_{eq} equal to the expression for the equilibrium warranted growth rate $s(\pi_{\text{eq}})\kappa_{\text{eq}}u_d$ and using the first equality in Eqn. (42) gives an expression for capital productivity,

$$\kappa_{\text{eq}} = \frac{1}{s(\pi_{\text{eq}})u_d}\left(\frac{a+n}{1-b} + \delta\right). \tag{45}$$

The equilibrium profit rate is $r_{\text{eq}} = \pi_{\text{eq}}\kappa_{\text{eq}}u_d$, which we can now write as

$$r_{\text{eq}} = \frac{\pi_{\text{eq}}}{s(\pi_{\text{eq}})}\left(\frac{a+n}{1-b} + \delta\right). \tag{46}$$

Because $\pi_{\text{eq}} = 1/(1+d)$, we can use Eqn. (34) to write

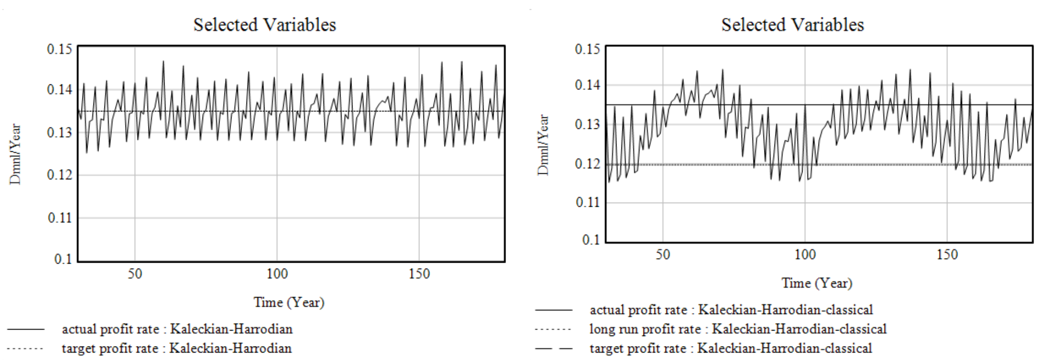
$$\frac{\pi_{\text{eq}}}{s(\pi_{\text{eq}})} = \frac{1}{s_p + s_w d}. \tag{47}$$

Substituting into Eqn. (46) then gives an expression for the profit rate

$$r_{\text{eq}} = \frac{1}{s_p + s_w d} \left(\frac{a+n}{1-b} + \delta \right). \tag{48}$$

This is a version of the famous "Cambridge equation" (Pasinetti, 1962), with a few differences. First, while we distinguish profit and wage income, we do not distinguish between profit-earning (capitalist) and wage-earning (worker) households.[8] We further break from convention by explicitly including depreciation. Finally, we take the Kaldor-Verdoorn productivity relationship into account.

Every parameter on the right-hand side of Eqn. (48) is exogenously specified in the model: technological parameters (a, b, d); a demographic parameter (n); depreciation (δ); and saving behavior (s_p and s_w). This means that r_{eq} is fully determined by these parameters. Yet, in the model, firms target an exogenously specified profit rate r^*, as in Eqn. (36). The resulting tension between the rate determined by the Cambridge equation and firms' desired rate is the source of the long waves generated by the model. This can be seen in Fig. 4, where the profit rate oscillates between firms' target rate r^* and the rate consistent with the Cambridge equation, r_{eq}.



(a) Kaleckian-Harrodian model

(b) Kaleckian-Harrodian-classical model

Figure 4: Profit rate in the models with and without conflict-based distribution, with target and equilibrium profit rate; time series extend over a 150 year period, from $T = 30$ years to $T = 180$ years

5.2 Political economy

The tension between the equilibrium and target profit rates opens political economy channels for influencing growth and distribution. From Eqn. (48), we find the usual Kaldorian result that higher saving leads to a lower equilibrium profit rate when Harrod's natural rate of growth is held fixed. This happens because some variable must adjust to bring the warranted rate of growth – which increases with saving – into alignment with the natural rate of growth. For Kaldor it was the profit share. In the model presented in this paper, it is

[8] Specifically, we do not assume that the profits are arising from workers' savings are returned to them in full. Pasinetti (1962) does make that assumption when he sets workers' profit income equal to their savings multiplied by the profit rate.

capital productivity, through Eqn. (45). In either case, the result of increased saving is a decline in the equilibrium profit rate.

Profit rates can be maintained if firms can manipulate the natural rate. In fact, there is nothing "natural" about the natural rate, and firms regularly attempt to raise it. One way is to raise n, thereby circumventing the labor constraint. This can be done through, e.g., a combination of offshoring (Crinò, 2009), immigration (Rodriguez, 2004), extending working lives (Vickerstaff, 2010), and bringing children into the workforce (Pollack et al., 1990). If those efforts are successful, then the equilibrium rate becomes firms' target rate $r_{\text{eq}} = r^*$ and n becomes the accommodating variable. Labor productivity growth will rise in this case, due to the Kaldor-Verdoorn law. Substituting g_{eq} from Eqn. (44) into Eqn. (35a) for labor productivity growth at the neutral distribution gives

$$\hat{\lambda}_{\text{eq}} = \frac{a + bn}{1 - b}. \tag{49}$$

A further channel to influence the equilibrium profit rate is through technology. The parameters a, b, and d can be seen as characterizing a "technological regime" (e.g., see Setterfield, 1997, 2002). Through R&D and lobbying efforts, firms have some measure of influence over the regime. From Eqn. (46), were either a or b to rise through a change in technological regime, the result would be a rise in r_{eq}. For any positive level of saving out of wages, a fall in d – that is, a shift in the neutral distribution away from wages and toward profits – will also increase r_{eq}. From Eqns. (35), the fall in d initially drives capital productivity downward, while accelerating labor productivity growth. The result is capital deepening. As long as a and c do not change, equilibrium productivity growth will be the same as before, but the levels and the functional income distribution will be different. If there is *dissaving* out of wages, e.g. because of household indebtedness (see Kim, 2013; Kim et al., 2019), then an increase in the equilibrium profit share will lead to a fall in the equilibrium profit rate.

Firms' scope to intervene in the technological trajectory is limited by the stage of development of the "techno-economic paradigm" (Perez, 2010). For example, if the manufacturing sector is mature and most of the additional jobs are in the service sector, then b is likely to be small (Magacho and McCombie, 2018). Absent a radical breakthrough, R&D can moderate, but not fundamentally change, the trend towards slowing productivity growth.

Throughout the paper we have made the neo-Kaleckian assumption that firms enjoy a substantial degree of monopoly. They are largely free to set prices, constrained only by the ability of labor to organize. Their freedom expands considerably if they can remove that constraint. Thus, efforts to weaken unions starting in the 1980s allowed firms greater flexibility in setting prices to provide desired rates of return. In the model, weak labor bargaining power allows firms to meet their target rate of return by raising the profit share. Because the income distribution then shifts towards profits relative to the neutral distribution, capital productivity will rise and labor productivity growth will slow, from Eqns. (35). That will continue until capital productivity is sufficiently high that the target profit rate can be met at the neutral distribution. The warranted growth rate will be higher than the natural growth rate, but as that does not feed back into firm costs, demand for labor will outstrip the growth rate of the working-age population. That imbalance must be accommodated. Because wages are expected to be stagnant in this case, "pull" factors are weak, but there may be "push" factors, as low wages drive additional household members into the workforce or lead existing workers to work longer hours and take on additional jobs.

We note that the political economy channels in the model arise from specific assumptions.

Among the most important is the form of the investment function in Eqn. (8). In this paper, the investment function depends solely on utilization. If it were expanded to include dependence on the net profit rate (Shaikh, 2016), then the equilibrium expression for the profit rate would depend on the interest rate, opening an additional channel for political contention.

A second key assumption is that output is constrained only by the availability of labor and capital. The economy may also be constrained by the supply of raw materials. Indeed, access to inexpensive raw materials, whether fertile land or fossil fuels, has historically been an important concern within political economy. This concern has a biophysical basis. While value added by extractive sectors is typically small in high-income countries, biophysical economists have long argued that material constraints have been under-appreciated (e.g., Ayres et al., 2013). Extractive sectors tend to have high forward linkages (Cahen-Fourot et al., 2020), so constraints on those sectors have a disproportionate impact on the economy as a whole. Moreover, with the increasing urgency of climate change, a new channel is opening: the political economy of keeping carbon-intensive fossil fuels in the ground (Benedikter et al., 2016). Constraints on material and energy inputs would erode the profits of non-extractive firms through rising material input costs, driving natural resource-saving technological change (Kemp-Benedict, 2018).

5.3 The equilibrium vs. the short and long run

The equilibrium of the model presented in this paper represents neither the short nor the long run. As shown in Fig. 4b, at any moment the economy might be moving towards or away from equilibrium, depending on firms' target rate of profit relative to the equilibrium level. A classical economist, observing that pattern, might suggest a "center of gravitation" for the profit rate somewhere between the equilibrium and target levels. Over the short run, the economy is executing business cycles, which can differ in detail from one cycle to the next. The analytical role of the equilibrium is to provide an anchor – a true "center of gravitation" – that pulls the economy towards a particular configuration. When the equilibrium configuration conflicts with the desires of economic actors, it triggers reactions. These may operate through exogenous political economy channels, as discussed above, or through endogenous dynamics.

Reactions to conflict can drive oscillating behavior, as in Fig. 4, or changes in the equilibrium configuration itself. The equilibrium configuration is determined both by saving behavior and technological potential, neither of which is fixed. As noted above, technological potential varies over the life cycle of a techno-economic paradigm. The end of one paradigm and the onset and development of the next depend on the available niche technologies (Geels, 2002) and choices made by governments, firms, financiers, and consumers (Perez, 2010). The exhaustion of an existing paradigm can be made endogenous, as shown by Setterfield (1997); as the economy becomes locked into a particular techno-economic regime, dwindling opportunties for productivity growth gradually diminish the Verdoorn coefficient.

Two further considerations illustrate the difference between the equilibrium and actual economic outcomes. First, as noted by Blatt (1983b), if economic dynamics are nonlinear, then long-run averages generally will not coincide with the equilibrium. Second, while we emphasize cycles in this paper, the model can exhibit collapse. Specifically, if firms do not believe that a downturn is likely to turn around (in the model, if θ is too large), then utilization can enter an unstable downward trajectory – a depression. No mechanism in the model can pull the economy out of the depression, opening the possibility for extraordinary government intervention.

These observations suggest the conclusion of Kaldor (1972) that history matters more than equilibrium. In the model presented in this paper, unspecified but implicit historical processes determine both the equilibrium and deviations from it. The model therefore supports the notion of "growth regimes" proposed by Setterfield (1997, 2002). In common with Setterfield (2002, p. 277), the center of gravity acts as a "weak attractor", while the actual path (the traverse) affects the attractor's conditions and position.

6 Conclusion

The model presented in this paper combines insights from both the post-Keynesian and classical/neo-Marxian literatures. As discussed in the Introduction, recent work on post-Keynesian Kaleckian-Harrodian models has shown that abandoning the Keynesian stability condition, whether in the long run (Skott, 2012) or the short run (Fazzari et al., 2013) can tame the Harrodian instability that arises under adaptive expectations for "animal spirits" (Hein, 2014, p. 32). The challenge then becomes containing short run instability. Fazzari et al. (2013) impose a hard constraint on the labor supply; in this paper we assume a hard constraint on capacity utilization and implement an indirect constraint on labor supply through conflict pricing.

In Kaleckian-Harrodian models, firms are able to target a desired level of capacity utilization, leading to convergence of Harrod's actual and warranted growth rates. The classical/Marxian literature (e.g., Foley, 2003; Julius, 2005) points out that cost share-induced technological change (Dutt, 2013) can bring Harrod's actual and natural growth rates into alignment. This paper has shown how a combination of these mechanisms can lead to convergence of Harrod's actual, warranted, and natural growth rates in a Kaleckian-Harrodian-classical model. The processes are endogenous – equilibrium stability is derived rather than imposed.

The resulting model, which has nonlinear dynamics, exhibits business cycles of a few years in length and "long waves" several decades in length. The long waves arise from conflict pricing behavior when firms' target profit rate is higher than the equilibrium profit rate arising from the Cambridge equation. When growth in labor demand exceeds the (exogenous) growth in labor supply, wage earners are able to demand higher wages, pulling the profit rate downwards towards the equilibrium rate; that drives faster labor productivity growth, which brings growth in labor demand below the growth in supply, allowing firms to adjust prices towards their target.

The model presented in this paper addresses a long-standing challenge in macroeconomics: explaining persistent long-run growth in the face of inherent instability and the agency of economic actors. Models that assume full capacity utilization, whether classical, Kaldorian/Robinsonian, or neoclassical, set the instability problem aside. Kaleckian models allow for under-utilization, but avoid instabilities by making long-run expectations ("animal spirits") exogenous and capacity utilization endogenous. Kaleckian-Harrodian models embrace instability and allow for firm agency in targeting a desired level of capacity utilization. In this paper we offer a version of such a model. In common with Fazzari et al. (2018), we link labor constraints to productivity growth, but go further by endogenizing distribution through cost share-induced technological change. In contrast to Fazzari et al. (2018) and others (e.g., Serrano et al., 2019; Fiebiger and Lavoie, 2019), we do not assume an autonomous source of demand. Instead, the economy recovers from a downturn due to firms' expectations that the downturn will eventually reverse.

The resulting combination of post-Keynesian and classical/neo-Marxian theory looks notably Kaldorian. Kaldor's (1961) "stylized facts" of constant capital productivity and steady labor productivity growth (that is, Harrod-neutral technological change) characterize the model's equilibrium. Moreover, to the extent that the economy remains close to full capacity utilization, higher saving propensities lead to both higher equilibrium growth rates and lower profit rates. Nevertheless, in keeping with a further Kaldorian (1972) theme, the economy follows a historical trajectory that can depart substantially from the equilibrium, and the equilibrium itself is not immutable. The model thus accommodates a sequence of economic regimes (Setterfield, 1997, 2002) and offers multiple entry points for exploring questions of political economy in a dynamic and fluctuating economic environment.

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A A generalized classical-evolutionary model of cost share-induced technological change

While firms in most industries cannot observe a demand curve for their output (Coutts and Norman, 2013), they are not indifferent to the costs of their inputs. When the cost of one input rises, firms can be expected to favor technological innovation that saves on that input. Cost-induced technological change was a theme of the classical economists (Kurz, 2010) and it was reinvigorated by Hicks (1932). Subsequent developments led to the concept of a "technical progress function" (Kaldor, 1961; Kennedy, 1964). Samuelson (1965) recast the technical progress function in terms of a production function, but ultimately this development ran into conceptual difficulties (e.g., see Nordhaus, 1973) and was eventually abandoned. More recently, endogenous growth theories enabled Acemoglu (2002) to revisit the concept of induced technological change from a neoclassical perspective, resulting in his theory of "directed technological change", while neo-Marxian theorists have provided a range of potential models (Dutt, 2013).

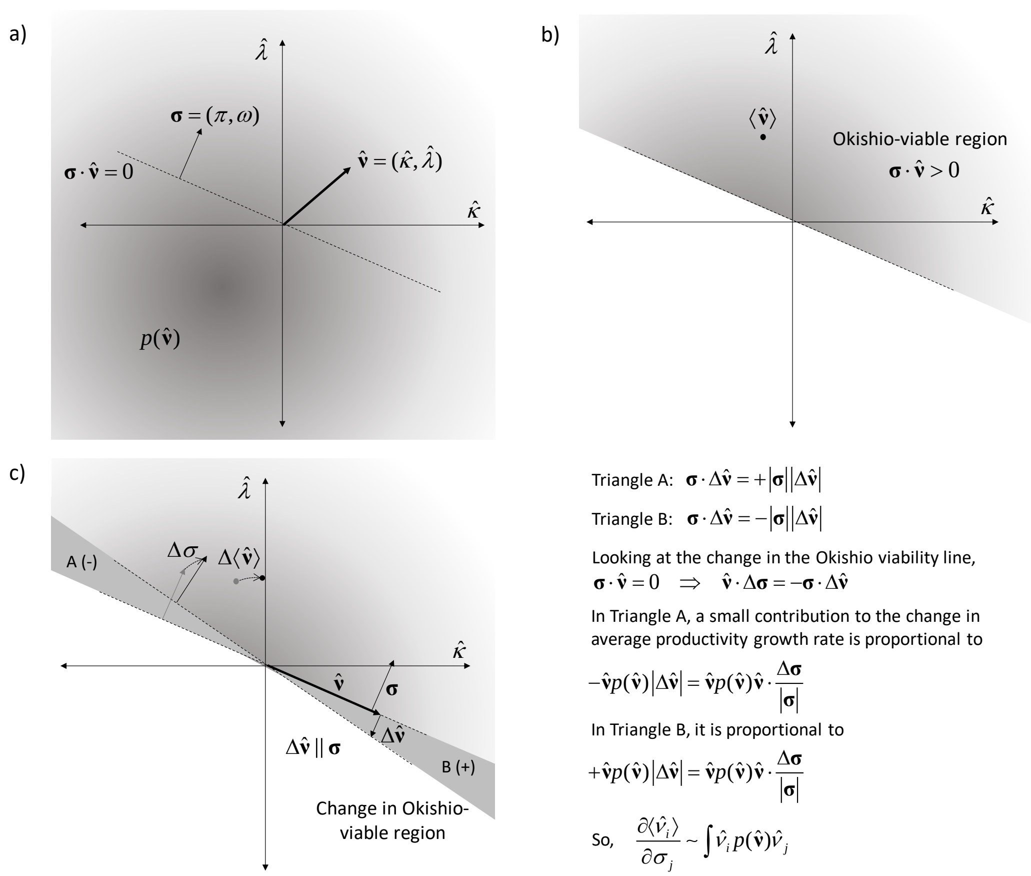
Among the neo-Marxian theories is an evolutionary theory due to Duménil and Lévy (1995, 2010). They discarded the neoclassical assumption that firms maximize profits with knowledge of an external technological frontier in favor of an evolutionary (Nelson and Winter, 1982) assumption that firms continually perform a random search for profitable innovations in the vicinity of their current technology. Firms are assumed to make the search under conditions of fixed prices and wages, in the expectation of short-run excess profits. Thus, the Okishio (1961) viability criterion applies at the point at which firms decide whether to innovate: any innovation must increase the firm's profit rate at fixed wages and prices. Subsequently, in a Marxian catch-22, those excess profits are lost as wages rise and profits fall through demands from labor to share the gains from productivity growth and from competitors driving prices downward. The process then begins again.

The evolutionary theory of Duménil and Lévy (1995, 2010) was shown by Kemp-Benedict (2019) to constrain the possible functional relationship between cost shares and productivity change. The proof is not intuitive, so we motivate it in this paper by reference to Fig. A.1. Graph (a) in the figure shows coordinate axes for rates of productivity growth for labor ($\hat{\lambda}$) and capital ($\hat{\kappa}$), where the "hat" indicates a growth rate. The productivity growth rates form a vector $\hat{\boldsymbol{\nu}} = (\hat{\kappa}, \hat{\lambda})$, while the profit share (π) and wage share (ω) form a vector of cost shares $\boldsymbol{\sigma} = (\pi, \omega)$. The probability of discovering a potential innovation that would raise productivity by $\hat{\boldsymbol{\nu}}$, denoted $p(\hat{\boldsymbol{\nu}})$, is indicated by shading in the figure.

The Okishio viability criterion can be shown to imply that viable innovations must satisfy $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} > 0$ (Kemp-Benedict, 2019, p. 7). Graph (b) shows the region where the criterion is satisfied. The border of the region is the dotted line shown in Graph (a), which satisfies the condition $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} = 0$. As firms randomly discover innovations and implement those that satisfy the Okishio viability criterion, an average tendency will emerge as the expected value $\langle \hat{\boldsymbol{\nu}} \rangle$ of the productivity growth rate over the viable region where the expectation is determined by the probability of discovery $p(\hat{\boldsymbol{\nu}})$, indicated by the density of shading in the figure.

While the Okishio viability criterion can be stated in a compact form, the expected value of the productivity growth rate depends on the probability function $p(\hat{\boldsymbol{\nu}})$. Unless this is known – and it generally will not be – it is not possible to calculate the average productivity growth rate from first principles. However, as shown by Kemp-Benedict (2019), it is possible to constrain the functional relationship between average productivity growth and cost shares without knowing the probability distribution. Suppose that the cost shares change by a small

Figure A.1: The innovation probability distribution (indicated through a shaded density plot), the Okishio viability region, and the change in average productivity growth



amount $\Delta\boldsymbol{\sigma}$. The Okishio viability line will then rotate, as shown in Graph (c). While the probability distribution of discovery does not change, the location of the Okishio viability region shifts, leading to a change in the expected value of the productivity growth rate. This is a key step in the derivation, as we now explain.

The difference in the average productivity growth rate can be calculated by averaging over the small gray triangles in Graph (c). Because it is a difference, no other part of the productivity growth space enters into the calculation. Moreover, because the triangles are small (in the continuous limit, infinitesimal), we can make some approximations that simplify the calculations. The two triangles enter with opposite signs: triangle A is no longer in the Okishio viability region as a consequence of the rotation of the Okishio viability line, so it enters with a negative sign; triangle B has entered the Okishio viability region, and enters with a positive sign.

The width of the triangle grows linearly with $\hat{\boldsymbol{\nu}}$ as it gets farther from the origin. In fact, as indicated by the text in Fig. A.1, it is proportional to $\hat{\boldsymbol{\nu}} \cdot \Delta\boldsymbol{\sigma}$, so when calculated as the expectation of $\hat{\boldsymbol{\nu}}$ over Triangles A and B, the terms are of the form $\hat{\boldsymbol{\nu}} p(\hat{\boldsymbol{\nu}})\hat{\boldsymbol{\nu}} \cdot \Delta\boldsymbol{\sigma}$. This means that the change in the expected value of $\hat{\nu}_i$ with respect to a change in σ_j includes terms like $\hat{\nu}_i p(\hat{\boldsymbol{\nu}})\hat{\nu}_j$. Since the order of the terms being multiplied does not matter, this is equal to $\hat{\nu}_j p(\hat{\boldsymbol{\nu}})\hat{\nu}_i$ Each term is therefore symmetric in i and j. Moreover, the triangles in Graph (c) represent small deviations in the neighborhood of the Okishio viability line, on which the Okishio viability criterion implies $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} = 0$. To first order, we can therefore impose the approximate condition $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} \simeq 0$, so that summing the product of σ_i or σ_j with each term gives (close to) zero as the result. Finally, multiplying on the left and right by an arbitrary vector $\boldsymbol{x}$ gives a set of terms like $p(\hat{\boldsymbol{\nu}})(\boldsymbol{x} \cdot \hat{\boldsymbol{\nu}})^2$, which is positive. Taken together, these properties give the following

Result. *The matrix of partial derivatives (the Jacobian) expressing the change in the expected value of productivity with respect to a cost share is positive semi-definite, with a null vector equal to* $\boldsymbol{\sigma}$.

To recapitulate: We assume that firms continually carry out a directed but essentially random search for marginal productivity gains in the vicinity of their current technology. The expected value of the vector of productivity growth rates is taken with respect to an unknown probability of discovering an innovation that yields a particular combination of productivity growth rates over a known viability region given by the Okishio viability criterion. While the probability distribution is not known, the change in the expected value of productivity growth for input i with respect to a change in cost share j has certain properties that are independent of the probability distribution. Specifically, the matrix of partial derivatives of productivity growth with respect to cost share (the Jacobian matrix) is symmetric and positive definite. It also has a null vector given by the cost shares.[9]

Before continuing, we acknowledge that the cost shares must sum to one. That is, for n inputs,

$$\sum_{i=1}^{n} \sigma_i = 1. \tag{A.1}$$

This condition must be applied at some point when applying this model. Indeed, it could have been applied earlier, when deriving the result about the Jacobian matrix. However, it is best not to do that, because it hides the underlying symmetry and positive semi-definiteness of the matrix. The model is easier to manipulate if the cost shares are first treated as independent and then the constraint that they sum to one is introduced at the end. We illustrate the point with an example in which there are two inputs to production.

With two inputs to production, capital and labor, the conditions derived in Kemp-Benedict (2019) and sketched above are quite constraining. Positive-definiteness implies

$$\frac{\partial\langle\hat{\kappa}\rangle}{\pi}, \frac{\partial\langle\hat{\lambda}\rangle}{\omega} > 0, \tag{A.2}$$

while symmetry implies

$$\frac{\partial\langle\hat{\kappa}\rangle}{\omega} = \frac{\partial\langle\hat{\lambda}\rangle}{\pi}. \tag{A.3}$$

[9] We note that the neo-Marxian models explored by Dutt (2013) do not satisfy these criteria.

Applying the null vector condition then implies

$$\pi\frac{\partial\langle\hat{\kappa}\rangle}{\pi} + \omega\frac{\partial\langle\hat{\kappa}\rangle}{\omega} = 0, \tag{A.4}$$

$$\pi\frac{\partial\langle\hat{\lambda}\rangle}{\pi} + \omega\frac{\partial\langle\hat{\lambda}\rangle}{\omega} = 0. \tag{A.5}$$

Combining these conditions with the symmetry condition shows that there is only one independent partial derivative. The null vector conditions also imply that the productivity growth rates are homogeneous of order zero in the cost shares.

We use the flexibility that remains in the system of equations by proposing a formula for the cost share-induced component of capital productivity growth that is homogeneous of order zero in cost shares,

$$\hat{\kappa} = e - c\frac{\omega}{\pi}. \tag{A.6}$$

Then from the conditions above we find that labor productivity growth must be of the form

$$\hat{\lambda} = a + c\ln\frac{\omega}{\pi}. \tag{A.7}$$

Aside from the Kaldor-Verdoorn term and defining e in terms of the neutral distribution d, these are the expressions used in the model for cost share-induced technical change in Eqns. (35). They follow from taking the expectation of productivity growth with respect to a probability of discovery over a region defined by the Okishio (1961) viability criterion. While we do not know the probability distribution of discovery, the model places strong constraints on the functional relationship between productivity growth rates and cost shares. We proposed a specific functional form for labor productivity growth consistent with those constraints and then derived the functional form for capital productivity growth by applying the constraints.

Post-Keynesian models often assume that labor productivity is driven by growth rather than costs, as captured by the Kaldor-Verdoorn law (Kaldor, 1966; Verdoorn, 1949, 2002). Kemp-Benedict (2019) showed that the evolutionary theory of Duménil and Lévy (1995, 2010) is compatible with the Kaldorn-Verdoorn law. In the model, we add to Eqn. (A.7) a term proportional to the investment rate. Thus, labor productivity growth in the model depends on investment rates and cost shares, while capital productivity growth depends on cost shares through Eqn. (A.6).