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Testing fundamentalist–momentum trader financial cycles. An empirical analysis via the Kalman filter

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Abstract

This paper proposes an empirical test for Minskyan financial cycles in asset prices, driven by the interaction of fundamentalist and momentum traders. Both agents' beliefs about the future are unobserved and can be modelled in a state space model. We use the Kalman filter to identify the two behavioral rules and evaluate whether the conditions for the existence of cycles hold. The model is estimated for equity and housing prices for France, Germany, UK and the USA, for the period 1970-2017, with annual and quarterly data. We find robust empirical support for the existence of endogenous financial cycles in equity markets for all countries and for France, UK and USA for housing markets.

Keywords: Financial cycles, House prices, Share prices, Minsky, Momentum traders, Kalman filter.

JEL codes: C32, E32, G40.

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1 Introduction

During the Great Moderation, standard macroeconomic models paid limited attention to financial cycles. Borio (2014) criticizes that the New Keynesian dynamic stochastic general equilibrium paradigm has regarded finance as a veil and consequently it has interpreted financial crises as exogenous shocks. By contrast, since the global financial crisis, models that allow for financial instability and cycles have gained prominence (Lavoie and Daigle, 2010; Nishi, 2012; Nikolaidi, 2014; Mian et al., 2017; Jordà et al., 2016; Stockhammer et al., 2019a; Kohler, 2019). They build on Minsky's financial instability hypothesis (Minsky, 1985) and behavioral finance (Shiller, 2003), which regard financial cycles and market inefficiency as the outcome of endogenous forces.

Minsky emphasizes the role of financial factors in a capitalist economy, characterized by the gradual emergence of endogenous financial fragility which eventually turns the boom into a bust (Ferri and Minsky, 1992; Vercelli, 2000). Nikolaidi and Stockhammer (2017), in a recent survey of Minskyian theory, identify two families of Minsky models. In the first, the dynamics emerge from the interaction of financial factors (usually debt or the interest rate) and a real variable (typically investment). A second family describes cycles as the outcome of the interaction to two asset pricing strategies on financial markets. This latter family overlaps with behavioral economics models (Lavoie and Daigle, 2010; Franke and Westerhoff, 2017).

The existing empirical literature on the financial instability hypothesis is sparse and focuses on the first family. Schroeder (2009), Mulligan (2013), Nishi (2016), and Davis et al. (2019) seek to identify the hedge, speculative and Ponzi states of a firm's financial condition for different countries and economic sectors.[1] Other studies have explored the impact of debt on aggregate demand (Palley, 1994; Kim, 2013, 2016). Stockhammer et al. (2019a; 2019b) formally test whether financial-real interactions give rise to endogenous cycles. As financial variables, they consider the interest rate as well as business and household debt. However, there are no empirical Minsky studies that incorporate an active role for asset prices with the crucial role of the expectation formation behavior of the agents. This paper will deal with the second group, specifically the momentum trader models.

Momentum trader models suggest that there is heterogeneity in the expectation formation on financial markets. The underlying behavioral rules can be grouped into those of fundamentalists and momentum traders (also: extrapolative traders). Under certain conditions (Beja and Goldman, 1980) the interaction between the two will generate cycles in asset prices. This argument is in line with behavioral economics which analyses changes in asset price based on behavioral heuristics rather changes in fundamentals. This theory emphasizes psychological elements in the decisions of traders such that price booms rooted in feedback mechanisms rather than changes in fundamentals can arise (Schleifer and Summers, 1990; Shiller, 2003; Vikash et al., 2015).

Importantly, these expectation rules, by their nature, cannot be directly observed but they will cause a response in the observed data. The contribution of this paper is to provide an empirical test for endogenous financial cycles[2] that emerge from the interaction of the two latent expectations rules. To achieve this, we use the Kalman filtering in a state space model with the aim of explaining the dynamics of asset prices in a context of an unobserved component model. Kalman-filtering is a recursive dynamic procedure used to estimate time dependent structural parameters of linear systems. It is used routinely in economics to estimate output gaps and the NAIRU (Boone, 2000; Rusticelli, 2014) and to decompose the trend and cyclical components of the GDP and other economic time series (De Winter et al., 2017; Klinger and Weber, 2019). Many authors follow Harvey (1989) and model the cycle as an autoregressive process by imposing the assumption that the polynomial autoregressive coefficients have complex roots (Bulligan et al. 2019; Galati et al. 2016, De Winter at al. 2017). This paper takes a different approach: we do not use a standard trend-cycle decomposition. Rather we define two expectation rules in asset pricing where the extrapolators overshoot based on observed past asset prices. We do not assume or impose the existence of cycles, but estimate coefficients for two behavioral rules and then check whether these meet the criteria for cyclical fluctuations (i.e. complex roots). In our model, cyclical fluctuations are a possible outcome. If they exist, they arise from the interaction of the two expectation rules. Momentum traders and fundamentalists are like the two blades of scissors that only together generate cycles.

[1] The indebtedness of firms is expressed in Minsky's categorization of firms as the hedge, speculative and Ponzi ones. Based on the relationship between cash flow and debt service requirements, firms gradually shift from hedge to speculative and Ponzi regimes, thereby generating over-indebtedness and higher financial fragility.

[2] We use the term endogenous cycles to describe systems with complex roots, i.e. we include damped oscillations as well as closed orbits.

We estimate the parameters associated with the two expectation rules to assess the presence of financial cycles and the relative shares of the two economic agents in the market. After the estimation the iterative Kalman filter algorithm is used to extract the unobserved states by performing forward recursion over the state-space model. We also implement a Monte Carlo bootstrap analysis to evaluate whether the cyclical conditions hold. A precondition for using the Kalman filter is that the model is linear. This is a shortcoming in our context as some momentum trader models are non-linear (Ryoo, 2010; Westerhoff, 2006a), in particular the share of fundamentalist and momentum traders may be endogenous (Hommes, 2006; Franke, 2008; De Grauwe, 2008; 2012). Beaudry et al. (2017) find (in a different context) that estimating linear models of non-linear processes can bias the estimated eigenvalues toward stability. Our model should be interpreted as a linear approximation and as a first step.

The model is estimated for the UK, France, Germany and the USA using the times series of equity and house prices over the period 1970-2017 with annual data as well as quarterly data. We analyze equity prices because they play a key role in Minsky models and because they are frequently used as key asset prices in macroeconomic analysis. The choice of house prices is due to the increasing interest in real estate prices in the Minskyan framework since the global financial crisis (Ryoo, 2016). Our results provide evidence of financial fluctuations in the equity market for the UK, France, Germany and the USA, with the highest price overshooting in economies with market-based financial systems, respectively the UK and the USA. Regarding house prices, we find robust evidence of cyclical fluctuations in the UK, France and the USA but not in Germany, with the highest price overshooting in the USA.

The paper is organized as follows. Section 2 reviews both the relevant theoretical and empirical literature. Section 3 presents the model and clarifies the conditions under which oscillations arise. Section 4 presents data and our econometric approach. Section 5 discusses the estimation results for equity prices and section 6 for house prices. Section 7 concludes with final considerations and directions for future research.

2 Review of the relevant literature

Since the 1980s, followers of the post-Keynesian school of economics have developed the economic ideas of Hyman Minsky in formal mathematical models. However, despite the great number of theoretical studies (see for example Taylor and O'Connell, 1985; Vercelli, 2000; Foley; 2003; Charles, 2008, Ryoo, 2010; 2012; 2013; Nishi, 2012; Kohler, 2019 among others), there are few empirical studies on the financial instability hypothesis. Section 2.1 revisits the theoretical and empirical papers on Minsky's theory. In section 2.2 we review the behavioral theory which highlights the role of heuristic behavior that can give rise to instability and fluctuations and the empirical literature on heterogeneous agents models.

2.1 Minskyan Financial Cycles

As Minsky has not provided a canonical formal model of his financial instability hypothesis, it has been formalized and interpreted in different ways. Minsky models can be grouped into debt cycles and asset price cycles. In the first, the dynamics of debt or interest rate is central in the analysis with no role assigned to asset prices (see for example Charles, 2008, Fazzari et al., 2008 and Nikolaidi, 2014). In the second group, asset prices play the key role (see for example, Taylor and O'Connell 1985 and Ryoo, 2016). In the standard version of the debt cycles, the model consists of a pro-cyclical debt ratio and a long-term negative effect of debt on investment which interact to generate cycles (Stockhammer, 2019). This idea is developed using diverse mechanisms and theoretical foundations: we can list the Kalecki-Minsky models, Kaldor-Minsky models, Goodwin-Minsky models, credit rationing models, endogenous target debt ratio models and Minsky-Veblen models. Among asset prices cycles we can distinguish between the equity price Minsky models (Taylor and O'Connor (1985); Ryoo, 2010, 2013) and the real estate price Minsky models (Ryoo, 2016). Within this group, asset price cycles are characterized by the speculative activity of agents based on expected capital gains that lead to an unsustainable bullish period which ultimately turns into a bust. In this class of models, two behavioral rules based on different forms of expectation formation interact, sometimes referred to as fundamentalist and momentum traders, with momentum traders providing the overshooting force. The interaction between the stabilizing of fundamentalists and the destabilizing of chartists speculators generates oscillation dynamics (Chiarella and Di Guilmi, 2011; Ryoo 2010, 2013; Sordi and Vercelli, 2012).

A small but growing body of literature has empirically examined the impact of financial variables on aggregate demand or their ability to cause crises. Palley (1994) and Kim (2013; 2016) estimate vector autoregressive (VAR) and vector error correction (VEC) models with GDP and household debt and report positive short-run feedback effects and negative long-run feedback effects of household debt on output. Greenwood-Nimmo and Tarassow (2016), with a policy-oriented Minsky model, examine the implications of monetary and macro-prudential shocks for aggregate financial fragility using a sign restricted VAR model.

The existing studies all focus on the interaction of the goods market and financial markets as the source of instability or cyclical phenomena. Moreover, Palley (1994), Kim (2013; 2016) and Greenwood-Nimmo and Tarassow (2016) do not test explicitly for endogenous cycles. Only recently, Stockhammer et al. (2019a) explicitly test the real-financial interaction mechanism and evaluate whether it gives rise to endogenous cycles. They start from a reduced form system of simultaneous equations in which a real variable and a financial variable interact with each other. Two conditions guarantee endogenous oscillations in a debt-burdened growth: complex eigenvalues and a negative sign of the off-diagonal coefficients of the Jacobian matrix. This means that from the interaction between the two state variables of the system an increase in one variable (the real one) induces an acceleration of the second variable (the financial one) which in turn drags down the first. They find evidence for financial-real interactions at high frequencies between GDP and interest rate and a low frequency between GDP and business debt. No evidence between GDP and household debt interaction is found. In the same vein, Stockhammer et al. (2019b), with historical macroeconomic data, estimate a Vector Autoregressive Moving Average model, to investigate whether business cycles are driven by corporate debt or by mortgage debt. They find that the USA economy has experienced corporate debt-driven Minsky cycles over the sample period. For the UK the leverage ratio is pro-cyclical, but no robust evidence for debt- burdened growth is found. Again, the estimation using mortgage debt yields no evidence for mortgage debt-driven Minsky cycles.

In summary, all the empirical works discussed above explore the empirics of Minskyan financial fragility but none of these studies account for the fundamental role played by asset prices. Minsky (1975) claimed that the decision to invest in equity markets would inevitably lead to speculative endogenous behaviours (Knell, 2014). The speculative behavior, stimulated by the euphoria of agents, would eventually turn the boom into the bust. In order to fill this gap in the Minsky literature, we empirically examine whether the asset prices dynamics in a context of behavioral heterogeneity is the driver of cyclical behavior. While we interpret the momentum trader model as one incarnation of Minsky models, the notion that speculative behavior can drive asset price dynamics has also been pioneered and is analytically further developed by behavioral economics.

2.2 Speculative behavior in behavioral economics

Theoretical studies in which the speculative thinking among investors plays a fundamental role in the determination of asset prices have an historical background in economics. Beja and Goldman (1980) in their seminal work present a dynamic model of the asset prices process in a disequilibrium setting. They distinguish between fundamentalist and speculative traders who act on their perception of the current price trend, i.e. they take into account information (past prices) which is unrelated to economic fundamentals. The speculation on the asset price-trend generates endogenous instabilities and oscillations in the price. Beja and Goldman (1980) thus prepares the ground for behavioral theory (Schleifer and Summers, 1990; Shiller, 2003; Vikash et al., 2015) and a variety of heterogeneous agents models (see e.g. Hommes, 2006 and Franke, 2008 for an overview).

Since the global financial crisis, the behavioral argument has received increasing attention and some of its insights have been incorporated in macroeconomic models. These theoretical studies range from the Behavioral New-Keynesian Models (BNKM) (De Grauwe 2008, 2012; Bofinger et al., 2013) to the linear and non-linear dynamic models of speculative market in a disequilibrium setting (Westerhoff, 2006a; 2006b; Lines and Westerhoff, 2006; Dieci and Westerhoof, 2012).[3] Despite the different paradigms, all these works allow for heterogeneity among agents. With regard to the BNKM, De Grauwe (2008; 2012) and Bofinger et al. (2013) highlight the role of heuristics in real and financial market. The agents may use fundamentalist or extrapolative rules to form their expectations. Fundamentalists act on the basis of fundamental information and process information

[3] The non-rational behavior is formalized assuming different behavioral biases. In De Grauwe (2008; 2012) momentum traders extrapolate variable of interest from the past into the future considering observed past values. The same in Westerhoff (2008a) with different autoregressive process. In Westerhoff (2006a; 2006b) and Lines and Westerhoff (2006) extrapolators base their beliefs on the observed past period and fundamental value.

rationally. In contrast, extrapolators base their expectations on past dynamics. They show by means of numerical simulation how the extrapolative formation rules of agents produce waves of optimism and pessimism in an endogenous way thus providing an explanation of the observed oscillation. In contrast to the paper by Beja and Goldman (1980), these authors introduce a time-variant selection mechanism à la Brock and Hommes (1998), as agents evaluate the performance of the rules and may switch strategy. Parallel to these, Westerhoff (2006a; 2006b; 2008a) and Lines and Westerhoff (2006) present more general disequilibrium dynamic models. Building on the multiplier-accelerator models of Samuelson (1939) they show how economic activity endogenously depends on extrapolative and mean-reverting behavior, thus emphasizing the role of heuristics in the generation of the business cycle. Dieci and Westerhoff (2012) analyze the house price dynamics in a nonlinear speculative discrete time dynamic model. Total demand for housing is created as an interaction between real and speculative demand, where the real demand decreases in price while the speculative demand is driven by price dynamics and depends on extrapolative and mean-reverting speculative strategies.

In contrast to the considerable number of theoretical studies, the empirical literature is rather limited and there is no consensus on the estimation methodology. Franke and Westerhoff (2017) note two approaches: direct and indirect. The first method employs surveys to measure the sentiments of a specific group of the population, typically the momentum traders, and thus explain their behavior. The second considers a model as a whole and strives to estimate all its parameters in one effort. With reference to the latter we can distinguish between two types of inference methods. In the first, key structural features of agent-based models can be estimated directly. Depending on the complexity of the models, we can list the nonlinear least squares, the maximum and quasi-maximum likelihood among others (Kukacka and Barunik, 2017; Chiarella et al., 2014; Westerhoff and Reitz, 2003): in line with the work of Frankel and Froot (1990) in our work the fraction of the two types is fixed in time. In the second method, estimation based on simulating artificial data from the model is used instead. The most frequently used estimation method is the method of simulated moments (MSM), (Franke and Westerhoff, 2011; Franke and Westerhoff, 2012). Estimation by MSM means searching for the parameter values of a model that minimize the distance between the simulated and the empirical counterparts. Through simulation runs it is possible to depict phenomena which are consequence of behavioral biases, such as volatility clustering, long memory effects, and a herding behavioral predisposition.

Empirical works of this type have been applied to different markets, such as equities, housing and foreign exchange market. Chiarella et al. (2014), Lof (2012; 2015) and Hommes and Veld (2017) show that heuristics perform very well in describing the dynamics of the stock market prices. Westerhoff and Reitz (2003) and De Jong et al. (2010) analyze the exchange rates market. In general, these works suggest that sentiment dynamics are important in explaining stylized facts observed in financial time series and in replicating observed anomalies in financial markets.

Along this line of research, our paper highlights the heterogeneity among agents and seeks to empirically identify the different evaluation behavior.[4] The behavioral models mentioned above do not provide evidence of cycles emerging directly from the data as a consequence of behavioral heuristics. The present paper proposes an estimation methodology for the empirical validation of endogenous cycles which has not yet been explored in the literature. We consider the beliefs of the agents as unobserved state components from which, through a state space model formulation, the endogeneity of fundamentalist-momentum trader cycles can be directly evaluated from the data. To achieve this, we use the maximum likelihood estimation (MLE). Unlike the indirect simulated-based estimation, as for the MSM, with MLE direct analytically estimation techniques are feasible. Differently from previous studies, we work in a state space model. Numerical techniques trough the Kalman filter algorithm are applied so that the maximized value of the log likelihood function can be reached and parameters can be recovered. Besides the tractability of the model, the main advantage of this framework is that, through filtering information on unobserved states, it is able to test whether behavioral rules lead to the cyclical dynamics in the observed asset prices.

[4] We should be clear that the 'identification' of these different behavioral rules is based on the theoretical framework of speculative Minsky cycles and Behavioral Finance as discussed in section 2. There is nothing intrinsic in the decomposition of the asset price series into a stochastic and an autoregressive process that would render them fundamentalists and momentum traders. Rather it is the particular theoretical framework that enables this interpretation. In this sense the 'identification' is ultimately contingent on the chosen theoretical frame and thus an 'interpretation' of the data.

3 The model

In this section we present the model and describe the proposed modelling strategy. We assume that the evolution of the asset price P_t, for equity asset and housing price, is determined by the following equation (See for example Ter Ellen and Verschoor, 2017 and Westerhoff, 2008b):

$$P_t = P_{t-1} + \gamma D^f_{t-1} + (1-\gamma)\, D^m_{t-1} \tag{1}$$

where D^f_{t-1} and D^m_{t-1} are the weighted (excess) demand of different types of agents, fundamentalists and momentum traders respectively. The weights γ and $1-\gamma$ are the proportions of fundamentalists and extrapolative agents in the housing and equity market.[5]

The demand functions can be specified as the difference between the current asset price and the expected asset price under fundamentalist ($P_t^{e,\,f}$) or extrapolative ($P_t^{e,\,m}$) expectations:

$$D^f_{t-1} = P_t^{e,\,f} - P_{t-1}$$

$$D^m_{t-1} = P_t^{e,\,m} - P_{t-1}$$

The fundamentalists believe that the asset price may temporarily deviate from the fundamental value, ${P_t}^f$. However, they also believe that the price will eventually converge to the fundamental value. Their demand for asset price is proportional to the difference between the market prices and the fundamental value. So the fundamentalist expectation can be defined in the following way

$$P_t^{e,\,f} = P_{t-1} + \lambda\left({P_t}^f - P_{t-1}\right) \quad 0 \leqslant \lambda \leqslant 1 \tag{2}$$

where λ measures the speed of reversion of the market price to the fundamental value. One implication of this is that, in the case of asset price boom or bust, fundamentalists expect market prices to revert to the fundamental value.

As to the momentum traders, we define their expectation behaviour in the following way

$$P_t^{e,\,m} = P_{t-1} + \beta\,(P_{t-1} - P_{t-2}) \qquad \beta \geqslant 0 \tag{3}$$

where β denotes the actual extrapolation parameter which captures the agent's price overshooting From Eq. (3), when the asset price is above (below) its value at previous time, it follows that the economic agent optimistically (pessimistically) believes in a further price increase (decrease). This form of expectation can be defined as a form of speculation on the current price trend based on the extrapolation of past prices rather than by fundamental news.[6]

If we substitute D^f_{t-1} and D^m_{t-1} in Eq. (1), we obtain

$$P_t = P_{t-1} + \gamma\left(P_t^{e,\,f} - P_{t-1}\right) + (1-\gamma)\,(P_t^{e,\,m} - P_{t-1})$$

from which

$$P_t = (1-\gamma)\,P_{t-1} + \gamma P_t^{e,\,f} + (1-\gamma)\,P_t^{e,\,m} - (1-\gamma)\,P_{t-1}$$

So at the end the observed asset price, P_t, for equity asset and housing price, is equal to the weighted sum of two unobserved expectations components

$$P_t = \gamma P_t^{e,\,f} + (1-\gamma)\,P_t^{e,\,m} \tag{4}$$

Now we can construct our state space model. In the context of the unobserved component model, agents' behavioral beliefs are unobserved state variables that have to be specified in a parametric stochastic form. To make our model tractable and to reach a feasible cyclical analysis (see Appendix A), we assume that the

[5] Momentum traders and extrapolative traders are used synonymously.

[6] Eqs. (2) and (3) are mostly used in Heterogeneous Agent Based Model literature (HABM). See for example Franke (2008) among others.

fundamentalists believes that the convergence to the fundamental value will take place next period. We follow Levy (2010 p. 8 and 9) and Franke (2008, p. 8) and assume that $\lambda = 1$, so to obtain from Eq. (2)

$$P_t^{e,f} = P_t{}^f \tag{5}$$

where $P_t{}^f$, the fundamental value, is determined following the random walk process (Franke, 2008)

$$P_t{}^f = P_{t-1}^f + \varepsilon_t \tag{6}$$

where ε_t is the individual disturbance term which is normally distributed with mean zero and variance σ_ε^2. With this simplification, fundamentalist traders can be called dogmatic fundamentalists. They believe that the stock price accurately reflects the asset's fundamental value. However, this assumption will be relaxed in the empirical analysis to see if the cyclical conditions are sensitive to it.

Substituting Eq. (4) in Eq. (3), the state equation for the extrapolators can be rewritten in the following way

$$P_t{}^m = \gamma(1+\beta) P_{t-1}^{e,f} + (1-\gamma)(1+\beta) P_{t-1}^{e,m} - \gamma\beta P_{t-2}^{e,f} - \beta(1-\gamma) P_{t-2}^{e,m}$$

We set

$$\begin{aligned} a_{21} &= \gamma(1+\beta) \\ a_{22} &= (1-\gamma)(1+\beta) \\ a_{23} &= -\gamma\beta \\ a_{24} &= -\beta(1-\gamma) \end{aligned} \tag{7}$$

such that, using Eq. (5), in a parametric stochastic form we finally obtain

$$P_t^{e,m} = a_{21} P_{t-1}^f + a_{22} P_{t-1}^{e,m} + a_{23} P_{t-2}^f + a_{22} P_{t-2}^{e,m} + \eta_t \tag{8}$$

where η_t is the individual disturbance term which is normally distributed with mean zero and variance σ_η^2.

Eqs. (6) and (8) are the so called state equations. Together with the observed asset price (Eq. (4)), they represent our state space model system. With this modelling strategy, we can reveal the nature and the cause of the dynamic movement of observed variables in an effective way. In fact, with a state space model it is possible to explain the behavior of an observed variable by examining the internal dynamic properties of the unobserved components.

An essential feature of any state space model is that the state equation must be a first-order stochastic difference equation (Enders, 2016). In our model the observation equation of the state space model is

$$P_t = \begin{pmatrix} \gamma & 1-\gamma & 0 & 0 \end{pmatrix} \begin{pmatrix} P_t^f \\ P_t^{e,m} \\ P_{t-1}^f \\ P_{t-1}^{e,m} \end{pmatrix} \tag{9}$$

Taking into account Eq. (8) and Eq. (6) with $\lambda = a_{11} = 1$, we have the transition equation of the state space model

$$\begin{pmatrix} P_t^f \\ P_t^{e,m} \\ P_{t-1}^f \\ P_{t-1}^{e,m} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_{t-1}^f \\ P_{t-1}^{e,m} \\ P_{t-2}^f \\ P_{t-2}^{e,m} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \eta_t \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

In a compact form, we define

$$P_t = HZ_t \tag{11}$$

$$Z_t = AZ_{t-1} + \delta_t \qquad \delta_t \sim \mathrm{N}(0, Q) \tag{12}$$

where P_t is the observable asset price,

$$Z_t = \begin{pmatrix} P_t^f \\ P_t^{e,m} \\ P_{t-1}^f \\ P_{t-1}^{e,m} \end{pmatrix}$$

is the state vector,

$$H = \begin{pmatrix} \gamma & 1-\gamma & 0 & 0 \end{pmatrix}$$

is the measurement matrix,

$$A = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

is the transition matrix and δ_t is the vector containing the state disturbance of unobserved components, normally distributed with mean zero and variances collected in the diagonal matrix Q.

The dynamic of the system is given by the transition equation which describes the evolution of the vector of unknown latent variables. Eigenvalues analysis can be performed to study the conditions for oscillations in our two-dimension discrete dynamic system associated with the two unobserved beliefs.[7] We obtain the associated characteristic equation considering the following determinant of the transition matrix:

$$\begin{vmatrix} a_{11}-\lambda & 0 & 0 & 0 \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = 0$$

First of all, we have the following two eigenvalues

$$\lambda_4 = a_{11} = 1 \in \Re \qquad \lambda_3 = 0$$

In addition, regarding the other two eigenvalues, they must satisfy

$$\begin{vmatrix} a_{22}-\lambda & a_{24} \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - a_{22}\lambda - a_{24} = 0$$

from which

$$\lambda_{1,2} = \frac{a_{22} \pm \sqrt{a_{22}^2 + 4a_{24}}}{2}$$

In order to have an oscillating behavior, these two last eigenvalues have to be complex, so that we require

$$\Delta = a_{22}^2 + 4a_{24} < 0$$

i.e.:

$$a_{24} < -\frac{a_{22}^2}{4} \tag{13}$$

When this is the case:

$$\lambda_{1,2} = \frac{a_{22}}{2} \pm i\frac{\sqrt{-\left(a_{22}^2 + 4a_{24}\right)}}{2} = a + ib$$

where i is the imaginary unit and a and b are real numbers. a is called the real part of the complex number and ib is the imaginary part. The complex number in the Cartesian form $a \pm ib$ can be written in the equivalent

[7] See Appendix A.

trigonometric form $\rho(\cos\omega \pm i\sin\omega)$. The positive number $\rho = \left(a^2+b^2\right)^{\frac{1}{2}}$ is called the modulus of the complex number (Gandolfo, 2009).

In order to have oscillations of constant amplitude we require

$$\rho = 1$$

i.e.:

$$\sqrt{\left(\frac{a_{22}}{2}\right)^2 + \frac{-\left(a_{22}^2+4a_{24}\right)}{4}} = 1$$

from which

$$a_{24} = -1$$

Inserting in Eq. (13)

$$-2 < a_{22} < 2$$

Then, the conditions to have oscillating behavior of constant amplitude are

$$a_{24} = -1 \qquad -2 < a_{22} < 2$$

If the condition in Eq. (13) is respected, with $-1 < a_{24} < 0$ (length of eigenvalues < 1) we have damped oscillations. With $a_{24} < -1$ (length of eigenvalues > 1) we have explosive oscillations. Summarizing we have an oscillating system if

$$|a_{11}| \leq 1 \qquad \forall a_{21}, \forall a_{23} \qquad a_{24} < -\frac{a_{22}^2}{4} \tag{14}$$

4 Data and econometric approach

The dataset consists of four OECD countries: the UK, France, Germany and the USA. We consider equity prices and housing prices with annual data from 1970 to 2017. We will repeat our estimations with quarterly data (1970Q1-2017Q4) as a robustness check. For all the four countries, the source for equity and house price series is the OECD database. We use deflated series for all the variables. House prices and equity prices series are deflated by the GDP deflator, which is taken from the Federal Reserve Economic Database for all the countries.[8]

In our model the driving forces behind the evolution of economic variables are not observable. In fact, asset price dynamics depend on the behavior of heterogeneous agents. In a context of the unobserved components model, the estimation problem can be solved with the Kalman filter approach in a state space model formulation. The state space model and the Kalman filter go hand-in-hand: to use the Kalman filter, we write the model in state space form. Then the recursive Kalman filter algorithm is used in calculating the optimal estimator of the state variables and in estimating the model parameters. Precisely, the parameters of the model are estimated by maximum likelihood using the prediction error decomposition approach where the one-step prediction and updating equations are calculated in a state space form using the iterative Kalman filtering.[9] Given the vector prediction errors and the variance-covariance matrix of the system, the log likelihood can be maximized.[10] In other words, the Kalman filter allows to construct the likelihood function associated with a state space model to estimate the parameters of unobservable variables. In our case, this econometric methodology seems to be the most appropriate as it aims to model latent factors (agents' expectation rules) that cannot be measured directly but lead to the response in observed data (asset prices). After the estimation, the iterative Kalman filter algorithm (also called one-sided filter) is used to extract the extrapolative and the fundamentalist states. In this case, unobserved states at period t are obtained using all information up to period t but without future observations. As the one-sided Kalman filter differs from the Kalman smoother (also called two-sided filter),

[8] For the econometric analysis all the series are transformed in log levels.

[9] See Appendix B

[10] The estimation procedure has been implemented with Matlab programming codes.

which also uses (forecasted) future states, Hamilton (2018) criticizes the use of such forecasted observations in the context of the Hodrick-Prescott filter.[11] However the one-sided Kalman filter is not subject to this criticism.

In the econometric analysis we first assume $a_{11} = 1$ for the fundamentalists; this assumption will later be relaxed. For the momentum traders, the coefficients a_{21}, a_{22}, a_{23} and a_{24} are estimated. To obtain oscillations, conditions in Eq. (14) have to be respected. Moreover, we estimate γ to obtain the proportion of fundamentalists and momentum agents. Once we obtain our estimation results, with a_{22}, a_{24} and γ it is possible to obtain β using Eq. (7).

From Eq. (7), it follows that

$$\begin{cases} a_{21} + a_{23} - \gamma = 0 \\ a_{22} + a_{24} + \gamma = 1 \end{cases}$$

These linear equality constraints for constrained likelihood objective function maximization have been imposed to obtain two values of β that differ for the sign. Considering Eq. (3), the positive value for price overshooting has been chosen. In our baseline model, the coefficients associated with the percentage of momentum traders and fundamentalists are fixed in time. However, these assumptions can be relaxed. In fact, it is possible to construct a time-varying linear state-space model. We leave the integration of time varying parameters to future work.

5 Estimation results for equity prices

Table 1 reports the maximum likelihood estimates of a_{22}, a_{24}, γ and β for equity prices in the UK, France, Germany and the USA for the period 1970-2017. The estimate of the model's parameters with the cyclical conditions and the log-likelihood with the sample size are given in the four columns headed by the country name. All the estimated coefficients are statistically significant at the 1% level. For all countries the size and signs of a_{22} and a_{24} respect conditions for oscillations ($a_{22}^2 < -4a_{24}$). Specifically, we find damped fluctuations as ($-1 < a_{24} < 0$) also holds.

Looking at the percentage of the two different types of agent in the financial market, for France and Germany we find that fundamentalists (γ) are the minority. In France and Germany, the momentum traders correspond to 71% and 54% respectively while the fundamentalists are estimated to be 29% and 46%. The opposite holds for the UK and the USA. In the UK, 75% of the agents are estimated to be fundamentalists while the 25% are momentum traders. In the USA, 69% of agents are estimated to be fundamentalists and 31% extrapolators. Nevertheless, the percentage of the extrapolators is sufficiently high to have a significant impact on observed prices.

From these results, it is possible to obtain the value of β, i.e. the price overshooting of the momentum agents. For the UK and the USA, the percentage of momentum traders is lower than Germany and France, however the momentum traders' price overshooting is higher. The highest price overshooting is in the UK ($\beta = 3$), followed by the USA ($\beta = 2.1$), Germany ($\beta = 0.8$) and France ($\beta = 0.4$).

Overall, in all the countries considered, the obtained results provide empirical support for Minsky's hypothesis of the existence of financial cycles in equity prices as a consequence of the different expectation rules defined in our model.

Table 1 also presents diagnostic tests for serial independence, homoscedasticity and normality of the residual of the models. In state space models, these tests are applied to what are known as the standardised prediction errors, which are defined as

$$e_t = \frac{v_t}{\sqrt{F_t}}$$

where v_t are the one-step ahead prediction errors obtained from the Kalman filter procedure. F_t is the variance of the one-step prediction errors v_t. The assumption of independence of the residuals is examined with the Ljung-Box Q-Test. The assumption of residual homoscedasticity is checked with the Engle test. Finally, the

[11] Note that there is a close connection between the Kalman smoother and the Hodrick-Prescott filter. The Hodrick-Prescott filter is a two-sided filter, usually calculated using the Kalman smoother for the state-space model. See Hamilton (2018) for a detailed discussion on the negative implications of using the Kalman smoother in HP filter.

Kolmogorov–Smirnov test is used to test the null hypothesis that residuals come from a normally distributed population.

The diagnostic tests suggest that residuals are well behaved for all four countries. The Ljung Box–Q statistic for autocorrelation, fails to reject the null hypothesis that the residuals are not autocorrelated. There is no sign of heteroscedasticity. Finally, the normality test statistic is lower than the critical value, so results indicate no rejection of the null hypothesis of normality.

Table 1: Estimation via Kalman filter for equity prices (annual data)

	France	Germany	UK	USA
a_{22}	1.0263***	0.9765***	1.0276***	0.9844***
	(0.0005)	(0.0007)	(0.0007)	(0.0009)
a_{24}	-0.3146***	-0.4308***	-0.7707***	-0.6663***
	(0.0029)	(0.0026)	(0.0047)	(0.0011)
γ	0.29***	0.46***	0.75***	0.69***
	(0.0027)	(0.0024)	(0.0039)	(0.0013)
$1-\gamma$	0.71***	0.54***	0.25***	0.31***
	(0.0027)	(0.0024)	(0.0039)	(0.0013)
β	0.4	0.8	3	2.1
σ_ε	0.1702***	0.1302***	0.1401***	0.1482***
	(0.0397)	(0.0332)	(0.0187)	(0.0140)
σ_η	0.2158***	0.2413***	0.1428***	0.1258***
	(0.0197)	(0.0230)	(0.0273)	(0.0289)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-likelihood	12.4351	16.3028	27.3701	28.6854
Sample size	48	48	48	48
Sample	1970-2017	1970-2017	1970-2017	1970-2017
	Autocorrelation Test			
Pvalue	0.26	0.38	0.98	0.78
CValue	37.56	37.56	37.56	37.56
Stat	23.40	21.26	8.74	14.76
	Heteroscedasticity Test			
Pvalue	0.26	0.57	0.40	0.26
CValue	31.41	31.41	31.41	31.41
Stat	23.39	18.18	20.93	23.45
	Normality Test			
Pvalue	0.25	0.50	0.07	0.50
CValue	0.13	0.13	0.13	0.13
Stat	0.10	0.08	0.12	0.08

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Cv and *Stat* are respectively the critical value and the test statistics.

To check the robustness of our estimation results for cyclical conditions, we re-estimate the transition matrix relaxing the assumption of $a_{11} = 1$ (instantaneous convergence to the fundamental value). The cyclical estimates of the unrestricted model are listed in Table 2. First, for all four countries, $a_{11} = 1$ is very close to 1. Second, cyclical conditions are not sensitive to relaxing the assumption of $a_{11} = 1$. In particular, we have damped fluctuations with $(-1 < a_{24} < 0)$.

Table 2: Estimation results for equity prices (annual data) [unrestricted model]

	France	Germany	UK	USA
a_{11}	0.99880***	0.9877***	0.9726***	0.9917***
	(0.0400)	(0.0141)	(0.0168)	(0.0160)
a_{22}	1.1128***	1.1477***	1.0742***	1.1248***
	(0.0113)	(0.0928)	(0.0013)	(0.0530)
a_{24}	-0.8746***	-0.6523***	-0.8424***	-0.9085***
	(0.0965)	(0.1001)	(0.0003)	(0.0534)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Up to now, we have checked for the existence of cycles using the maximum likelihood estimates. In small samples, to assess the precision of the estimates we can rely on resampling techniques. Among these, the bootstrap method chooses random samples with replacement from the sample data to estimate the parameters of interest. We now use this method to quantify how strongly the data support the presence of endogenous cycles. The procedure is applied to the standardized innovations following the Monte Carlo bootstrap procedure for state space models (Stoffer and Wall, 1991; 2004). The algorithm involves the following steps: firstly, we construct the standardised innovations using the prediction errors obtained from the Kalman filter procedure. From it, we sample with replacement. Secondly, we construct the new data set using the bootstrap sample. Using the bootstrap data set, we obtain the bootstrap estimation of the cyclical conditions. We repeat these steps 1000 times, obtaining a bootstrapped set of parameter estimates. From the bootstrap distribution we finally calculate the mean values of a_{22} and a_{24} which are compared with the gaussian maximum likelihood estimates. Together with the mean values we report the confidence intervals to assess the precision of the cyclical conditions. Table 3 shows the results. For all four countries, the bootstrapped coefficients are very similar in sign and size to those presented in Table 1 and cyclical conditions hold. Moreover, for all countries, the bootstrapped 95% confidence intervals of cyclical condition $(a_{22}^2 + 4a_{24})$ do not include zero, thus we can claim that the condition for oscillation hold at the 95% level.

Table 3: Bootstrap results for equity prices (annual data)

	France	Germany	UK	USA
a_{22}	1.0862 [1.0535, 1.1189]	0.9001 [0.8660, 0.9342]	1.0741 [1.0341, 1.1141]	1.0001 [0.9642, 1.0360]
a_{24}	-0.3733 [-0.4066, -0.3400]	-0.3601 [-0.3721, -0.3481]	-0.8241 [-0.8641, -0.7841]	-0.6901 [-0.6687, -0.7115]
$a_{22}^2 + 4a_{24}$	-0.0360 [-0.0674, -0.0046]	-0.6303 [-0.6720, -0.5886]	-1.7272 [-1.7974, -1.6570]	-1.7602 [-1.8313, -1.6891]
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*

Notes: The estimate are computed using 1.000 bootstrap sample.
In square brackets the bootstrapped 95% confidence interval.

As a further robustness test of our model, we report estimation results using quarterly data. This gives as a larger sample, but potentially more noise. Table 4 report the maximum likelihood estimates of a_{22}, a_{24}, and γ for equity prices in the UK, France, Germany and the USA, for the period 1970Q1-2017Q4. As for annual data, in all the countries considered, the signs of a_{22}, and a_{24} meet conditions for oscillation. In particular, we have damped fluctuations ($-1 < a_{24} < 0$) with a_{22} in the range consistent with oscillations ($a_{22}^2 < -4a_{24}$). Moreover, all the estimated coefficients are statistically significant at 1% statistical level. The highest price overshooting is in the UK ($\beta = 1.4$), followed by the USA ($\beta = 1.1$), France ($\beta = 0.9$) and Germany ($\beta = 0.7$).

Table 4 also reports diagnostic tests. The normality assumption of the residuals is rejected in all four countries. While we fail to reject the assumption of no autocorrelation for Germany, the UK and the USA, it is rejected in France. Residuals do not seem to suffer from heteroscedasticity.

Table 5 reports results for the unrestricted model, which does not impose the assumption of instantaneous price adjustment for fundamentalists. Again the conditions for oscillations holds, i.e. they are not sensitive to the relaxing the assumption of $a_{11} = 1$. In fact, $a_{11} = 1$ is close to one in all cases.

Table 6 presents results for the baseline specification based on bootstrap analysis. The coefficient estimates are very similar to our baseline model and, importantly, the bootstrap analysis confirms that the conditions for oscillations hold.

Overall, results with quarterly data are consistent with those obtained from annual data, in particular they provide further empirical support for the existence of endogenous financial cycles in asset prices as a consequence of the different behavioral rules defined in our model. Also in this case, we find the lowest share of momentum traders, but the highest degree of price overshooting in the UK and the USA. However diagnostic statistics deteriorate with quarterly data. This may be due to the quarterly series containing more complicated time structures that our simple model does not adequately represent. We thus regard the results with annual data as more reliable.

Table 4: Estimation via Kalman filter for equity prices (quarterly data)

	France	Germany	UK	USA
a_{22}	1.0349***	1.1478***	1.1380***	0.9324***
	(0.0489)	(0.1147)	(0.0007)	(0.0473)
a_{24}	-0.5052***	-0.4976***	-0.6635***	-0.5075***
	(0.0566)	(0.0663)	(0.0004)	(0.0683)
γ	0.47***	0.35***	0.53***	0.57***
	(0.0274)	(0.0735)	(0.0003)	(0.0393)
$1-\gamma$	0.53***	0.65***	0.47***	0.43***
	(0.0274)	(0.0735)	(0.0003)	(0.0393)
β	0.9	0.7	1.4	1.1
σ_ε	0.0505***	0.0934***	0.0752***	0.0626***
	(0.0028)	(0.0138)	(0.0003)	(0.0068)
σ_η	0.0044***	0.0823***	0.0004***	0.0761***
	(0.0007)	(0.0139)	(0.0000)	(0.0101)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-likelihood	152.38	194.804	216.147	231.249
Sample size	192	192	192	192
Sample	1970Q1-2017Q4	1970Q1-2017Q4	1970Q1-2017Q4	1970Q1-2017Q4
	Autocorrelation Test			
Pvalue	0.00	0.12	0.14	0.11
CValue	37.56	37.56	37.56	37.56
Stat	137.36	27.36	26.79	27.86
	Heteroscedasticity Test			
Pvalue	0.76	0.99	0.39	0.99
CValue	31.41	31.41	31.41	31.41
Stat	15.23	5.72	21.05	7.25
	Normality Test			
Pvalue	0.01	0.002	0.001	0.001
CValue	0.065	0.065	0.065	0.065
Stat	0.075	0.084	0.104	0.109

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Cv and *Stat* are respectively the critical value and the test statistics.

Table 5: Estimation results for equity prices (quarterly data) [unrestricted model]

	France	Germany	UK	USA
a_{11}	0.9957***	0.9913***	0.9889***	1.0000***
	(0.0042)	(0.0052)	(0.0116)	(0.0029)
a_{22}	1.0892***	0.9432***	1.0136***	1.1305***
	(0.0518)	(0.0712)	(0.1450)	(0.2978)
a_{24}	-0.4852***	-0.4280***	-0.6482***	-0.7513**
	(0.0460)	(0.1135)	(0.0844)	(0.2920)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table 6: Bootstrap results for equity prices (quarterly data)

	France	Germany	UK	USA
a_{22}	1.0500	1.1610	1.1500	1.1329
	[1.0133 - 1.0867]	[1.1218, 1.2002]	[1.1114, 1.1886]	[1.0345 - 1.2313]
a_{24}	-0.5200	-0.5110	-0.6800	-0.7029
	[-0.5361, -0.5039]	[-0.5286, -0.4934]	[-0.7012, -0.6588]	[-0.8013, -0.6045]
$a_{22}^2 + 4a_{24}$	-0.9745	-0.6961	-1.3975	-1.2800
	[-1.0245, -0.9245]	[-0.7394, -0.6528]	[-1.4587,-1.3363]	[-1.3378, -1.2222]
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*

Notes: The estimate are computed using 1.000 bootstrap sample.

In square brackets the bootstrapped 95% confidence interval.

6 Estimation results for house prices

Results for house prices (with annual data) are summarized in Table 7. For all countries we find that both the sizes and the signs of a_{22} and a_{24} respect conditions for fluctuations. We have damped fluctuations for all the four countries considered ($-1 < a_{24} < 0$), with a value for France and USA near to minus one, likely to generate almost constant amplitude cycles. For the UK, France and the USA, both a_{22} and a_{24} are statistically significant at the 1% level. For Germany, the conditions for cycles are satisfied, but the coefficient estimates are not statistically significant.

For the UK, Germany and the USA the estimated share of fundamentalists (γ) is substantially higher than that of momentum traders. For the UK, 69% of agents are fundamentalists and the remaining 31% are extrapolators. In Germany, the momentum agents account for 30% while the fundamentalists are estimated to be 70%. In the USA, 74% of agents are estimated to be fundamentalists and 26% are extrapolators. Only for France do we find similar proportion for the momentum traders (51%) and fundamentalists (49%).

Again we can calculate the extent of price overshooting. We find the highest price overshooting in the USA with a value of β equal to 3.7. This value is followed by the price overshooting in the UK with France ($\beta = 1.9$) and in Germany ($\beta = 0.2$).

Table 7 also reports diagnostic tests. The Ljung-Box Q-Test suggests that the residuals do not show significant evidence of autocorrelation for the UK, France and Germany, but for the USA, indicates the presence of serial correlation (at the 1% level). Finally, for all the four countries, the test fails to reject the null hypothesis of normality and homoscedasticity.

Overall, we find evidence for Minsky cycles on housing markets for the UK, France and the USA. For Germany, the point estimates for parameter suggest the presence of cyclical dynamics, however the relevant parameters are not statistically significant. Qualitatively speaking, these differences seem to be confirmed in the observed price's series of the four countries: unlike the UK, France and the USA, the house price fluctuation in Germany is less evident (See Appendix C).

Subsequently, we compare the obtained cyclical conditions with maximum likelihood estimates of the unrestricted model and bootstrap results. Table 8 reports results for the unrestricted model. Results for a_{11} are very close to one and cyclical conditions hold for France, the UK and the USA. Germany is the only case where we find that cyclical conditions are not respected.

Table 9 reports the results of the bootstrap analysis. Coefficient estimates are close to those of the baseline specification. Importantly the cyclical conditions hold. Table 9 also reports a confidence interval for the cyclical condition, which suggests that the condition holds at least at the 95% level.

Table 7: Estimation via Kalman filter for house prices (annual data)

	France	Germany	UK	USA
a_{22}	1.5102***	0.3580*	0.8991***	1.2195***
	(0.0894)	(0.1935)	(0.0036)	(0.0220)
a_{24}	-0.9968***	-0.0583	-0.5924***	-0.9599***
	(0.0009)	(0.1928)	(0.004)	(0.0083)
γ	0.49***	0.70***	0.69***	0.74***
	(0.0885)	(0.0016)	(0.0032)	(0.0252)
$1-\gamma$	0.51***	0.30***	0.31***	0.26***
	(0.0885)	(0.0016)	(0.0032)	(0.0252)
β	1.9	0.2	1.9	3.7
σ_ε	0.0621***	0.0305***	0.0830***	0.0374***
	(0.0018)	(0.0025)	(0.0074)	(0.0032)
σ_η	0.0376***	0.0000	0.0670***	0.0673***
	(0.0037)	(0.000)	(0.0172)	(0.0061)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*yes*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*yes*	*yes*	*yes*
Log-likelihood	67.8361	111.4751	56.297	77.1056
Sample size	48	48	48	48
Sample	1970-2017	1970-2017	1970-2017	1970-2017
	Autocorrelation Test			
Pvalue	0.27	0.22	0.018	0.000
CValue	37.56	37.56	37.56	37.56
Stat	23.35	24.37	35.34	85.17
	Heteroscedasticity Test			
Pvalue	0.65	0.58	0.25	0.62
CValue	31.41	31.41	31.41	31.41
Stat	17.03	18.01	23.75	17.41
	Normality Test			
Pvalue	0.50	0.50	0.23	0.50
CValue	0.13	0.13	0.13	0.13
Stat	0.07	0.07	0.10	0.08

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Cv and *Stat* are respectively the critical value and the test statistics.

Table 8: Estimation results for house prices (annual data) [unrestricted model]

	France	Germany	UK	USA
a_{11}	0.9654***	0.9244***	0.9970***	0.9637***
	(0.0134)	(0.0552)	(0.0123)	(0.0050)
a_{22}	0.4034***	0.3448**	1.2602***	1.4441***
	(0.0749)	(0.1735)	(0.0733)	(0.0308)
a_{24}	-0.1438**	-0.02298	-0.9382***	-0.9699***
	(0.0670)	(0.1727)	(0.0518)	(0.0054)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*no*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*no*	*yes*	*yes*

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table 9: Bootstrap results for house prices (annual data)

	France	Germany	UK	USA
a_{22}	1.4700	0.3087	0.9001	1.2000
	[1.4241, 1.5159]	[0.3065, 0.3109]	[0.8517, 0.9485]	[1.1604, 1.2396]
a_{24}	-0.9600	-0.0087	-0.5901	-0.9400
	[-0.9302, -0.9898]	[-0.0065, -0.0109]	[-0.5570, -0.6232]	[-0.9108, -0.9692]
$a_{22}^2 + 4a_{24}$	-1.6791	0.0617	-1.5501	-2.3200
	[-1.7479, -1.6103]	[0.0277, 0.0957]	[-1.6154, -1.4848]	[-2.4072, -2.2328]
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*no*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*no*	*yes*	*yes*

Notes: The estimate are computed using 1.000 bootstrap sample.

In square brackets the bootstrapped 95% confidence interval.

As robustness check we also estimate our model with quarterly data for the period 1970Q1-2017Q4. Results are summarized in Table 10. For France, the UK, and the USA a_{22} and a_{24} are statistically significant at the 1% level. For Germany only a_{24} is statistically significant. We find that for France, the UK, and the USA both the sizes and the signs of a_{22} and a_{24} respect conditions for fluctuations. In particular, we have damped fluctuations. For Germany, the point estimates suggest the absence of cyclical dynamics with no positive price overshooting. This is consistent with results for annual data where we found statistically significant evidence for cycle for the UK, France and the USA. We find the highest price overshooting in the US with a value of β equal to 1. This value is followed by the price overshooting in the UK ($\beta = 0.7$) and France ($\beta = 0.6$). The results with quarterly data indicate higher shares of momentum traders for France, the UK and the USA than those obtained with annual data. However these results come with a caveat as diagnostic tests for the normality, homoscedasticity and independence of the residuals suggest that assumptions are not met in France and the USA. Moreover, homoscedasticity and normality assumptions are rejected in Germany and in the UK respectively. This means that results may not be reliable. We suspect that our simple model does not fully capture the adjustment dynamics for quarterly data.

The unrestricted model results (Table 11) and the bootstrap results (Table 12) confirm the model's robustness. The cyclical conditions are hold for France, the UK and the USA.

The results with quarterly data are consistent with those obtained from annual data as regards our main question, the existence of endogenous financial cycles in asset prices. However, again diagnostic statistics deteriorate with quarterly data and we regard the results with annual data as more reliable.

Comparing these results for the house market to those for the equity market, we find similarities. With the exception of Germany, we find robust empirical evidence for Minsky's hypothesis of the existence of financial cycles in a context of different expectations in asset prices. We notice a lower share of extrapolative agents but higher price overshooting in the UK and the USA, the two advanced financial asset market-oriented economies. The cyclical dynamics are thus driven by the speculative expectations of a minority of market participants.

Moreover, the obtained results confirm the importance of considering the house prices affected by the presence of speculative forces that can generate cyclical fluctuations. The same forces of behavioral strategy that drive international financial markets also have the potential to affect other markets, like the housing market. In fact, it does not appear possible to explain the boom and bust in terms of fundamentals such as construction costs (Shiller, 2007).

Table 10: Estimation via Kalman filter for house prices (quarterly data)

	France	Germany	UK	USA
a_{22}	1.5180***	0.0689	1.3857***	1.0799***
	(0.1006)	(0.0825)	(0.0768)	(0.0483)
a_{24}	-0.6105***	0.3965***	-0.6056***	-0.5527***
	(0.0829)	(0.0882)	(0.0471)	(0.0315)
γ	0.10***	0.53***	0.22***	0.47***
	(0.0284)	(0.0203)	(0.0359)	(0.0301)
$1-\gamma$	0.90***	0.47***	0.78***	0.53***
	(0.0284)	(0.02036)	(0.0359)	(0.0301)
β	0.6	n.a.	0.7	1
σ_ε	0.0377***	0.0167***	0.0365***	0.0131***
	(0.0067)	(0.0017)	(0.0038)	(0.0006)
σ_η	0.0051***	0.0080***	0.0005***	0.0003***
	(0.0011)	(0.0021)	(0.0006)	(0.0006)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*no*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*no*	*yes*	*yes*
Log-likelihood	610.278	584.184	450.666	655.165
Sample size	192	192	192	192
Sample	1970Q1-2017Q4	1970Q1-2017Q4	1970Q1-2017Q4	1970Q1-2017Q4
	Autocorrelation Check			
Pvalue	0.007	0.1059	0.5260	0.000
CValue	37.56	37.56	37.56	37.56
Stat	38.76	28.14	18.93	115.08
	Heteroscedasticity Check			
Pvalue	0.019	0.039	0.7034	0.000
CValue	31.41	31.41	31.41	31.41
Stat	35.20	32.34	16.21	71.25
	Normality Check			
Pvalue	0.001	0.0847	0.001	0.001
CValue	0.065	0.065	0.065	0.065
Stat	0.165	0.0609	0.132	0.152

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Cv and *Stat* are respectively the critical value and the test statistics.

n.a. = no positive price overshooting.

Table 11: Estimation results for house prices (quarterly data) [unrestricted model]

	France	Germany	UK	USA
a_{11}	0.9959***	0.9985***	0.9975***	0.9993***
	(0.0005)	(0.0101)	(0.0005)	(0.0075)
a_{22}	1.49293***	0.0891	1.0276***	0.8326***
	(0.0119)	(0.0738)	(0.0705)	(0.0159)
a_{24}	-0.5724***	0.1355	-0.5763***	-0.3759***
	(0.0089)	(0.0910)	(0.0532)	(0.0505)
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*no*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*no*	*yes*	*yes*

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table 12: Bootstrap results for house prices (quarterly data)

	France	Germany	UK	USA
a_{22}	1.5001	0.2987	1.3000	1.1000
	[1.4536, 1.5466]	[0.2695 - 0.3279]	[1.2581, 1.3419]	[1.0624, 1.1376]
a_{24}	-0.5999	0.1713	-0.5200	-0.5700
	[-0.61856, -0.5813]	[0.1660, 0.1766]	[-0.5361, -0.5039]	[-0.5876, -0.5524]
$a_{22}^2 + 4a_{24}$	-0.1493	0.7852	-0.3900	-1.0700
	[-0.1820, -0.1166]	[0.7529, 0.8175]	[-0.4268, -0.3532]	[-1.1223, -1.0177]
	Cyclical Conditions			
$[-1 < a_{24} < 0]$	*yes*	*no*	*yes*	*yes*
$[a_{22}^2 < -4a_{24}]$	*yes*	*no*	*yes*	*yes*

Notes: The estimate are computed using 1.000 bootstrap sample.

In square brackets the bootstrapped 95% confidence interval.

7 Conclusions

This paper has proposed a test of asset price cycles based on the interaction of fundamentalist and momentum expectation rules. Both expectation rules are unobservable. The proposed model is formulated in a state space form and the parameters are estimated using the Kalman filter. We find robust empirical evidence for the presence of financial cycles in asset prices. Specifically, we find evidence of financial cycles in the equity market for the UK, France, Germany and the USA. For the housing market we find strong evidence for the UK, France and the USA. We also find that there are higher shares of fundamentalist traders, but that momentum traders' price expectations overshoot more in market-based financial systems, namely the UK and the USA. Housing markets have similar shares of fundamentalist traders as equity markets in the UK and USA, but higher shares in France and Germany.

The results have both theoretical and empirical implications, contributing to the literature in two main aspects. Firstly, for debates in the Minskyan literature, our results support speculative Minskyan cycles in equity and real estate prices. This goes beyond the existing empirical Minsky literature which has so far only investigated debt cycles, but not asset prices cycles.

Secondly, our results support behavioral economics, where heuristic decisions of agents are considered as a source of instability and fluctuations in the economy (De Grauwe, 2012; Franke and Westerhoff, 2017). In this regard, the contribution of the present paper is to provide an analytical framework that allows to estimate the effect of heuristic behavior with macroeconomic data. Our results highlight the fundamental role of heterogeneous expectations in generating fluctuations both in the equity market (Beja and Goldman, 1980) and in the housing market (Dieci and Westerhoff, 2012; Bofinger et al., 2013). In other words, our results contrast with the standard theoretical approach to asset price fluctuations, based on rational expectations and market "fundamentals". Our findings are fully in line with the notion that price changes are not explained by an economic fundamental variation, but by the use of heuristics (Shiller, 2003).

Our results are based on a specific model with simplifying assumptions. We want to highlight several possible extensions of the model. First, the most important simplification is the linear nature of the model. A key step for future research is thus to allowing time-varying shares of traders. Specifically, such a model could allow for an endogenous change of the share of fundamentalists and momentum traders conditional on their previous performance, (see for example Franke, 2008; Ter Ellen and Verschoor, 2017; Lux, 2018 and Lux and Zwinkels, 2018). This would also help to overcome the issue that estimates from a linear model may bias the eigenvalues toward stability (see Beaudry et al. (2017)). For this reason, using nonlinear methods and higher frequency sampling will help to enrich the analysis for more complex dynamics such as limit cycles, quasi-periodic cycles or chaos. An extended Kalman filter or the unscented Kalman filter algorithm could be used for these nonlinear extensions. Second, our model includes only two behavioral rules, which may not fully capture actual behaviors. Future research could integrate other behavioral rules in the framework proposed so as to improve the approximation of the asset price dynamics. Finally, our model (in line with the efficient market hypothesis) makes no substantive explanation of the fundamental value, but only assumes that it follows a random walk. Future empirical analysis could go beyond that by including exogenous variables that influence the fundamental variable. For example, the profit for the equity market or the household income for housing prices. All these extensions would represent a worthwhile improvement in the analysis, but would require a substantial change in the estimation strategy. The main contribution of this paper is that it offers a relatively simple statistical analysis of unobserved behavior that can give rise to cyclical fluctuations.

The evidence presented in this paper suggests endogenous cyclical dynamics in financial asset markets. These financial cycles are likely to have real economic and social costs that occur not only to momentum traders. The main policy implication of this paper thus is, fully in line with the suggestions of Hyman Minsky, that the financial regulator needs to lean against the wind and counteract financial boom-bust cycles.

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References

Beaudry, P., Galizia, D., & Portier, F. (2017). Is the Macroeconomy Locally Unstable and Why Should We Care? *NBER Macroeconomics Annual*, 31(1), 479–530. doi: 10.3386/w22275

Beja, A., & Goldman, M. B. (1980). On the dynamic behavior of prices in disequilibrium. *The Journal of Finance*, 35(2), 235–248. doi: 1540-6261.1980.tb02151.x

Bulligan, G., Burlon, L., Delle Monache, & D., Silvestrini, A. (2019). Real and financial cycles: estimates using unobserved component models for the Italian economy. *Statistical Methods & Applications*, 28, 541–569. doi: 10.1007/s10260-019-00453-1

Bofinger, P., Debes, S., Gareis, J. & Mayer, E. (2013). Monetary policy transmission in a model with animal spirits and house price booms and busts. *Journal of Economic Dynamics and Control*, 37, 2862–2881. doi: 10.1016/j.jedc.2013.08.002

Boone, L. (2000). Comparing semi-structural methods to estimate unobserved variables. *OECD Working Papers*, No. 240. doi: 10.1787/18151973

Borio, C. (2014). The financial cycle and macroeconomics: What have we learnt? *Journal of Banking and Finance*, 45, 182–198. doi: 10.1016/j.jbankfin.2013.07.031

Brock, W. A., & Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22(8-9), 1235–1274. doi: 10.1016/S0165-1889(98)00011-6.

Charles, S. (2008). Teaching Minsky's financial instability hypothesis: a manageable suggestion. *Journal of Post Keynesian Economics*, 31(1), 125–138. doi: 10.2753/PKE0160-3477310106

Chiarella, C., & Di Guilmi, C. (2011). The financial instability hypothesis: A stochastic microfoundation framework. *Journal of Economic Dynamics and Control*, 35(8), 1151–1171. doi: 10.1016/j.jedc.2011.02.005

Chiarella, C., He, X.-Z. & Zwinkels, R. C. (2014). Heterogeneous expectations in asset pricing: Empirical evidence from the S&P500. *Journal of Economic Behavior and Organization*, 105, 1–16. doi: 10.1016/j.jebo.2014.03.003

Davis, L. E., de Souza J. P. A. & Hernandez, G. (2019). An empirical analysis of Minsky regimes in the US economy. *Cambridge Journal of Economics*, 43(3), 541–583. doi: 10.1093/cje/bey061

De Grauwe, P. (2008). Stock Prices and Monetary Policy. *CEPS Working Document*, No. 304, September. doi: 10.2139/ssrn.1337027

De Grauwe, P. (2012). Booms and busts in economic activity: A behavioral explanation. *Journal of Economic Behavior and Organization*, 83, 484–501. doi: 10.1016/j.jebo.2012.02.013

De Jong, E., Verschoor, W. F. C. & Zwinkels, R.C.J. (2010). Heterogeneity of agents and ex-change rate dynamics: evidence from the EMS. *Journal of International Money and Finance*, 2(8), 1652– 1669. doi: 10.1016/j.jimonfin.2010.05.007

De Winter, J., Koopman, S. J., Hindrayanto, I. & Chouhan, A. (2017). Modeling the business and financial cycle in a multivariate structural time series model. *DNB Working Papers*, 573, Netherlands Central Bank, Research Department. doi: 10.2139/ssrn.3057349

Dieci, R., & Westerhoff, F. (2012). A simple model of a speculative housing market. *Journal of Evolutionary Economics*, 22, 303–329. doi: 10.1007/s00191-011-0259-8

Enders, W. (2016). *Applied econometric time series*(4th ed.). Supplementary Manual. New York, Wiley.

Fazzari, S., Ferri, P. & Greenberg, E. (2008). Cash flow, investment, and Keynes-Minsky cycles. *Journal of Economic Behavior and Organization*, 65(3–4), 555–572. doi: 10.1016/j.jebo.2005.11.007

Ferri, P., & Minsky, H. P. (1992). Market processes and thwarting systems. *Structural Change and Economic Dynamics*, 3(1), 79–91. doi: 10.1016/0954-349X(92)90027-4

Foley, D. (2003). Financial fragility in developing economics. In A.K.Dutt and J. ros (ed.) Development Economics and Structural Macroeconomics: Essays in Honor of Lance Taylor Cheltenham: edward elgar. doi: 10.4337/9781781950081.00018

Franke, R. (2008). Models of asset price dynamics from Beja-Goldman to Brock-Hommes: A synopsis of specification details. University of Kiel.

Franke, R., & Westerhoff, F. (2011). Estimation of a structural stochastic volatility model of asset pricing. *Computational Economics,* 38(1), 53–83. doi: 10.1007/s10614-010-9238-7

Franke, R, & Westerhoff, F. (2012). Structural stochastic volatility in asset pricing dynamics: Estimation and model contest. *Journal of Economic Dynamics and Control*, 36(8), 1193-1211. doi: 10.1016/j.jedc.2011.10.004

Franke, R, & Westerhoff, F. (2017). Taking stock: rigorous modeling of animal spirits in macroeconomics. *Journal of Economic Surveys*, 31(5), 1152–1182. doi: 10.1111/joes.12219

Frankel, J. A., & Froot, K.A. (1990). Chartists, fundamentalists, and trading in the foreign exchange market. *American Economic Review*, 80(2), 181–185.

Gandolfo, G. (2009). *Economic Dynamics* (4th ed.). Springer.

Galati, G., Hindrayanto, I., Koopman, S. J., & Vlekke, M. (2016). Measuring financial cycles in a model-based analysis: Empirical evidence for the United States and the euro area. *Economics Letters*, 145, 83-87. doi: 10.1016/j.econlet.2016.05.034.

Greenwood-Nimmo, M., & Tarassow, A. (2016). Monetary shocks, macroprudential shocks and financial stability. *Economic Modelling*, 56, 11–24. doi: 10.1016/j.econmod.2016.03.003

Hamilton, J. D. (2018). Why You Should Never Use the Hodrick-Prescott Filter. *Review of Economics and Statistics*, 100(5), 831-843. doi: 10.1162/rest-a-00706

Harvey, A. C. (1989). *Forecasting, structural time series models and the Kalman filter.* Cambridge University Press.

Hommes, C. H. (2006). Chapter 23. Heterogeneous Agent Models in Economics and Finance. *Handbook of Computational Economics*, 1109–1186.

Hommes, C., & in 't Veld, D. (2017). Booms, busts and behavioural heterogeneity in stock prices. *Journal of Economic Dynamics and Control*, 80, 101–124. doi: 0.1016/j.jedc.2017.05.006

Kim, Y. K. (2013). Household debt, financialization, and macroeconomic performance in the United States, 1951–2009. *Journal of Post Keynesian Economics*, 35(4), 675-694. doi: 10.2307/23469854

Kim, Y. K. (2016). Macroeconomic effects of household debt: an empirical analysis. *Review of Keynesian Economics*, 4(2), 127-150. doi: 10.4337/roke.2016.02.01

Klinger, S., & Weber, E. (2019). GDP-Employment Decoupling in Germany. *Structural Change and Economic Dynamics*, doi: 10.1016/j.strueco.2019.10.003

Knell, M. (2014). Schumpeter, Minsky and the financial instability hypothesis. *Journal of Evolutionary Economics*, 25(1), 293–310. doi: 10.1007/s00191-014-0370-8

Kohler, K. (2019). Exchange rate dynamics, balance sheet effects, and capital flows. A Minskyan model of emerging market boom-bust cycles. *Structural Change and Economic Dynamics*, 51, 250-273. doi: 10.1016/j.strueco.2019.09.006

Kukacka, J., & J. Barunik (2017). Estimation of financial agent-based models with simulated maximum likelihood. *Journal of Economic Dynamics and Control*, 85, 21-45. doi: 10.1016/j.jedc.2017.09.006

Jordà, O., Schularick, M. & Taylor, A.M., (2016). The great mortgaging: housing finance, crises and business cycles. *Economic Policy,* 31, 107–152. doi: doi.org/10.1093/epolic/eiv017

Lavoie, M., & Daigle, G. (2010). A behavioural finance model of exchange rate expectations within a stock-flow consistent framework. *Metroeconomica*, 62(3), 434–458. doi: 10.1111/j.1467-999X.2010.04116.x

Levy, M. (2010). Chapter from book: Complex systems in finance and econometrics. Selected entries from the Encyclopedia of Complexity and Systems Science, 1-21.

Lines, M., & Westerhoff, F. (2006). Expectations and the Multiplier-Accelerator Model. In: Puu T., Sushko I. (eds) *Business Cycle Dynamics*, Springer, Berlin, Heidelberg.

Lof, M. (2012). Heterogeneity in stock prices: A STAR model with multivariate transition function. *Journal of Economic Dynamics and Control*, 36(12), 1845–1854. doi:10.1016/j.jedc.2012.06.006

Lof, M. (2015). Rational speculators, contrarians, and excess volatility. *Management Science*, 61(8), 1889–1901. doi: 10.1287/mnsc.2014.1937

Lux, T. (2018). Estimation of agent-based models using sequential Monte Carlo methods. *Journal of Economic Dynamics and Control*, 91, 391-408. doi: 10.1016/j.jedc.2018.01.021

Lux, T. & Zwinkels, RCJ. (2018). Empirical validation of agent-based models. *Handbook of computational economics*, 4, 437-488. doi: 10.1016/bs.hescom.2018.02.003

Mian, A., Sufi, A. & Verner, E. (2017). Household debt and business cycles worldwide. *The Quarterly Journal of Economics*, 132(4), 1755 – 1817. doi: 10.1093/qje/qjx017

Minsky, H. (1975). *John Maynard Keynes.* Columbia University Press, New York.

Minsky, H. (1985). The Financial Instability Hypothesis: A Restatement. In: P Arestis and T Skouras (eds): Post Keynesian Economic Theory. A Challenge to Neo Classical Economics. Sussex: Weatsheaf Books

Mulligan, R. (2013). A sectoral analysis of the financial instability hypothesis. *The Quarterly Review of Economics and Finance*, 53(4), 450–459. doi: 10.1016/j.qref.2013.05.010

Nikolaidi, M. (2014). Margins of safety and instability in a macrodynamic model with Minskyan insights. *Structural Change and Economic Dynamics*, 31, 1–16. doi: 10.1016/j.strueco.2014.07.001

Nikolaidi, M., & Stockhammer, E. (2017). Minsky models: a structured survey. *Journal of Economic Surveys*, 31(5), 1304–1331. doi: 10.1111/joes.12222

Nishi, H. (2012). A dynamic analysis of debt-led and debt-burdened growth regimes with minskian financial structure. *Metroeconomica*, 63(4), 634–660. doi: 10.1111/j.1467-999X.2012.04158.x

Nishi, H. (2016). An empirical contribution to Minsky's financial fragility: evidence from non-financial sectors in Japan. *Cambridge Journal of Economics*, 43(3), 585–622. doi: 10.1093/cje/bey031

Palley, T. (1994). Debt, Aggregate Demand, and The Business Cycle: an Analysis in the Spirit of Kaldor and Minsky. *Journal of Post Keynesian Economics*, 16, 371–390. doi: 10.1080/01603477.1994.11489991

Rusticelli, E. (2014). Rescuing the Phillips curve. O*ECD Journal: Economic Studies*, 2014.1, 109-127. doi: 10.1787/19952856

Ryoo, S. (2010). Long waves and short cycles in a model of endogenous financial fragility. *Journal of Economic Behavior and Organization*, 74(3), 163–186. doi: 10.1016/j.jebo.2010.03.015

Ryoo, S. (2012). The paradox of debt and minsky's financial instability hypothesis. *Metroeconomica*, 64(1), 1–24. doi: 10.1111/j.1467-999X.2012.04163.x

Ryoo, S. (2013). Minsky cycles in Keynesian models of growth and distribution. *Review of Keynesian Economics*. 1(1), 37–60. doi: 10.4337/roke.2013.01.03

Ryoo, S. (2016). Household debt and housing bubble: a Minskian approach to boom-bust cycles. *Journal of Evolutionary Economics*, 26(5), 971–1006. doi: 10.1007/s00191-016-0473-5

Samuelson, P. (1939). Interactions between the Multiplier Analysis and the Principle of Acceleration. *The Review of Economics and Statistics*, 21(2), 75-78.

Schleifer, A., & Summers, L. (1990). The Noise Trader Approach to Finance. *Journal of Economic Perspectives*, 4(2), 19-34. doi: 10.1257/jep.4.2.19

Schroeder, S. (2009). Defining and detecting financial fragility: New Zealand's experience. *International Journal of Social Economics*, 36(3), 287–307. doi: 10.1108/03068290910932765

Shiller, Robert J. (2003). From efficient markets theory to behavioral finance. *Journal of Economic Perspectives*, 17(1), 83-104. doi: 10.1257/089533003321164967

Shiller, Robert J. (2005). *Irrational exuberance.* (2nd ed.). Princeton University Press, Princeton.

Shiller, Robert J. (2007). Understanding recent trends in house prices and home ownership. *Cowles foundation discussion paper*, no.1630. Yale University, New Haven. doi: 10.3386/w13553

Sordi, S., & Vercelli, A. (2012). Heterogeneous expectations and strong uncertainty in a Minskyian model of financial fluctuations. *Journal of Economic Behavior and Organization*, 83(3), 544–557. doi: 10.1016/j.jebo.2012.05.012

Stockhammer, E., (2019). An update on Kalecki–Minsky modelling. *European Journal of Economics and Economic Policies: Intervention*, 16(2), 179-192. doi: 10.4337/ejeep.2019.02.04

Stockhammer, E., Calvert Jump, R., Kohler, K., & Cavallero, J. (2019a). Short and medium-term financial-real cycles: An empirical assessment. *Journal of International Money and Finance*, 94, 81–96. doi: 10.1016/j.jimonfin.2019.02.006

Stockhammer, E., Gouzolis, G., & Calvert Jump R. (2019b). Debt driven business cycle in historical perspective: The case of the USA (1889-2015) and UK(1882-2010), *Working Papers PKWP1907*, Post Keynesian Economics Society (PKES).

Stoffer, D., & Wall, K. (1991). Bootstrapping State-Space Models: Gaussian Maximum Likelihood Estimation and the Kalman Filter. *Journal of the American Statistical Association*, 86, 1024-1033. doi: 10.2307/2290521

Stoffer, D., & Wall, K. (2004). Resampling in state space models. In A. Harvey, S. Koopman, & N. Shephard (Eds.), State Space and Unobserved Component Models: Theory and Applications (pp. 171-202). Cambridge: Cambridge University Press.

Taylor, L., & O'Connell, S.A. (1985). A Minsky crisis. *Quarterly Journal of Economics*, 100(Supplement), 871–885. doi: 10.1093/qje/100.Supplement.871

Ter Ellen, S., & Verschoor, W. (2017). Heterogeneous beliefs and asset price dynamics: a survey of recent evidence, Chapter from book: Uncertainty, expectations and Asset Price Dynamics. Springer

Vercelli, A. (2000). Structural financial instability and cyclical fluctuations. *Structural Change and Economic Dynamics*, 11(1-2), 139–156. doi: 10.1016/S0954-349X(00)00015-1

Vikash, R., Xiaoming, X. & Imad, A. M. (2015). Neoclassical finance, behavioral finance and noise traders: A review and assessment of the literature. *International Review of Financial Analysis*, 41, 89–100. doi: 10.1016/j.irfa.2015.05.021

Westerhoff, F. (2006a). Nonlinear Expectation Formation, Endogenous Business Cycles and Stylized Facts. *Studies in Nonlinear Dynamics and Econometrics*, 10(4). doi: 10.2202/1558-3708.1324

Westerhoff, F. (2006b). Samuelson's multiplier–accelerator model revisited. *Applied Economics Letters*, 13, 89–92. doi: 10.1080/13504850500390663

Westerhoff, F. (2008a). Heuristic expectation formation and business cycles: A simple linear model. *Metroeconomica*, 59(1), 371-390. doi: 10.1111/j.1467-999X.2007.00290.x

Westerhoff, F. (2008b). The use of agent-based financial market models to test the effectiveness of regulatory policies. *Journal of Economics and Statistics*, 228(2-3), 195–227. doi: 10.1515/jbnst-2008-2-305

Westerhoff, F., & Reitz, S. (2003). Nonlinearities and cyclical behaviour: The role of chartists and fundamentalists. *Studies in Nonlinear Dynamics and Econometrics*, 7(4), 1–13. doi: 10.2202/1558-3708.1125

Appendix A

Let us consider a discrete system

$$U = [u_i(t)] = \begin{bmatrix} u_1(t) \\ . \\ u_i(t) \\ . \\ u_r(t) \end{bmatrix} \in \Re^{rx1}$$

where

$$u_i(t) : \Re \to \Re \qquad i = 1, ..., r \quad t \in [0, T]$$

We assume that:

Hp.1) functions $u_i(t)$ can be described by their values assumed in discrete time.

Introducing the vector

$$U_j = [u_i(t_j)] \quad t_j = j\Delta t \quad j = 1, 2, ..., n \quad n\Delta t = T$$

Hp.2) the values at time t_j can be expressed by the values assumed at previous times $t_{j-1}, ..., t_{j-R}$ where R is the memory's degree.

Introducing the vector

$$[\mathbf{U}_j] = \begin{bmatrix} U_j \\ U_{j-1} \\ . \\ U_{j-(R-1)} \end{bmatrix}$$

the condition assumed by the second hypothesis can be expressed by

$$[\mathbf{U}_j] = \begin{bmatrix} U_j \\ U_{j-1} \\ . \\ U_{j-(R-1)} \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} U_{j-1} \\ U_{j-2} \\ . \\ U_{j-R} \end{bmatrix} = [\mathbf{A}] [\mathbf{U}_{j-1}] \qquad j = R+1, ..., N$$

where

$$[\mathbf{U}_k] \in \Re^N \quad [\mathbf{A}] \in R^{NxN} \quad N = rR$$

It should be noted that it is necessary to know the state vector at the first R-times to activate the recursive law. Assuming in the previous equation $j = 1, ..., N$ (that amounts to assume that the state vector is known at R previous times), the previous recursive law can be expressed by

$$\begin{aligned} &\mathbf{U}_2 = \mathbf{A}\mathbf{U}_1 \\ &\mathbf{U}_3 = \mathbf{A}^2\mathbf{U}_1 \\ &... \\ &\mathbf{U}_j = \mathbf{A}^j\mathbf{U}_1 \end{aligned}$$

Let $\mathbf{V}$ and $\mathbf{D}$ be the matrix of the eigenvectors and eigenvalues of the matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1} \quad \mathbf{V}\mathbf{V}^{-1} = \mathbf{I}$$

so that

$$\mathbf{U}_j = \mathbf{V}\mathbf{D}^j\mathbf{V}^{-1}\mathbf{U}_1$$

Also, the behavior of the recursive law is entirely described by the values of the eigenvalues

$$\lambda_i \quad i = 1, ..., N$$

When $\lambda_i \in \Re$, $i = 1, ..., N$, the system is constant if $\lambda_i = 1 \quad \forall i$, monotonic increasing if $\lambda_i > 1$ for one i, monotonic decreasing if $\lambda_i < 1$ for one i.

In order to have an oscillating behavior it is necessary that

$$\lambda_i \in C \quad i = 1, ..., N$$

Moreover, the behavior depends on the modulus ρ of the complex eigenvalues. Amplitude will increase, remain constant or decrease if, respectively, ρ is greater than equal or smaller than unity.

Now let us consider $r = 1$ and $R = 2$

$$u_j = \alpha u_{j-1} + \beta u_{j-2}$$

so that

$$\begin{bmatrix} u_j \\ u_{j-1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} u_{j-1} \\ u_{j-2} \end{bmatrix}$$

with

$$\mathbf{A} = \begin{bmatrix} \alpha & \beta \\ 1 & 0 \end{bmatrix}$$

We consider

$$\det \begin{bmatrix} \alpha - \lambda & \beta \\ 1 & -\lambda \end{bmatrix} = \lambda^2 - \alpha\lambda - \beta = 0$$

so that the eigenvalues are

$$\lambda_{1,2} = \frac{\alpha \pm \sqrt{\alpha^2 + 4\beta}}{2}$$

In order to have an oscillating behavior, the eigenvalues have to be complex so that

$$\Delta = \alpha^2 + 4\beta < 0$$

$$\beta < -\frac{\alpha^2}{4} \tag{A.1}$$

When this is the case:

$$\lambda_{1,2} = \frac{\alpha}{2} \pm i \frac{\sqrt{-\left(\alpha^2 + 4\beta\right)}}{2} = a + ib$$

where i is the imaginary unit and a and b are real numbers. a is called the real part of the complex number and ib is the imaginary part. The complex number in the Cartesian form $a \pm ib$ can be written in the equivalent trigonometric form $\rho\left(\cos\omega \pm i \sin\omega\right)$. The positive number $\rho = \left(a^2 + b^2\right)^{\frac{1}{2}}$ is called the modulus or absolute value of the complex number (Gandolfo, 2009).

In order to have oscillations of constant amplitude we require

$$\rho = 1$$

i.e.:

$$\sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{-\left(\alpha^2 + 4\beta\right)}{4}} = 1$$

from which

$$\beta = -1$$

Inserting in Eq. (A.1)

$$-2 < \alpha < 2$$

Then, the conditions to have oscillating behavior of constant amplitude are

$$\beta = -1$$

and

$$-2 < \alpha < 2$$

If the condition in Eq. (A.1) is respected, with $-1 < \beta < 0$ (length of eigenvalues < 1) we have damped oscillations. With $\beta < -1$ (length of eigenvalues > 1) we have explosive oscillations.

Connecting to our model with $r = 2$ and $R = 2$, where $u_1 = p^f$ and $u_2 = p^m$, we have

$$\begin{pmatrix} u_{1,j} \\ u_{2,j} \\ u_{1,j-1} \\ u_{2,j-1} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} u_{1,j-1} \\ u_{2,j-1} \\ u_{1,j-2} \\ u_{2,j-2} \end{pmatrix}$$

$$\begin{vmatrix} a_{11}-\lambda & 0 & 0 & 0 \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = (a_{11}-\lambda) \begin{vmatrix} a_{22}-\lambda & a_{23} & a_{24} \\ 0 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -(a_{11}-\lambda)(\lambda) \begin{vmatrix} a_{22}-\lambda & a_{24} \\ 1 & -\lambda \end{vmatrix} = 0$$

The first two eigenvalues are

$$\lambda_4 = a_{11} \in \Re \qquad \lambda_3 = 0$$

Regarding the other eigenvalues, it should be noted that the problem is equivalent to the preceding case so that the system is oscillating if

$$|a_{11}| \leq 1 \qquad \forall a_{21}, \forall a_{23} \qquad a_{24} < -\frac{a_{22}^2}{4} \tag{A.2}$$

Appendix B

The Kalman filter is a recursive dynamic procedure for calculating the optimal estimator of the unobserved state vector. It is considered the best among the linear filters and one important advantage of using the state-space approach via the Kalman Filter is that stationarity of variables is not required. One limitation is that the state equation must be a first-order stochastic difference equation. However, it is often possible to rewrite a complicated dynamic process as a vector process (See for example Enders, 2016). The goal is to minimize the mean square prediction error of the unobserved state vector conditional of the observation of P_t.

The optimal forecasting rule has the form

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t \left(P_t - P_{t\,|\,t-1}\right)$$

where K_t is a weight that changes as new information becomes available, $Z_{t\,|\,t}$ denotes the forecast of state variable once P_t is realized while $Z_{t\,|\,t-1}$ and $P_{t\,|\,t-1}$ denote respectively the forecast of variables Z_t and P_t before P_t is realized.

Now we can select the optimal value of K_t to minimize the mean square prediction error at time t

$$\min_{k_t} E_t\big(Z_t - Z_{t\,|\,t}\big)^2 = \min_{k_t} E_t\big[Z_t - \big(Z_{t\,|\,t-1} + K_t\left(P_t - P_{t\,|\,t-1}\right)\big)\big]^2$$

Using Eq. (8) for the observable asset price, we obtain

$$\min_{k_t} E_t\big[Z_t - \big(Z_{t\,|\,t-1} + K_t\left(HZ_t - HZ_{t\,|\,t-1}\right)\big)\big]^2$$

$$\min_{k_t} E_t\big[(I - HK_t)\left(Z_t - Z_{t\,|\,t-1}\right)\big]^2$$

$$\min_{k_t} (I - HK_t)^2 E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2$$

Optimizing with respect to K_t we get

$$-2H\left(I - HK_t\right) E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2 = 0$$

Indicating with $\Gamma_{t\,|\,t-1} = E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2$, we obtain

$$-2H\left(I - HK_t\right)\Gamma_{t\,|\,t-1} = 0$$

Solving for K_t we obtain

$$K_t = \frac{H\ \Gamma_{t\,|\,t-1}}{H\ \Gamma_{t\,|\,t-1} H'}$$

Regrouping the equations, we obtain that

$$Z_{t\,|\,t-1} = AZ_{t-1\,|\,t-1} \tag{B.1}$$

$$\Gamma_{t\,|\,t-1} = A\,\Gamma_{t-1\,|\,t-1}A' + Q \tag{B.2}$$

$$P_{t\,|\,t-1} = HP_{t-1\,|\,t-1}$$

Equations (B.1) and (B.2) are the so-called prediction equations in the Kalman filtering. The other equations we need are the three updating equations which are

$$K_t = \Gamma_{t\,|\,t-1} H'(\psi_t)^{-1} \tag{B.3}$$

with

$$\psi_t = H\Gamma_{t\,|\,t-1}H'$$

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t\left(P_t - P_{t\,|\,t-1}\right) \tag{B.4}$$

$$\Gamma_{t\,|\,t} = (I - K_t H)\,\Gamma_{t\,|\,t-1} \tag{B.5}$$

In this case, the inference about Z_t is updated using the observed value of P_t.

We start with a specification information set with initial conditions $Z_{0\,|\,0}$ and $\Gamma_{0\,|\,0}$. Then we use the prediction equations (B.1) and (B.2) to obtain $Z_{1\,|\,0}$ and $\Gamma_{1\,|\,0}$. Once we observe P_1 we use the updating equations (B.3), (B.4), and (B.5) to obtain $Z_{1\,|\,1}$, $\Gamma_{1\,|\,1}$ and $P_{1\,|\,1}$. We next use this information to form $Z_{2\,|\,1}$ and $\Gamma_{2\,|\,1}$, then forecasts are updated and we continue to repeat this process until the end of the dataset.

Given the vector prediction errors $\mu_t = P_t - P_{t\,|\,t-1}$ and the variance-covariance matrix ψ_t, we can form the log-likelihood to be maximized and to estimate our parameters.

$$\log l = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left(\left|\psi_{t\,|\,t-1}\right|\right) - \frac{1}{2}\sum_{t=1}^{T}\mu_t{}'\left(\psi_{t\,|\,t-1}\right)^{-1}\mu_t$$

Appendix C

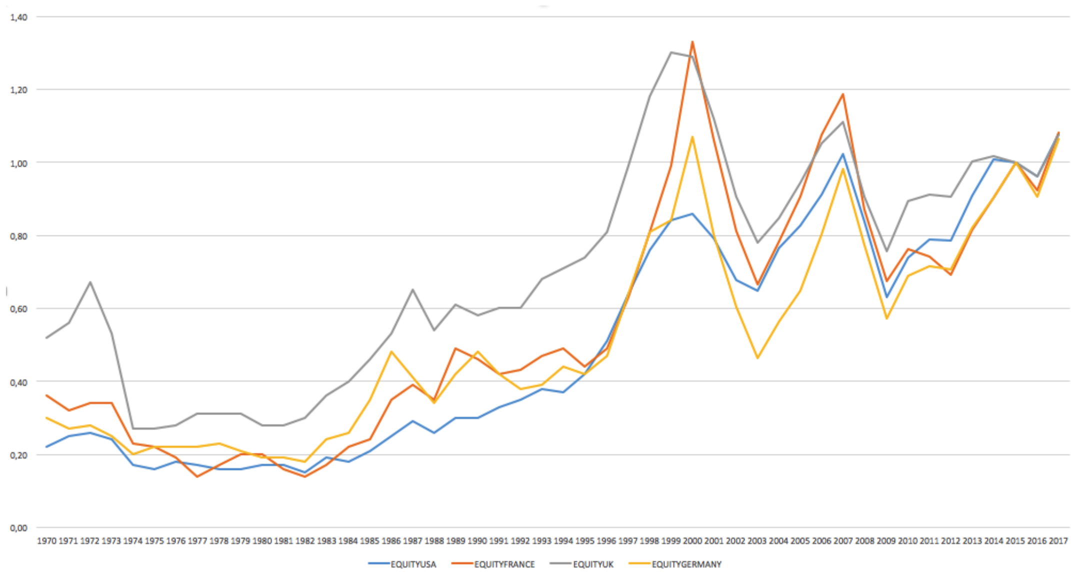

Figure C1: Real equity prices index (1970-2017).

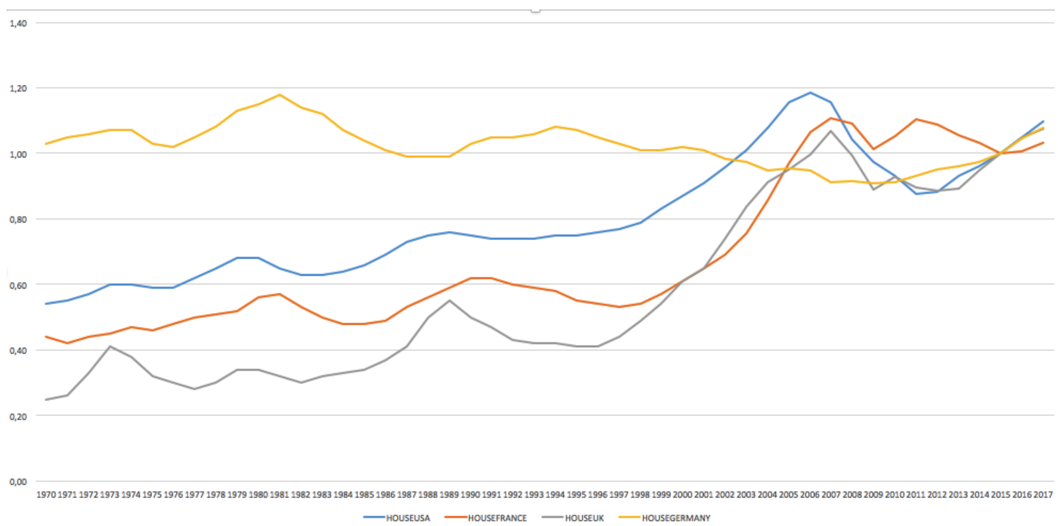

Figure C2: Real housing prices index (1970-2017).

Appendix D

The filtered estimate of the state variables has been obtained via the iterative Kalman filter algorithm. Filtered states are estimated states at period t, updated using all information up to period t. The results relative to the equity asset are reported in Figures D.3, D.4, D.5 and D.6. The results relative to housing price are reported in Figures D.7, D.8, D.9 and D.10.

In the figures below we have the filtered state variable of the fundamentalists (red), the filtered state variable of the extrapolative traders (blue), the observed asset prices (black) and the union of the three-time series. On the x-axis for the filtered states of equity prices, we have the time period from 1973 to 2017, because the first three years of the sample period correspond to the observations required to initialize the Kalman filter and for which the filtered states assume a value equal to zero. For the housing prices, in France, Germany and the USA we have the time period from 1972 to 2017. In the UK we have the time period from 1973 to 2017.

D.1 Equity Asset

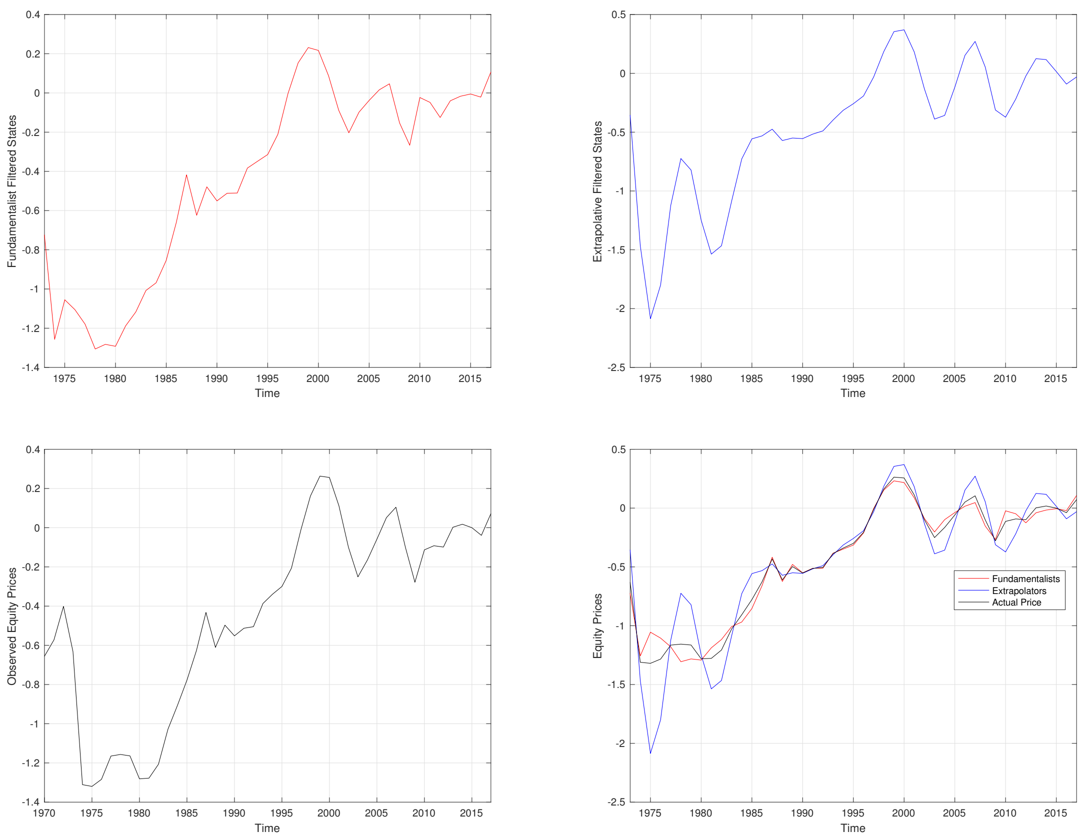

Figure D3: Filtered state variables (UK)

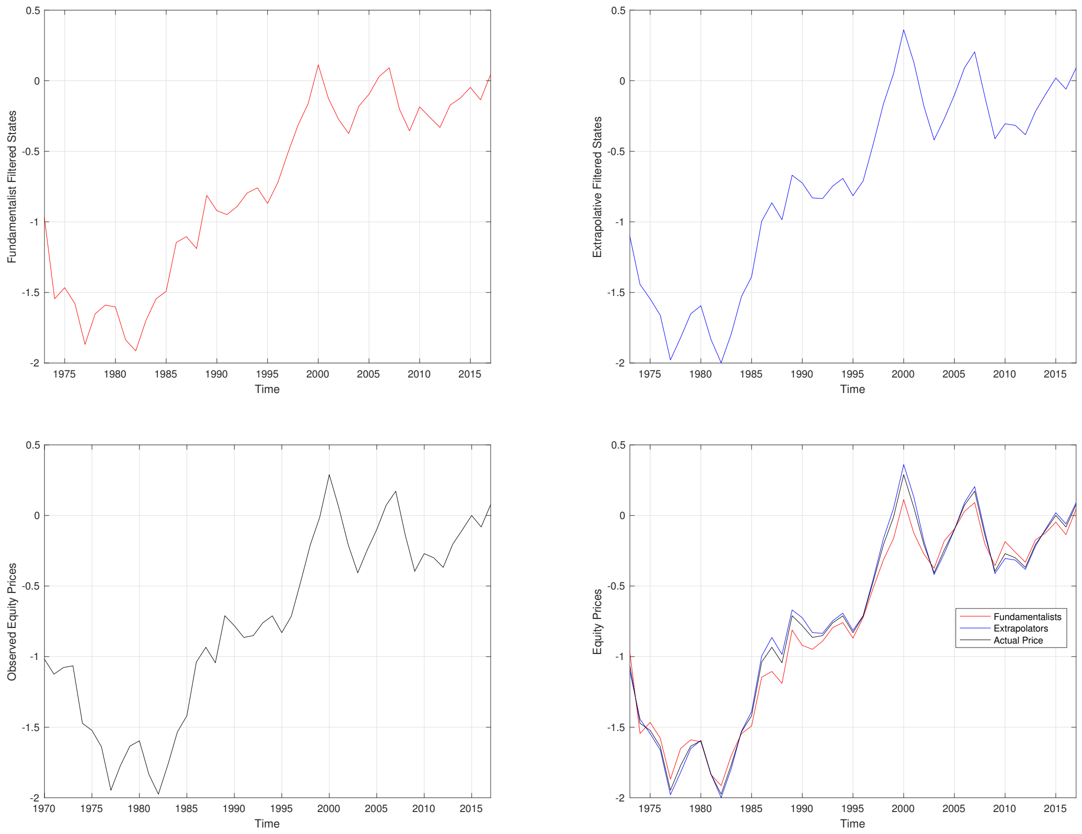

Figure D4: Filtered state variables (France)

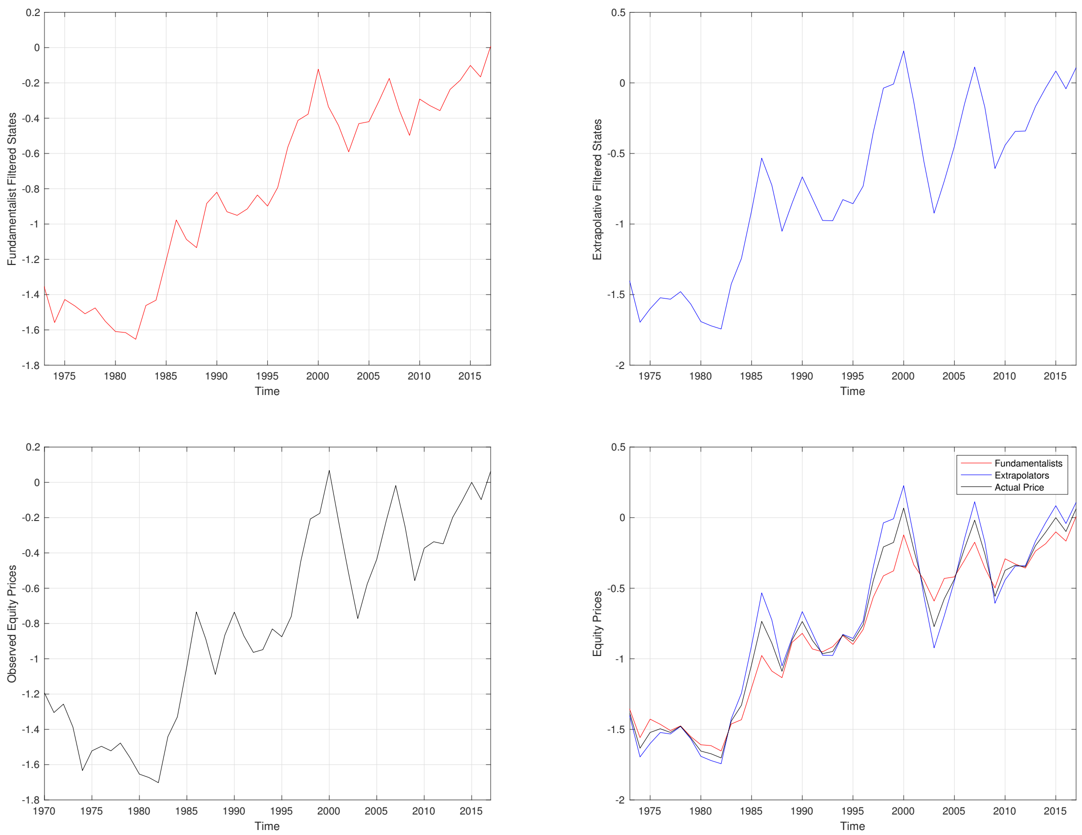

Figure D5: Filtered state variables (Germany)

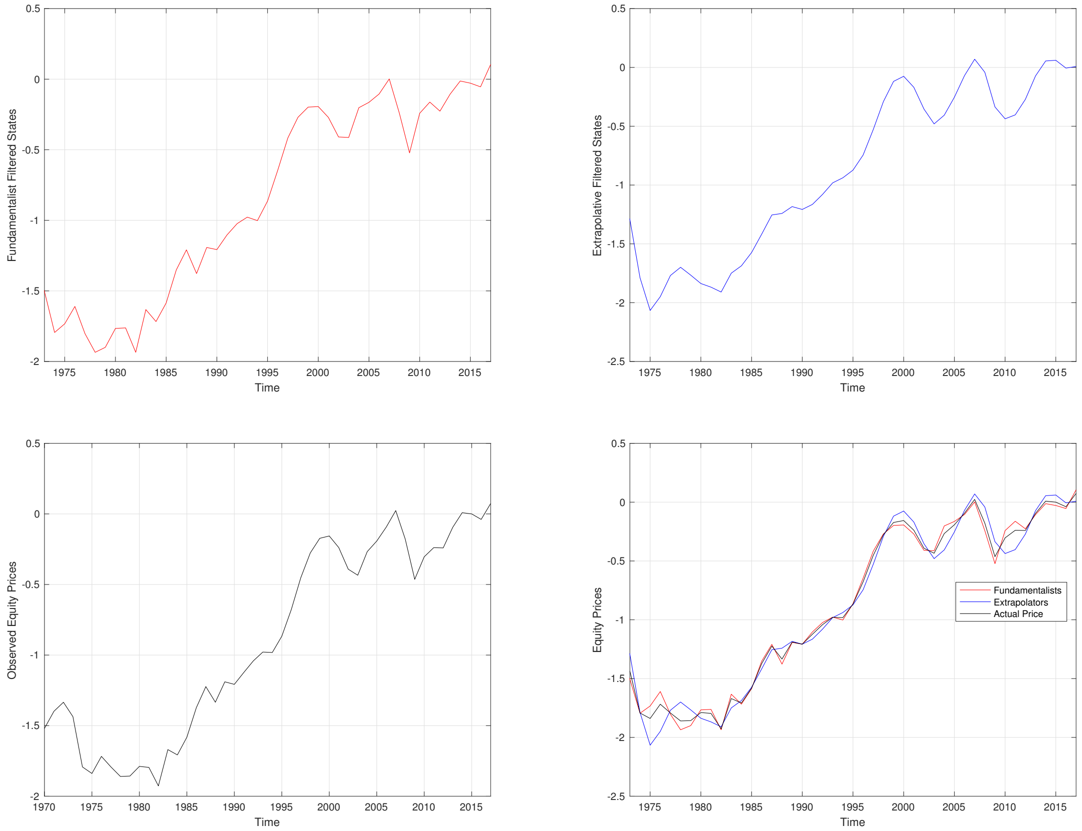

Figure D6: Filtered state variables (US)

D.2 Housing Price

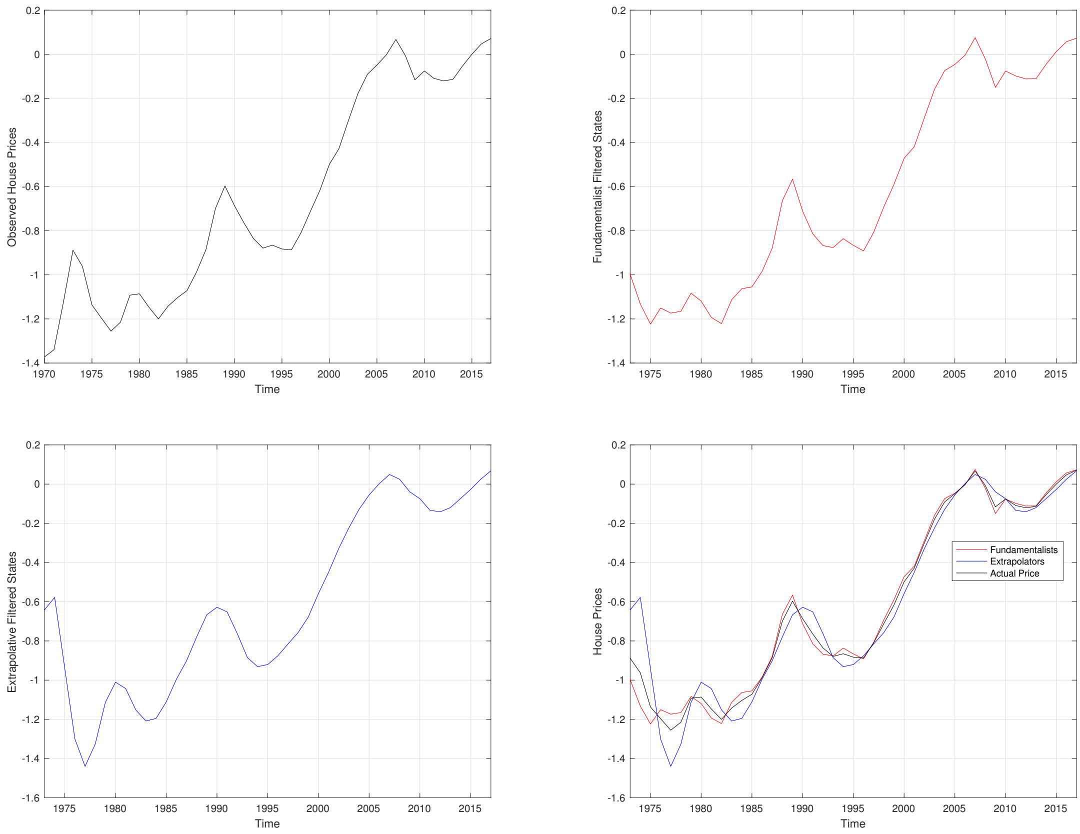

Figure D7: Filtered state variables (UK)

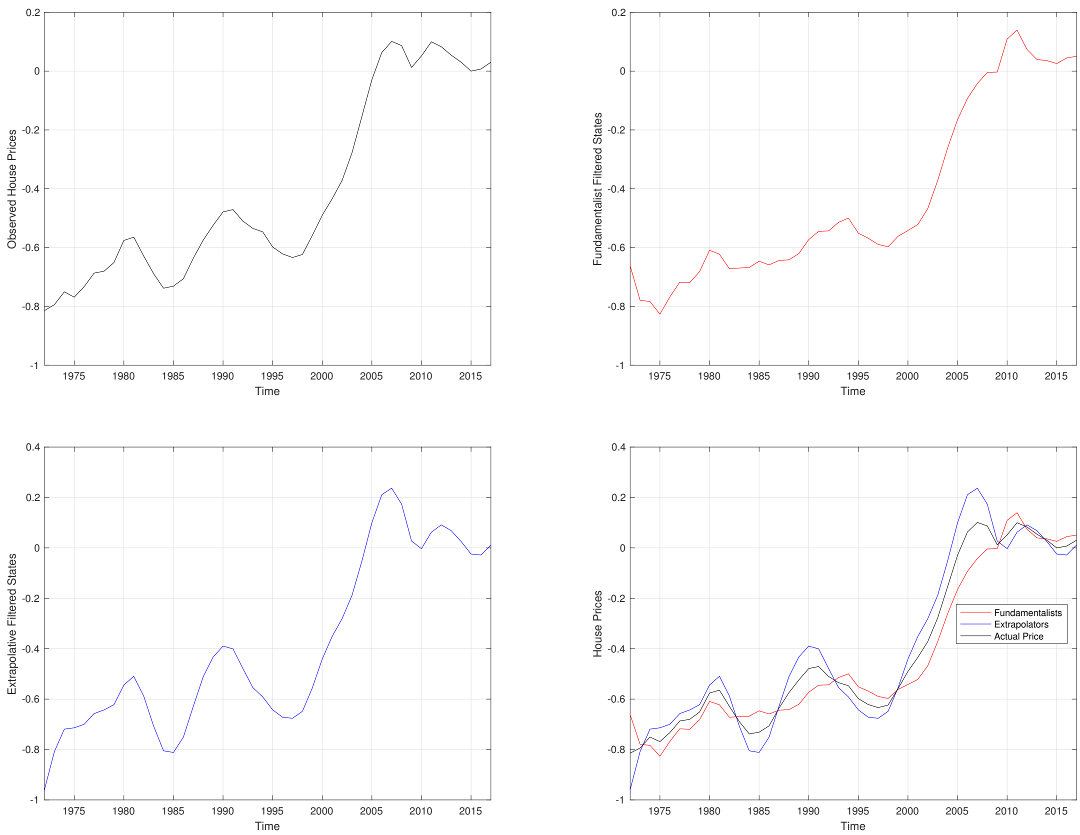

Figure D8: Filtered state variables (France)

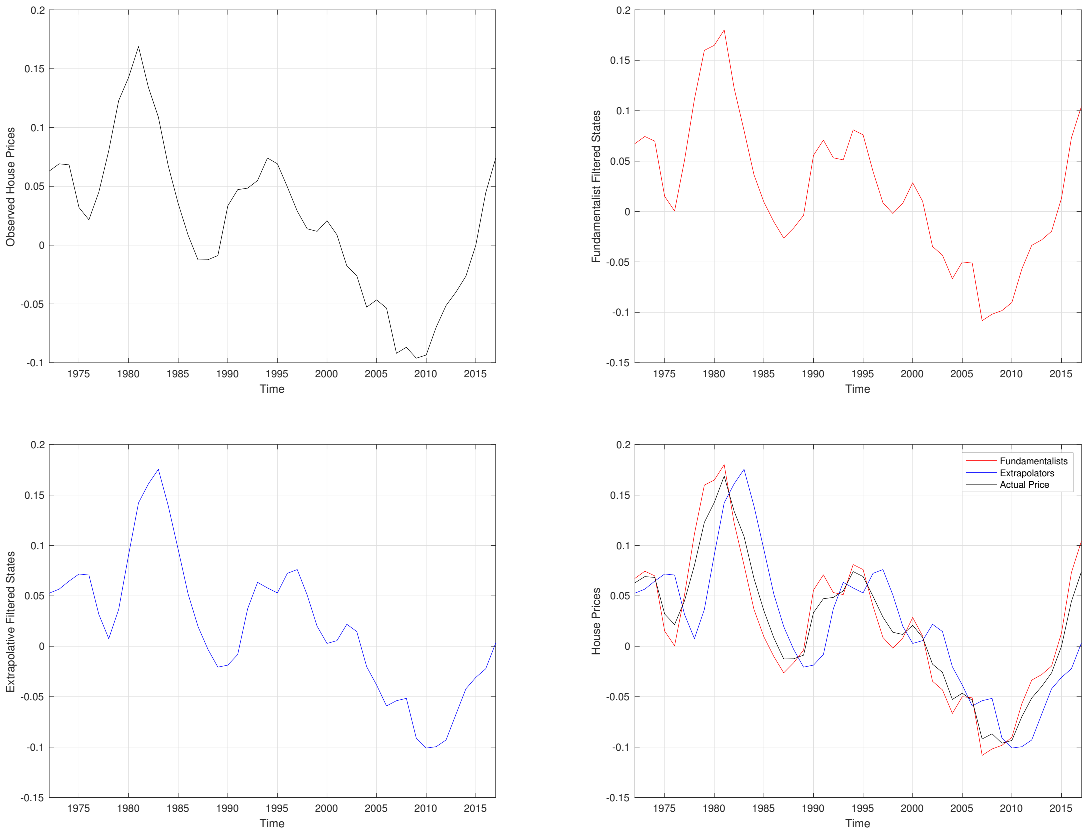

Figure D9: Filtered state variables (Germany)

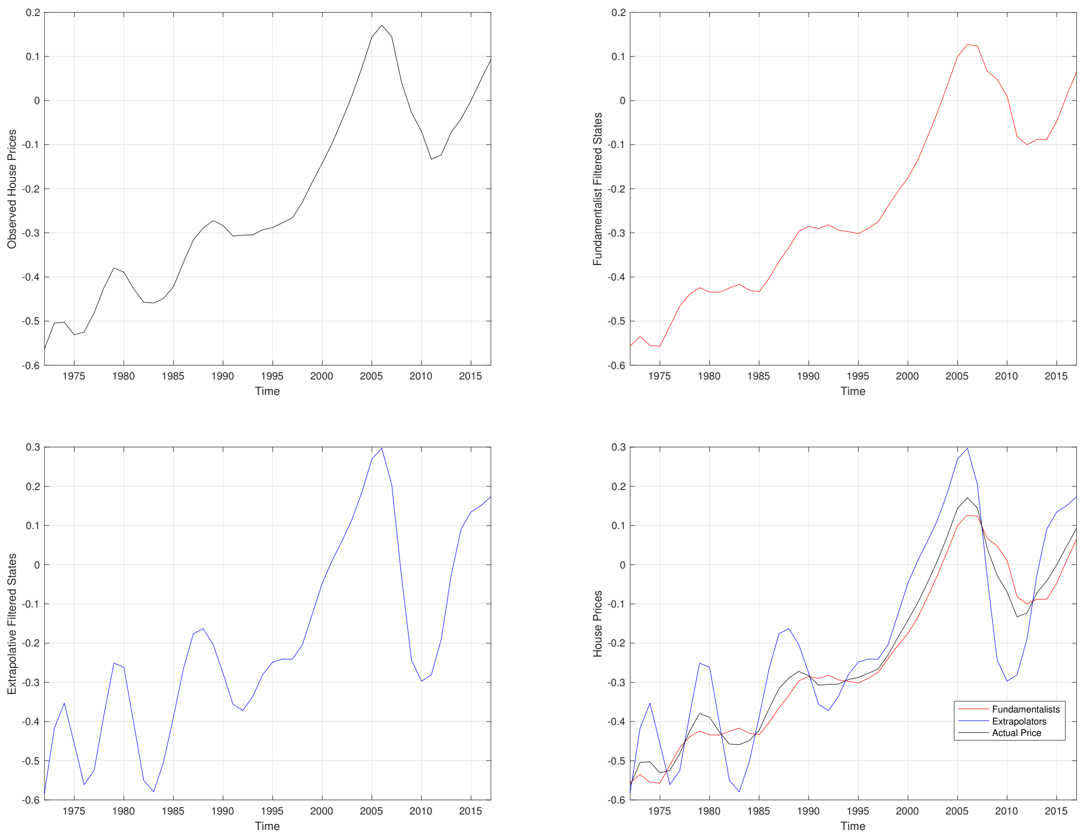

Figure D10: Filtered state variables (US)