WORKING PAPER 2108

Aggregate Demand Externalities, Income Distribution, and Wealth Inequality

Luke Petach and Daniele Tavani

June 2021, revised November 2021

POST-KEYNESIAN ECONOMICS SOCIETY

Aggregate Demand Externalities, Income Distribution, and Wealth Inequality

Luke Petach,[*] Daniele Tavani[†]

November 14, 2021

Abstract

We study a two-class model of growth and the distribution of income and wealth at the intersection of contemporary work in classical political economy and post-Keynesian economics. The key insight is that aggregate demand is an externality for individual firms: this generates a strategic complementarity in production and results in equilibrium underutilization of the economy's productive capacity, as well as hysteresis in real output. Underutilization also affects the functional distribution of income and the distribution of wealth: both the wage share and the workers' wealth share would be higher at full capacity. Consequently, fiscal allocation policy that achieves full utilization also attains a higher labor share and a more equitable distribution of wealth; while demand shocks have permanent level effects. Extensions look at hysteresis in the employment rate and growth. These findings are useful as an organizing framework for thinking through the lackluster economic record of the so-called Neoliberal era, the sluggish recovery of most advanced economies following the Great Recession, and the importance of fiscal policy in countering large shocks such as the Covid-19 pandemic.

Keywords: Externalities, Capacity Utilization, Factor Shares, Wealth Inequality.
JEL Codes: D25, D31, D33, D62, E12.

[*]Jack Massey College of Business, Belmont University. Email: Luke.Petach@Belmont.edu

[†]Corresponding Author. Department of Economics, Colorado State University. 1771 Campus Delivery, Fort Collins, CO 80523-1771, United States. Email: Daniele.Tavani@Colostate.edu.

1 Introduction

From 1980 onward the so-called "Neoliberal era" in the United States has been characterized by a slow but persistent atrophy of federal government economic activity. Figure (1) depicts the contours of what Stiglitz, Tucker, and Zucman (2020) call the "Starving State:" declining federal investment, federal consumption, and federal R&D expenditures as a share of gross domestic product, declining top marginal income tax rates, and declining effective corporate taxation. Predicated on the belief that *laissez faire* policies would deliver first-best results with respect economic growth and individual welfare—a belief abetted by macroeconomic theories depicting unemployment and under-utilization as the result of rational agents responding optimally to exogenous shocks, thereby *ex-ante* ruling out active macroeconomic management (e.g., Lucas, 1977; Kydland and Prescott, 1982)—politicians and policymakers spent decades cutting taxes, deregulating, and divesting.[1]

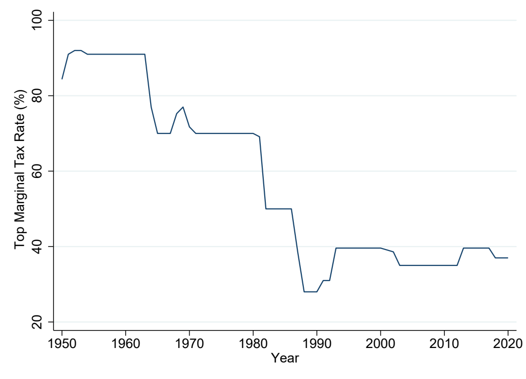
Despite the prestigious intellectual heritage of Neoliberalism's *laissez faire* policy orientation, macroeconomic performance in the Neoliberal era has been lackluster at best.[2] Figure (2) depicts several macroeconomic trends characteristic of the post-1980 period. Rising wealth inequality, a decline in labor's share in national income, decreased labor productivity growth, and a persistent downward trend in capacity utilization all obtain over the period.[3]

In response to the emergent macroeconomic trends depicted above, economists adopting classical-Marxian or post-Keynesian approaches to growth and distribution have largely studied: (a) the relationship between the functional distribution of income and capacity utilization (Nikiforos and Foley, 2012; Rada and Kiefer, 2015; Petach, 2020), (b) the relationship between long-run growth—both endogenous and exogenous—and the distribution of wealth (Zamparelli, 2017; Petach and Tavani, 2020), and (c) factors related to demand-driven growth and under-utilization (or under-employment) in the long-run (Allain, 2015; Setterfield, 2019; Petach and Tavani, 2019; Tavani and Petach, 2021; Fazzari, Ferri, and Variato, 2020).

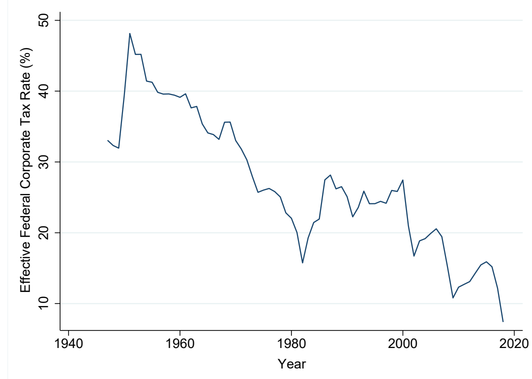
[1]Lucas (1977) articulates this implication of real business cycle theory quite clearly: "[B]y seeking an equilibrium account of business cycles, one accepts *in advance* rather severe limitations on the scope of governmental countercyclical policy which might be rationalized by the theory" (p. 25, italics added).

[2]Other familiar advocates of *laissez faire* include Nobel prize winners Milton Friedman, James Buchanan, and Friedrich Hayek. Several recent works attempt to give a history of the means by which the ideas of these thinkers grew to prominence among academics and policymakers. See MacLean (2017), Slobodian (2018), and Appelbaum (2019).

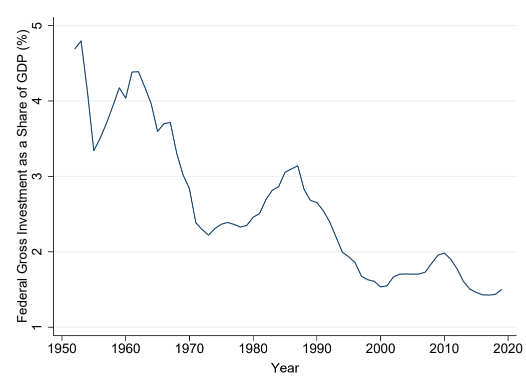
[3]Many authors have debated the possibility of a declining long-term trend in capacity utilization in the United States (Nikiforos, 2016, 2018, 2019; Girardi and Pariboni, 2019; Gahn and Gonzalez, 2019). Part of the so-called "utilization controversy" concerns the validity of the official Federal Reserve Board (FRB) measure of capacity utilization, due to the possibility of measurement error in the value reported by the FRB. Using two alternative measures of capacity utilization derived from the Census Bureau's Quarterly Survey of Plant Capacity Utilization—the "Full Utilization Rate" and the "National Emergency Utilization Rate", used in this paper—Gahn (2020) provides evidence that there is indeed a long-term downward trend in capacity utilization.



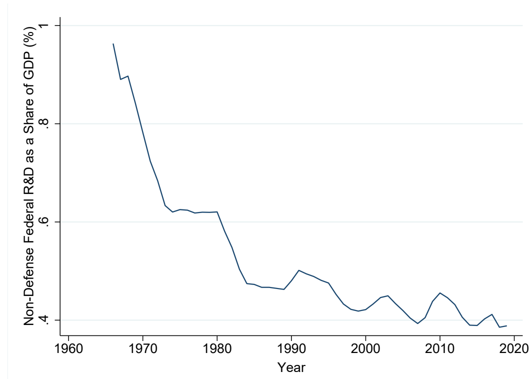
(a) Top Marginal Income Tax Rate



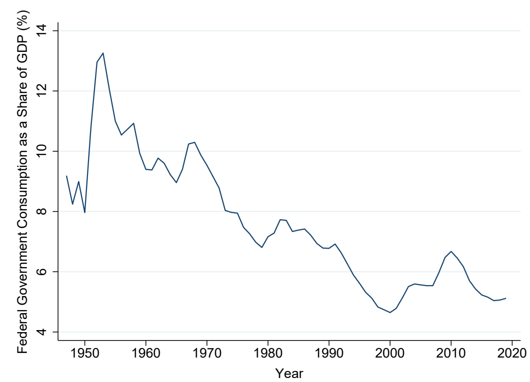
(b) Effective Corporate Tax Rate



(c) Federal Investment Spending



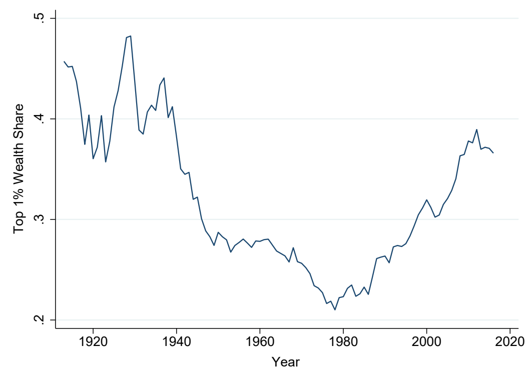
(d) Federal Non-Defense R&D



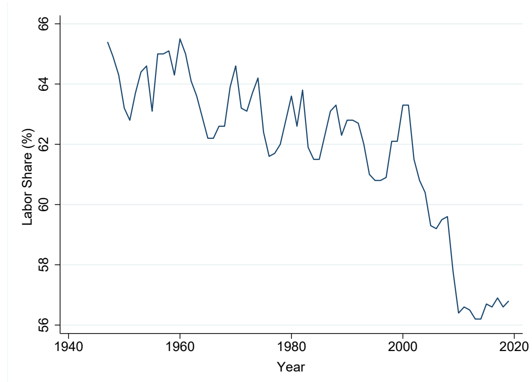
(e) Federal Consumption Expenditures

Figure 1: The Starving State

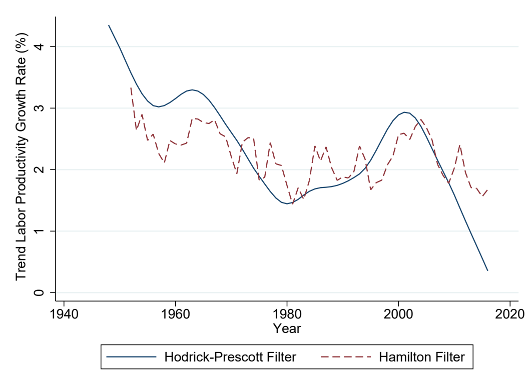
Notes: Marginal income tax rate data from the Tax Policy Center. Effective corporate tax rate measures tax receipts on corporate income as a percentage of the sum of tax receipts on corporate income and corporate profits after tax, obtained from the Federal Reserve Bank of St. Louis. Federal investment spending measures gross federal investment (FRED series A787RC1Q027SBEA) as a share of gross domestic product, obtained from the Federal Reserve Bank of St. Louis. Federal non-defense R&D measures federal investment in non-defense R&D (FRED series Y069RC1Q027SBEA) as a share of gross domestic product, obtained from the Federal Reserve Bank of St. Louis. Finally, federal consumption expenditures measures federal consumption (FRED series A957RC1Q027SBEA) as a share of GDP, also from the Federal Reserve Bank of St. Louis.



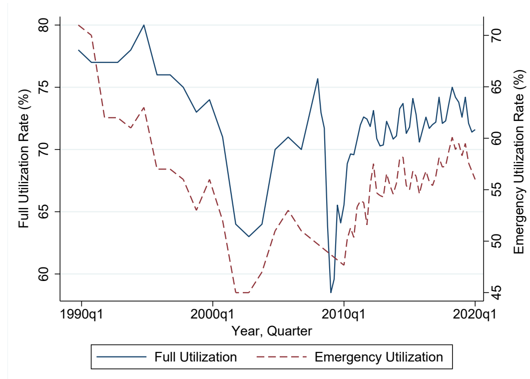
(a) Top 1% Wealth Share



(b) Wage Share



(c) Labor Productivity Growth



(d) Capacity Utilization

Figure 2: Macroeconomic Trends in the Neoliberal Era

Notes: Data on the the top 1% wealth share, wage share, labor productivity, and capacity utilization are from the World Inequality Database, the Bureau of Labor Statistics (wage share and labor productivity), and the Census Quarterly Survey of Plant Capacity Utilization, respectively. Labor productivity data plotted as the trend-component of the labor productivity series, using the Hodrick-Prescott Filter and the Hamilton Filter.

An interesting aspect of the non-mainstream literature on secular stagnation is the dichotomy, both methodological and regarding policy implications, between classical-Marxian approaches on the one hand, and post-Keynesian and Kaleckian approaches on the other. The former emphasize distributive conflict, are amenable to microeconomic foundations (see Marglin, 1984; Foley et al., 2019, for example), but ultimately assign no role to effective demand and activist policy in the long run; the latter embrace the role of effective demand and active economic management, but are usually built on ad-hoc assumptions about the behavioral grounds for individual action.[4]

[4] This point should not be over-emphasized, however. Even in micro-founded models, the choices of the key tradeoffs at stake are ultimately arbitrary. Our goal here is not to advocate for microeconomic foundations *tout court*, but to provide a *specific* microeconomic argument for the possibility of persistent under-utilization and its distributive implications.

On a more concrete level, few classical or Keynesian authors have explicitly studied either the simultaneous implications of activist fiscal policy for welfare and inequality or the relationship between the long-run underutilization of productive capacity and rising wealth inequality. Exceptions that are relevant for our analysis can be found in Ederer and Rehm (2020a,b), and Taylor et al. (2019), that address the evolution of wealth inequality in neo-Kaleckian models.[5] Ederer and Rehm (2020a,b) show that in the short-run greater wealth inequality (via an increase in the capitalist share of wealth) lowers the rate of capacity utilization because of capitalists' lower marginal propensity to consume. In the long run, the distribution of wealth is endogenous, but—with exogenous income distribution—a rise in the profit share will simultaneously raise the steady-state capitalist wealth share and lower the steady-state rate of capacity utilization, such that one should expect to observe a negative reduced-form correlation between capacity utilization and wealth inequality in the data. Taylor et al. (2019) introduce the dynamics of the distribution of wealth in a model of profit-led demand and profit-squeeze distribution with a reduced-form technical progress function that features both dynamic increasing returns and induced bias in technology. In their simulation framework, an adverse demand shock permanently increases wealth concentration, lowers the wage share, and reduces the long-run rate of utilization in the economy.

In this paper, we present an analytical model that bridges classical-Marxian and post-Keynesian insights and can be used to address underutilization and its implications for income distribution and wealth inequality in the Neoliberal era. In particular, we develop a stylized micro-to-macro model of utilization and accumulation, situated within modern work in the classical political economy tradition (Harris, 1978; Marglin, 1984; Michl, 2009; Foley et al., 2019), that also incorporates a role for externalities and coordination (Cooper and John, 1988), and therefore the concrete possibility of equilibrium underutilization of an economy's productive capacity. While our model builds on earlier work (Petach and Tavani, 2019; Tavani and Petach, 2021), it departs from it in two important ways. First, and this is the main contribution of this paper, we revisit the Pasinetti (1962) issue of workers' saving and the distribution of wealth in the context of our microeconomic foundations for the firm's choice of utilization. By going beyond the usual assumption of "hand-to-mouth" workers in classical models, this focus adds another dimension to evaluate inequality in the framework, and it highlights the effect of equilibrium underutilization and counteracting policies on the distribution of wealth. This is especially important given the renewed interest by the economics profession in questions pertaining to wealth inequality following years of path-breaking data collection by Thomas

[5] Michl (2009) studies the role of government debt and the distribution of wealth in a classical economy operating at full capacity. Our contribution can be seen as providing a bridge between his and the post-Keynesian approach.

Piketty, Emmanuel Saez and Gabriel Zucman (Piketty and Saez, 2003; Saez and Zucman, 2016; Piketty et al., 2018); and it is novel in the heterodox literature because of our focus on microeconomic foundations. Second, we focus explicitly on demand shocks —or alternatively shifts in Keynesian "animal spirits"— in generating path dependence or hysteresis in the economy. This second aspect is useful in thinking not only about secular trends of the kind already discussed above, but the more recent path-dependence displayed by economies like the United States and the European Union in the aftermath of the Great Recession of 2008-09 (Fatas, 2019), as well as its distributive implications.

In a nutshell, the argument is as follows. The fact that individual firms consider aggregate demand (i.e. aggregate capacity utilization) as a purely external effect, without taking into account the feedback from their choice on the economy-wide rate of utilization, implies that productive capacity will be under-utilized relative to the "full" utilization level that could be attained if firms coordinated among each others when choosing utilization, thereby internalizing the aggregate demand externality.[6] Excess capacity in product markets reduces labor market tightness via a decline in the employment rate, which depresses the wage share through the usual classical Phillips curve relation. As workers' income falls, their ability to accumulate wealth is also lessened: this, in turn, increases the concentration of wealth in the hands of the capitalist class. Similar to Ederer and Rehm (2020a,b), our model predicts an inverse reduced form correlation between capacity utilization and wealth inequality in the long run.[7] However, and unlike previous work, our model suggests that less-than-full utilization in equilibrium implies that both the labor share of income and the workers' wealth share are below what they would be if the economy operated at full capacity. Accordingly, in addition to stimulating employment and production, fiscal policy has an important secondary role: that of correcting equilibrium inequality by effectively redistributing toward workers. A key feature of our model is thus the elimination of the "efficiency-equity trade-off" that usually affects classical-Marxian growth models, where the main engine of accumulation and growth is the capitalist profit motive. Yet, this result highlights some important political economy implications: first, in the baseline exogenous growth model we show that balanced-growth firms' profits at full utilization and in equilibrium are the same in levels, but both the share of profits in total income and the capitalist share in total wealth are lower at full utilization; second, in an extension that features hysteresis through a reduced-form effect of aggregate demand on the growth rate, we show that firms' profits may even be higher at full utilization than in equilibrium, despite again

[6] In previous work (Petach and Tavani, 2019; Tavani and Petach, 2021), we emphasized how this way of thinking about the firm-level choice of utilization provides a rationale for an endogenous utilization rate in neo-Kaleckian economics. See also Franke (2020) and the literature on the "utilization controversy" already mentioned. Additionally, our argument implies a clear definition of "full" utilization as the coordinated solution.

[7] In Section (4) we show that this reduced-form correlation does indeed appear in the data.

lower profit share and lower capitalist share of wealth. These results may help shed some light on the aversion to activist fiscal policy to achieve full-employment by upper-income classes and businesses, which has been a constant in the US political arena at least since the reaction to the New Deal in the 1940s (Kalecki, 1943; Carter, 2020, Ch. 13). In particular, this aspect of the model reflects Kalecki (1943)'s insights on the political aspects of full employment, in that—despite no loss in, or even higher profits at the full utilization level of output—capitalists may resist policies designed to achieve full employment due to concerns about the effect on their strategic bargaining position vis-a-vis workers, as proxied by changes in distributional variables.

Moreover, we explicitly introduce the possibility of demand shocks, or shifts in animal spirits, entering the firm-level choice of capacity utilization. Contrary to the received wisdom according to which such shocks should display only temporary effects, we show that in fact there are permanent level effects not only on equilibrium real GDP, but also on the functional distribution of income and on the distribution of wealth. These effects are ultimately similar to the numerical findings in Taylor et al. (2019), but here are a result of explicit microeconomic foundations and can be derived in closed form. More importantly, this result points to the relevance of stabilization policies, in addition to allocation policy, to correct for the inefficiencies already described. It also helps to shed light on the sluggish recovery from the Great Recession of 2008-09, given the documented downward revisions of potential output in many high income economies (Fatas, 2019). Furthermore, it points the attention to the likelihood of long-lasting effects of large shocks —such as the Covid-19 recession— to the world economy absent appropriate corrective policy measures: if the world economies are prone to hysteresis, the chances of V-shaped recoveries without appropriate demand stimulus may be slim. Finally, by modeling aggregate demand as an externality, our paper provides a potential bridge between the non-mainstream literature and recent mainstream work on hysteresis and the persistence of aggregate demand shocks (Engler and Tervala, 2018; Farmer and Platonov, 2019; Cerra, Fatas, and Saxena, 2021).

2 The Model

The economic environment is as follows. We assume a one-sector economy with a large number of identical, competitive firms whose "entrepreneurs" make decisions about factor demands and the rate of utilization of installed capacity, and distribute income to workers and the owners of capital stock. Similarly to Foley et al. (2019), we assume that competition among entrepreneurs implies that they earn no pure rent for their services: the entrepreneurial compensation is subsumed into the firms' wage bill. There are then two types of households:

"workers" who earn wage income and interest income on capital stock, consume and save, and "capitalists" who only earn profit income that is distributed to them by entrepreneurs, consume and save.

2.1 Firms

Firms in the economy produce homogeneous output according to the Leontief technology $Y = \min\{uK, AL\}$, where L stands for labor demand, A is labor productivity, K denotes capital stock, and u is the rate of utilization of installed capacity. The output-capital ratio at full capacity is normalized to one for notational simplicity.

Operating capital equipment entails a user cost, which would not be incurred if a firm's machines remained unutilized. The user cost function generalizes the one proposed in Petach and Tavani (2019), which in turn built on Greenwood *et al.* (1988), and depends on own utilization and the firm's beliefs about the aggregate utilization rate (i.e. aggregate demand) in the economy, denoted by $\tilde{u}$, as follows:

$$\delta(u; \tilde{u}, \theta) = \frac{\beta}{\theta} u^{\frac{1}{\beta}} \tilde{u}^{-\frac{\gamma}{\beta}} \tag{1}$$

where γ, β are assumed to be positive and bounded above by 1. The restriction $\beta \in (0, 1)$ guarantees that the user cost function is convex in own utilization, which in turn ensures an interior solution for the firm's profit maximization problem. The parameter $\theta > 0$ is related the role of exogenous demand shocks or shocks to "animal spirits" in the economy, as will become clear below. As for the role of the dependence on aggregate utilization, the functional form proposed ensures that the firm has an incentive to utilize more when aggregate utilization (demand) increases: in fact, the marginal user cost declines in $\tilde{u}$, given that the second cross-partial derivative is negative ($\delta_{u\tilde{u}} < 0$). Everything else equal, higher aggregate utilization lowers the firm's user cost of utilizing more its plants. With regards to the parameter γ, we make use of the following restriction in what follows.

Assumption 1. *(Weak Strategic Complementarity) Throughout this paper, we assume* $\gamma \in (0, 1 - \beta)$.

This assumption ensures single-crossing, i.e. that the choice of utilization by the firm intersects the 45-degree line $u = \tilde{u}$ —which is an equilibrium requirement in the model, see Section 3— only once for strictly positive utilization rates. The economic interpretation of the requirement is that the extent of aggregate demand externalities, given by the ratio γ/β in equation (1), does not dominate the firm's internal cost structure $1/\beta$: the requirement can be rearranged as $(1 - \gamma)/\beta > 1$, which guarantees strict convexity of the user cost function when evaluated in equilibrium.[8]

Both the shock parameter θ and aggregate utilization $\tilde{u}$ (demand) are taken as a given by individual firms when maximizing profits. Also, they affect the user cost in similar ways, as an increase in either lowers the user cost everything else equal. The difference between the two will show up in equilibrium: while the shock parameter remains as such, the aggregate utilization rate is an endogenous variable in the model.

The firm's profit maximization problem requires, at each time period, to choose a rate of utilization of installed capacity to maximize the revenues minus wage costs minus the user cost:

$$\Pi = Y - wL - \delta(u; \tilde{u}, \theta)K \tag{2}$$

subject to the technological constraint $Y = \min\{uK, AL\}$ given $\tilde{u}$ and the real wage w. Formally, the solution amounts to use the Leontief requirement that firms will set effective capital uK equal to effective labor AL —so that labor demand will be equal to uK/A— to restate the problem as choosing utilization at the margin in order to balance the marginal revenue with the marginal cost of increasing the usage of machinery.[9] In practice, this way of thinking about the firms' problem amounts to impose the following sequence of events. First, the firm's entrepreneurs choose labor demand so as to equalize effective labor with effective capital; then, they choose the utilization rate given labor costs and their beliefs about aggregate utilization. If the corresponding profit is non-negative, a firm will undertake production; otherwise, it will remain idle.[10]

The firm's choice of utilization is in fact a best-response function to the aggregate utilization rate (aggregate demand) in the economy. In particular, the firm will utilize more—everything else equal—if aggregate utilization increases:

$$u(\omega; \tilde{u}, \theta) = [\theta(1-\omega)]^{\frac{\beta}{1-\beta}} \tilde{u}^{\frac{\gamma}{1-\beta}} \tag{3}$$

with $\partial u / \partial \tilde{u} > 0$. Petach and Tavani (2019) estimated a series of versions of equation (3) using sector-by-state US data: the estimates, which address endogeneity in various ways, provide

[8] If the assumption is violated locally, the firm-level choice of utilization may end up being S-shaped, leading to the possibility of multiple equilibria (Cooper and John, 1988). Palley (1990) explores this possibility in a Keynesian model with strategic complementarities. We leave the analysis of multiple equilibria to future research.

[9] An alternative would be to assume that firms *revenues* were increasing but concave on both own and aggregate utilization, while the user cost was linear in own utilization: Franke (2020) presents a model along these lines. For instance, suppose that the firm's profits in equation (2) were as follows: $\Pi = \beta^{-1} u^{\beta} \tilde{u}^{\gamma}(1-\omega)K - \theta uK$, with the same restrictions of Assumption 1. The choice of utilization would lead to $u = [\theta(1-\omega)]^{1/(1-\beta)} \tilde{u}^{\gamma/(1-\beta)}$ with very similar comparative statics to equation (3).

[10] We assume that the shutdown costs are zero. This assumption is made here to simplify the analysis, but is of no consequence to what follows provided that shutdown costs are a lump-sum and do not affect the choice of utilization at the margin.

strong and robust support for our model. Notice also that the choice of utilization in this model makes the firm-level demand for labor elastic to the unit labor cost ω, even though the underlying technology is Leontief. In fact, the firm-level labor demand is found by inserting the choice of utilization into the profit-maximizing proportions of capital and labor:

$$L = \frac{u(\omega; \tilde{u}, \theta)K}{A} \tag{4}$$

and is inversely related to real unit labor costs $\omega \equiv w/A$ and therefore to the real wage given that $\partial u/\partial \omega < 0$. The rationale is as follows: the choice of utilization equalizes the marginal revenue of higher utilization with the marginal user cost. If unit labor costs increase, firms' (absolute) revenues fall, and the firm can cut back on utilization in order to reduce the user cost. Despite the fixed-proportion technology, this mechanism produces a feedback from real wages to labor demand that operates similarly to factor substitution along a neoclassical production function.

Finally, the best-response function (3) makes it clear why the parameter θ can be interpreted as synthetically capturing the role of exogenous shocks or Keynesian "animal spirits." Independent on any other endogenous variable, firm-level utilization increases in θ.

Using the choice of utilization (3), we can then calculate the firm's profit function (i.e. the value of maximized profits) as:

$$\begin{aligned} \Pi &= (1-\beta)\theta^{\frac{\beta}{1-\beta}}(1-\omega)^{\frac{1}{1-\beta}}\tilde{u}^{\frac{\gamma}{1-\beta}}K \\ &= (1-\beta)(1-\omega)u(\tilde{u}, \omega; \theta)K \end{aligned} \tag{5}$$

where $r \equiv (1-\beta)(1-\omega)u(\tilde{u}, \omega; \theta)$ denotes the profit rate *net of the user cost* that will be distributed to households by the firms' entrepreneurs.

2.2 Households: Capitalists and Workers

We revisit the Pasinetti (1962) problem in the present context and distinguish between capitalist households (denoted by the superscript c) and worker households (denoted by the superscript w) in order to determine the accumulation of capital stock (wealth) in the economy. Neither type of household makes decisions about utilization: they take the utilization rate chosen by the entrepreneurs as a given. Capitalists earn net profit income $(1-\beta)u(1-\omega)K^c$, have log utility from consumption c^c, and discount the future at a rate $\rho^c > 0$, constant. Worker households earn wage income when active (that is when part of labor demand) $wL = \omega u(K^c + K^w)$ and net profit income on the wealth they own $(1-\beta)u(1-\omega)K^w$, have log utility from consumption c^w, and discount the future at a rate $\rho^w > \rho^c$. This assumption: (a) is *de facto* equivalent to assuming workers to have a lower propensity to save than capitalists,[11] and (b) synthetically captures the role of class and wealth holdings in determining the households' willingness to defer consumption.[12]

A major focus of this contribution is the distribution of wealth. Let the share of wealth accruing to capitalists be $K^c/(K^c + K^w) \equiv \phi \in [0, 1]$. Consider workers' income y^w. Starting from

$$\begin{aligned} y^w &= \tfrac{w}{A} u(K^c + K^w) + rK^w \\ &= \omega u \left(\frac{K^c + K^w}{K^w} \right) K^w + rK^w \\ &= \omega \frac{u}{1-\phi} K^w + rK^w \end{aligned}$$

we can write, using the very definition of the net profit rate provided in (5),

$$y^w = \frac{u(\tilde{u}, \omega; \theta)}{1-\phi} \left[\omega + (1-\beta)(1-\phi)(1-\omega) \right] K^w \tag{6}$$

As shown in Appendix A, intertemporal optimization under perfect foresight for both types of households gives the two classes' consumption Euler equations:

$$\frac{\dot{c}^c}{c^c} = (1-\beta)(1-\omega)u(\tilde{u}, \omega; \theta) - \rho^c \tag{7}$$

$$\frac{\dot{c}^w}{c^w} = \frac{u(\tilde{u}, \omega; \theta)}{1-\phi} \left[\omega + (1-\beta)(1-\phi)(1-\omega) \right] - \rho^w \tag{8}$$

where of course the rate of utilization satisfies equation (3).

3 Balanced Growth Equilibrium and Dynamics

An equilibrium growth path is defined by: (a) sequences of consumption and capital stock such that utility is maximized given the resource constraints for both classes; (b) demands for capital and labor such that profits are maximized; (c) a rate of utilization such that profits are maximized and (d) firms' beliefs are realized, so that $u(t) = \tilde{u}(t)\ \forall t$. The latter requirement ensures that firms' beliefs are mutually consistent, and is similar to the notion of Nash equilibrium. Moreover, balanced growth requires that, (d) for both classes, consumption and capital

[11] For empirical evidence on differential savings rates and their implications for growth, see Petach and Tavani (2021).

[12] The continuous time nature of our model and consequent issues with known terminal times for the households' planning horizons makes it difficult to justify differential saving propensities along the lines suggested by Michl (2009), namely that workers save for the life cycle while capitalists save for dynastic purposes. However, the difference in discount rates could be explained by appealing to the "perpetual youth" households in the Yaari (1965) or Blanchard (1985) models while letting capitalist dynasties last forever.

stock grow at the same rate: $\dot{c}^i/c^i = \dot{k}^i/k^i = g^i, i = \{c, w\}$. The equilibrium utilization rate is

$$u(\omega; \bar{\theta}) = \bar{\theta}(1-\omega)^{\frac{\beta}{1-\beta-\gamma}} \tag{9}$$

with the parameter $\bar{\theta} \equiv \theta^{\beta/(1-\beta-\gamma)}$ denoting the *aggregate*—as opposed to the individual firm effect θ—effect of exogenous shocks or "animal spirits" on aggregate demand/utilization, and $\partial u/\partial \omega < 0$. The two classes' equilibrium accumulation rates are then:

$$g^c = (1-\beta)u(\omega; \bar{\theta})(1-\omega) - \rho^c \tag{10}$$

$$g^w = \frac{u(\omega; \bar{\theta})}{1-\phi}\left[\omega + (1-\beta)(1-\phi)(1-\omega)\right] - \rho^w \tag{11}$$

At a balanced growth equilibrium, the economy-wide growth rate of capital stock is a weighted average of the two classes' accumulation rates, the weight being given by their respective shares in total wealth. From $g = \phi g^c + (1-\phi)g^w$, factoring and simplifying, we find:

$$g = u(\omega; \bar{\theta})[\omega + (1-\beta)(1-\omega)] - [\phi\rho^c + (1-\phi)\rho^w] \tag{12}$$

3.1 Dynamics of the Distribution of Wealth

The capitalist share of wealth evolves through a replicator equation (Samuelson and Modigliani, 1966; Zamparelli, 2017; Ederer and Rehm, 2020a,b):

$$\dot{\phi} = \phi[g^c - g]$$

which, using (10) and (12), simplifies to:

$$\dot{\phi} = \phi\left[(1-\phi)(\rho^w - \rho^c) - \omega u(\omega; \bar{\theta})\right] \tag{13}$$

3.2 Cyclical Growth Dynamics

To close the model, we build on previous work (Tavani and Petach, 2021) and embed the results of the utilization choice and the wealth-share dynamics into a simple classical growth cycle model. Following Goodwin (1967), assume that the real wage grows with the employment rate $e \equiv uK/AN$, where N is the total labor force, according to the usual classical wage-Phillips curve: $\dot{w}/w = f(e), f(0) < 0, f_e > 0, f_{ee} \geq 0$, and for now let labor productivity grow exogenously at a rate $\alpha > 0$. The wage share, then, changes over time with the difference between the growth rate of the real wage and the growth rate of labor productivity:

$$\dot{\omega} = [f(e) - \alpha]\omega \tag{14}$$

We will use the original Goodwin specification of a linear wage-Phillips curve in what follows: $f(e) = -\xi + \lambda e, \xi > 0, \lambda > 0$. Finally, log-differentiating the employment rate e, we find the following differential equation:

$$\begin{aligned} \frac{\dot{e}}{e} &= \left[\frac{\dot{u}}{u} + g - (\alpha + n)\right] \qquad (15) \\ &= \left\{-\frac{\beta}{1-\beta-\gamma}\frac{\omega}{1-\omega}[f(e)-\alpha] + \left[u(\omega;\bar{\theta})[1-\beta(1-\omega)] - (\phi\rho^c + (1-\phi)\rho^w)\right] - (\alpha+n)\right\} \end{aligned}$$

We thus have a three-dimensional dynamical system tracing the evolution of the the distribution of wealth ϕ, the functional distribution of income ω and the employment rate e as described by equations (13), (14), and (15). We first characterize the steady state and draw policy implications. A detailed local stability analysis is provided in Appendix C.

4 Steady State and Policy

A steady state is a triple $(\phi_{ss}, \omega_{ss}, e_{ss})$ such that $\dot{\phi} = \dot{\omega} = \dot{e} = 0$. We start with characterizing the long-run distribution of wealth. Equation (13) has two steady-states: the"dual" steady state $\phi_{ss} = 0$ (Samuelson and Modigliani, 1966); and the Pasinetti (1962) two-class steady state, which is the focus of our analysis and we write in preliminary form as follows:

$$\begin{aligned} 1 - \phi_{ss} &= \frac{u(\omega;\bar{\theta})\omega}{\rho^w - \rho^c} \\ &= \frac{\bar{\theta}(1-\omega)^{\frac{\beta}{1-\beta-\gamma}}\omega}{\rho^w - \rho^c} \end{aligned} \tag{16}$$

Notice first that, given that the numerator of the fraction in the right-hand side is positive, the assumption of a higher rate of time-preference on behalf of workers ($\rho^w > \rho^c$) guarantees a positive value for the two-class wealth distribution in steady state. This is analogous to the Pasinetti requirement that the workers' saving propensity be less than the capitalists' saving propensity. If on the other hand $\rho^w \leq \rho^c$, the only surviving steady state is the "dual" solution where there are no pure capitalists. Also, for ϕ_{ss} to be between zero and one, we need $\rho^w - \rho^c > u(\omega_{ss};\bar{\theta})\omega_{ss}$, which we will be assuming throughout. Furthermore, and as shown in Figure 3 below the nullcline (16) representing the workers' (capitalists') wealth share is hill-shaped (Ushaped) in the wage share, because there are two forces at play here. On the one hand, an increase in the wage share decreases equilibrium utilization; on the other hand, a higher wage share increases the funds available for capital accumulation by workers.

Next, the law of motion for the wage share pins down the steady state employment rate, tied up to labor productivity growth, as:

$$e_{ss} \; = \; f^{-1}(\alpha) \; = \; \frac{\xi + \alpha}{\lambda} \tag{17}$$

As it is standard in the literature on the growth cycle, the steady state employment rate is exogenous, given the exogeneity of labor productivity growth: see Section 6.1 for an extension of the model that relaxes this conclusion.

Finally, the law of motion for the employment rate can be used in order to solve for the long-run value of the wage share. We focus here on studying the two-class steady state: setting $\dot{e} = 0$ at $g = g^c$, we have:

$$1 - \omega_{ss} = \left[\frac{\rho^c + \alpha + n}{\bar{\theta}(1-\beta)} \right]^{\frac{1-\beta-\gamma}{1-\gamma}} \tag{18}$$

Note that: (a) as in the original Goodwin (1967) model, the wage share increases in the capitalist propensity to save: a reduction in the rate of time preference ρ^c increases capitalist accumulation and therefore the long-run share of wages. Moreover, (b) the steady state wage share is directly related to the shock parameter $\bar{\theta}$. Inserting the long-run value for income shares (18) in equation (9), we obtain the steady state utilization rate in equilibrium as a function of parameters only:

$$u_{ss} = \left[\frac{\theta(\rho^c + \alpha + n)}{1-\beta} \right]^{\frac{\beta}{1-\gamma}} \tag{19}$$

We can then plug equation (18) into (16) to obtain the long-run value of the wealth distribution in terms of parameters only. Repeated rearrangements and substitutions lead to:

$$1 - \phi_{ss} = \left[\frac{\rho^c + \alpha + n}{(1-\beta)(\rho^w - \rho^c)} \right] \left\{ \left[\frac{\bar{\theta}(1-\beta)}{\rho^c + \alpha + n} \right]^{\frac{1-\beta-\gamma}{1-\gamma}} - 1 \right\} \tag{20}$$

Inspection of the term in braces reveals that it is equal to the ratio of long-run factor shares $\omega_{ss}/(1 - \omega_{ss})$. In turn, and intuitively, this implies that any positive shock to the wage share will increase the workers' wealth share in the economy. In order to draw policy implications, we must establish whether the economy operates at full capacity. The analysis below shows that: (i) this is not the case, and (ii) it has implications for both wealth and income distribution.

4.1 "Full" Utilization

Consider a scenario in which firms choose utilization by coordinating among each other, that is under the additional constraint that $u = \tilde{u}$. This amounts to internalizing the aggregate demand externality, given that while aggregate utilization was endogenous for a firm acting in isolation, it becomes endogenous in the coordinated problem. Indeed, Foley (2016) refers to such a scenario as the *socially-coordinated solution*. The firm's profits to be maximized become:

$$\Pi = u(1-\omega)K - \frac{\beta}{\theta} u^{\frac{1-\gamma}{\beta}} K \tag{21}$$

and the resulting "full" utilization is

$$u^*(\omega;\bar{\theta}) = \bar{\theta}\left(\frac{1-\omega}{1-\gamma}\right)^{\frac{\beta}{1-\beta-\gamma}} \tag{22}$$

which is always higher than equilibrium utilization for a given wage share under Assumption 1. This finding has implications not only for the long-run functional distribution of income, but also for the long-run distribution of wealth. In fact, consider first the nullcline that relates the worker's share of wealth at full utilization to the wage share:

$$1-\phi^*_{ss} = \left(\frac{1}{1-\gamma}\right)^{\frac{\beta}{1-\beta-\gamma}} \left[\frac{\bar{\theta}(1-\omega)^{\frac{\beta}{1-\beta-\gamma}}\omega}{\rho^w - \rho^c}\right] \tag{23}$$

Next, the steady state wage share at full utilization can be found from:

$$1-\omega^*_{ss} = (1-\gamma)^{\frac{\beta}{1-\gamma}} \left(\frac{\rho^c+\alpha+n}{\bar{\theta}(1-\beta)}\right)^{\frac{1-\beta-\gamma}{1-\gamma}} \tag{24}$$

Using this expression we can solve for the full utilization rate in terms of parameters only:

$$u^*_{ss} = \left(\frac{1}{1-\gamma}\right)^{\frac{\beta}{1-\gamma}} \left[\frac{\theta(\rho^c+\alpha+n)}{1-\beta}\right]^{\frac{\beta}{1-\gamma}} \tag{25}$$

And the comparison with (19) makes it clear that $u^*_{ss} > u_{ss}$. Finally, inserting (24) into (23), we can then calculate the solution for the long-run distribution of wealth. Some algebra leads to an equation that is immediately comparable to (20):

$$1-\phi^*_{ss} = \left(\frac{1}{1-\gamma}\right)^{\frac{\beta}{1-\gamma}} \left[\frac{\rho^c+\alpha+n}{(1-\beta)(\rho^w-\rho^c)}\right] \left\{\left[\frac{\bar{\theta}(1-\beta)}{\rho^c+\alpha+n}\right]^{\frac{1-\beta-\gamma}{1-\gamma}} - 1\right\} \tag{26}$$

We can then state the following result, that has a very simple proof provided in Appendix B.

Proposition 1. *At a steady state of the model, both the wage share and workers' wealth share would be higher at full utilization than along the equilibrium path. Moreover, both the wage share and the workers' share of wealth permanently increase following a positive demand shock.*

A first implication of this result is that—by operating at less than full capacity—the long-run equilibrium position of this economy is also characterized by higher wealth inequality and lower wage share, as displayed in Figure 3. Thus, policies that push the economy toward full capacity will not only have the effect of raising real GDP, but also of increasing the workers' share of both income and wealth in the economy. This conclusion seems puzzling at first

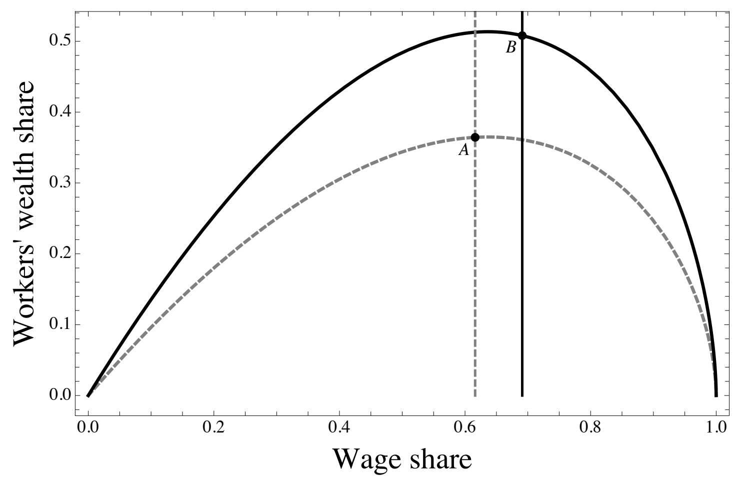


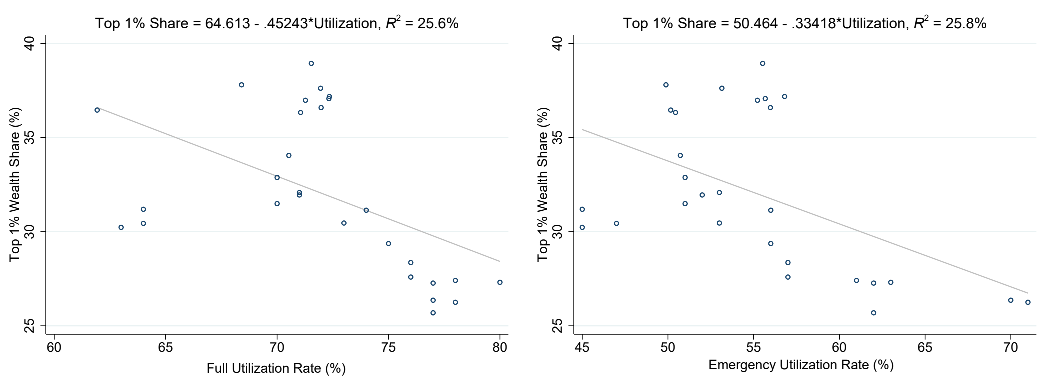
Figure 3: Factor shares and wealth distribution at the long-run equilibrium (point A) and at full utilization (point B). See Appendix D for a description of the parameter calibration.

glance: since both the wage and profit share on the one hand, and the capitalists' and workers' wage share on the other sum up to one, it must necessarily be the case that the higher wage share and workers' wealth share that this economy can attain along the full utilization path would occur at the detriment of the capitalists in the economy. However, one must notice that, at full utilization, the economy's *total profit income*, up to the initial value of capital stock, will be the same as in equilibrium, despite the capitalist shares in income and wealth being lower. This point can also be shown formally. Assume that the decentralized and the full-utilization path start with the same amount of capital stock $K(0)$, which is a given in both problems and can be chosen freely. In balanced growth, capital stock grows at $\alpha+n$ at both the full-utilization and the equilibrium path: moreover, always in balanced growth, the growth rate g is equal to the capitalist accumulation rate g^c, in turn equal to the difference between the net profit rate r and the capitalist discount rate ρ^c. At both the equilibrium and efficient path, the balanced growth condition requires that $g^c = r - \rho^c = \alpha + n$, which implies that (i) $r = \rho^c + \alpha + n$; and (ii) total profit income will be the same in both cases and, given the initial capital stock $K(0)$, equal to:

$$\Pi(t) = (\rho^c + \alpha + n)K(0)\exp\{(\alpha + n)t\} = \Pi^*(t) \tag{27}$$

We see here a first example of some interesting political economy implications of our model. Resistance to full-employment policy is insufficiently explained by appeals to profitability, as total profit income, up to the initial amount of capital stock, does not change between equilibrium and full utilization. We will see in Section 6.2 that, if the long-run growth rate is endogenous to aggregate demand, profits may even be higher at full utilization than in equilibrium.

A secondary implication is that —similarly to Ederer and Rehm (2020a,b)— there exists an inverse reduced-form correlation between capacity utilization and wealth inequality in the long-run, corresponding to the relationship between u and ϕ equation (16).[13] Figure 4 plots the share of wealth held by the top 1% against two alternative measures of capacity utilization[14]. In both cases capacity utilization and wealth inequality are negatively correlated. The slope of the naïve OLS estimates suggests that a one percentage-point increase in the rate of capacity utilization reduces the top 1% wealth share between 0.33 and 0.45 percentage points.



(a) Top 1% Share v. Full Utilization (b) Top 1% Share v. Emergency Utilization

Figure 4: Wealth Inequality and Capacity Utilization, 1989-2016

Notes: Capacity utilization data from the Census Quarterly Survey of Plant Capacity Utilization. Top 1% wealth share data from the World Inequality Database.

A third implication is that positive demand shocks have progressive and permanent level

[13] While in our model equilibrium utilization is endogenous to the wage share and the demand shock parameter, data is available directly on the utilization rate. Taken by itself, equation (16) describes a downward-sloping relation in the (u, ϕ) plane, and that motivates the naïve estimate and the corresponding plot it against the top wealth share in Figure 4.

[14] Full Utilization and Emergency Utilization from the Census Quarterly Survey of Plant Capacity Utilization.

effects not only on real GDP, but also on the distribution of income and wealth. Mechanically, an increase in the parameter $\bar{\theta}$ shifts the best-response function (3) upward, implying higher equilibrium utilization. This in turn lowers the capitalist wealth share, given that the two are inversely related. Moreover, it increases the long-run share of wages in national income. The intuition is the following: everything else equal, an increase in $\bar{\theta}$ fosters capital accumulation and employment in the short run. But the long-run growth rate, equal to the growth rate of labor productivity α, is constant: restoring the balanced growth condition $g = \alpha + n$ requires the profit rate to fall, which can only happen through an increase in the wage share. Given that workers' labor income has increased, their funds available to accumulation also increase, which leads to an increase in the workers' wealth share.

4.2 Allocation Policy

As done in previous work (Petach and Tavani, 2019) let us now introduce a fiscal authority into the model. The main goal of this section is to show that decentralizing the full utilization rate will imply not only a higher labor share but also a more progressive distribution of wealth. The government taxes firms lump-sum at the rate τ, and uses the tax proceedings to provide a user cost rebate σ to firms. The profits to be maximized are now given by $\Pi = uK(1-\omega) - \delta(u;\tilde{u})(1-\sigma)K - \tau$. Assume further that the government runs a balanced budget at all times, so that $\sigma\delta(u;\tilde{u})K = \tau$ always. The firm-level choice of utilization now fulfills

$$u(\omega;\tilde{u},\sigma,\theta) = \theta\left(\frac{1-\omega}{1-\sigma}\right)^{\frac{\beta}{1-\beta}}\tilde{u}^{\frac{\gamma}{1-\beta}} \tag{28}$$

and is increasing in the policy parameter σ. Imposing the equilibrium condition $u = \tilde{u}$ gives aggregate utilization as a function of government spending (and the wage share) as:

$$u(\omega;\sigma,\bar{\theta}) = \bar{\theta}\left(\frac{1-\omega}{1-\sigma}\right)^{\frac{\beta}{1-\beta-\gamma}} \tag{29}$$

which makes it clear that setting $\sigma = \gamma$ achieves the full utilization rate. The previous findings have already shown that an economy operating at full utilization also features a higher wage share and a higher workers' wealth share. Also, as shown in Tavani and Petach (2021), we can recover the fiscal multiplier as the ratio of the equilibrium response to the policy over the individual response. We can summarize these results in the following proposition, proven in Appendix B. Figure 3 provides a visual illustration of the long-run effects of the decentralization policy on income and wealth distribution.

Proposition 2. *A fiscal authority can implement the full utilization rate through a user cost rebate $\sigma = \gamma$. The balanced budget fiscal multiplier is equal to $m = (1-\beta)/(1-\beta-\gamma) > 1$. The policy increases the wage share of income and the workers' share of wealth.*

The result that fiscal policy that increases capacity utilization in the economy —up to the full utilization rate— also has progressive effects on the distribution of income is similar to the findings in Rada and Kiefer (2015). Their estimates suggest the so-called *demand regime*, that is the long-run dependence of utilization on the labor share, to be downward sloping or profit-led, and the same is true in our model as per equation (9). They also estimate the so-called *distributive curve*, that is the long-run relation between the labor share and utilization, and find it displays profit-squeeze in that it is upward sloping. The combination of profit-led demand and profit-squeeze distribution delivers progressive effects of a fiscal expansion. In our model, utilization is profit-led as per equation (9), but equation (18) pins down the long-run value of income shares, thus replacing the distributive curve. The progressive effects of fiscal policy arise through the balanced growth condition that requires the long-run accumulation rate to equal the growth rate of the effective labor force: $g = \alpha + n$. A fiscal expansion that achieves the full rate of utilization puts pressure on the accumulation rate g, but the long run growth rate $\alpha + n$ is constant: the wage share must increase in order to restore balanced growth.

5 Transitional Dynamics and Numerical Simulations

Appendix C provides a formal analysis of the local stability properties of the two-class steady state of the model, and shows that it is locally stable under mild conditions on parameters. This finding marks a difference with the benchmark Goodwin (1967) model where the struggle between the two classes leads to perpetual cycles in employment and distribution. As highlighted by Shah and Desai (1981) and van der Ploeg (1985), the disappearance of the Goodwin cycle in the long run arises when firms can counter the workers' wage demands following increases in employment through changes in the technique of production that substitute labor for capital. This mechanism is guaranteed in our model by the choice of utilization which, as shown above, makes labor demand elastic to the wage despite the Leontief technology.[15]

While the framework is simple enough that it can be studied analytically, it is interesting to visualize the transitional dynamics numerically in simulations. Appendix D provides details on the calibration used for these simulation rounds. Importantly, these exercises are meant to showcase the qualitative features of the transitional dynamics and *not* to accurately replicate

[15]The ultimate dampening of the cycle does not mean, however, that there won't be oscillations at all: the simulations shown below show that oscillations do in fact occur in the aftermath of a policy or demand shock, before the system converges to its new steady state.

or predict the quantitative behavior of actual economies. Moreover, it is interesting to visually compare the dynamics of the baseline model with the two extensions presented in the next section. Two numerical exercises are of interest: (1) the effect of a demand shock, and (2) a decentralization policy along the lines of Section 4.2 taking place at time zero.[16] The left panel of Figure 5 displays the transitional dynamics following of a 2.5% demand shock occurring at time zero; while the right panel plots the transitional dynamics of implementing the decentralization policy outlined above, again at time zero, supposing that the system was operating in laissez faire before the policy. In both plots, the economy starts in balanced growth equilibrium; and the three variables of interest are plotted as ratios of their value after the shock over their pre-shock value. The dotted line is flat at 100% to facilitate visualization. In both

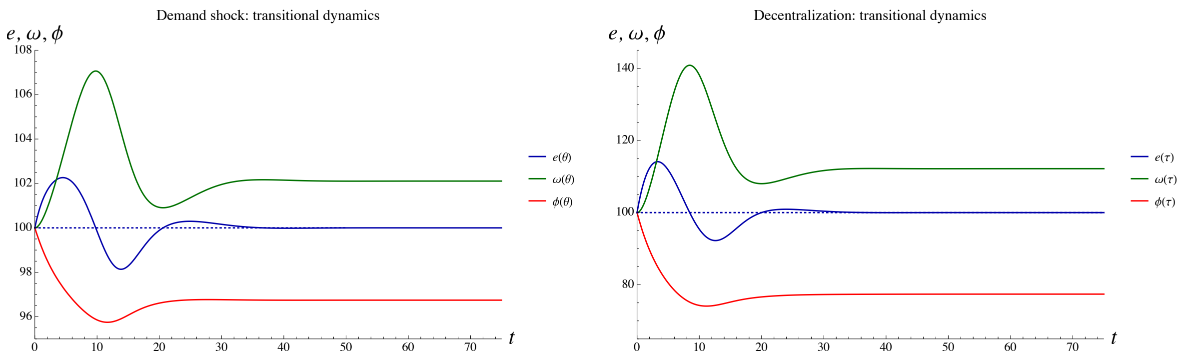


Figure 5: The effect of a demand shock (left) and of a fiscal policy shock (right).

cases, the wage share and the workers' wealth share overshoot before reaching their new steady state value; and the employment rate displays some cyclical fluctuations before returning to its long-run, exogenous value.

6 Extensions

6.1 Employment Hysteresis

A shortcoming of the analysis we detailed thus far is that hysteresis effects manifest only on the rate of capacity utilization but not on the employment rate which, squarely in the Goodwinian tradition, is basically a classical-Marxian version of an exogenously given NAIRU (Stockhammer, 2008). Yet, hysteresis appears to have been important in the aftermath of the Great Recession (Fatas, 2019); and it may prove relevant as the world economy recovers from the Covid-19 shock of 2020. A simple way to capture employment hysteresis amounts to extend

[16] The simulations are coded in *Mathematica* 12, and are available at https://danieletavani.com/code.

the classical Phillips curve $\dot{w}/w = f(e)$ in order to incorporate a feedback from utilization to real wage growth. To economize on notation, we assume that the intercept term in the wage-Phillips curve now depends on utilization as follows:[17]

$$\frac{\dot{w}}{w} = -\xi(u) + \lambda e, \;\; \xi_u > 0 \tag{30}$$

so that the steady state employment rate, always found by setting the change in the wage share equal to zero in equation (14), now depends directly on the utilization rate:

$$e_{ss} = \frac{\xi[u(\omega_{ss};\bar{\theta})] + \alpha}{\lambda} \tag{31}$$

Now, equilibrium unemployment will also be lower than at full capacity utilization, and demand shocks will have permanent effects on employment, too. The effects of a demand shock and decentralization on this version of the model are illustrated in the simulations in Figure 6. For these simulations, we used a linear $\xi(u)$ function: $\xi(u) = \xi_0 + \xi_1 u$: see Appendix D for the calibration used. In both cases, long-run employment permanently increases following both a

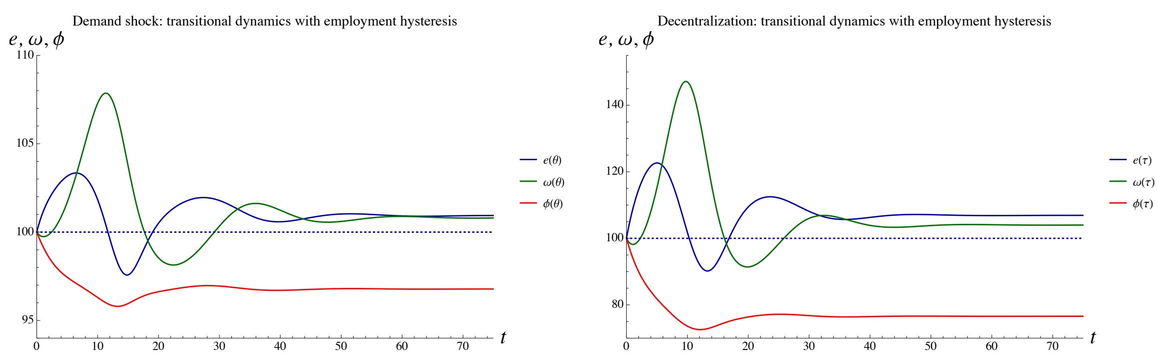


Figure 6: The effect of a demand shock (left) and of a fiscal policy shock (right) with employment hysteresis.

time-zero positive demand shock and a fiscal policy shock that decentralizes the full utilization rate. Moreover, the long-run increase in the wages share is smaller than in the baseline model; and the transitional dynamics displays stronger cyclicality in both the employment rate and the wage share before their new steady state is reached.

[17]The term ξ in the Phillips curve can be thought of as capturing the effect of price-inflation on real wages. In fact, the corresponding equation can be written as relating the growth rate of *nominal* wages $\dot{w}/w + \xi = \lambda e$ when ξ is taken as the inflation rate. Rendering $\xi = \xi(u)$ amounts to synthetically capture the role of stronger aggregate demand in determining price increases.

6.2 Growth Hysteresis

Finally, suppose that aggregate labor productivity growth is endogenous to the utilization rate and that this effect is external to the firm. Thus, assume $\alpha = \alpha(u)$ with $\alpha_u > 0$, while the Goodwin wage-Phillips curve is as in the baseline (Section 3.2). This extended model will again feature hysteresis in the employment rate and the possibility of achieving higher employment at full utilization, as displayed in the simulations in Figure 7, given that

$$e_{ss} = \frac{\xi + \alpha[u(\omega_{ss}); \bar{\theta}]}{\lambda} \tag{32}$$

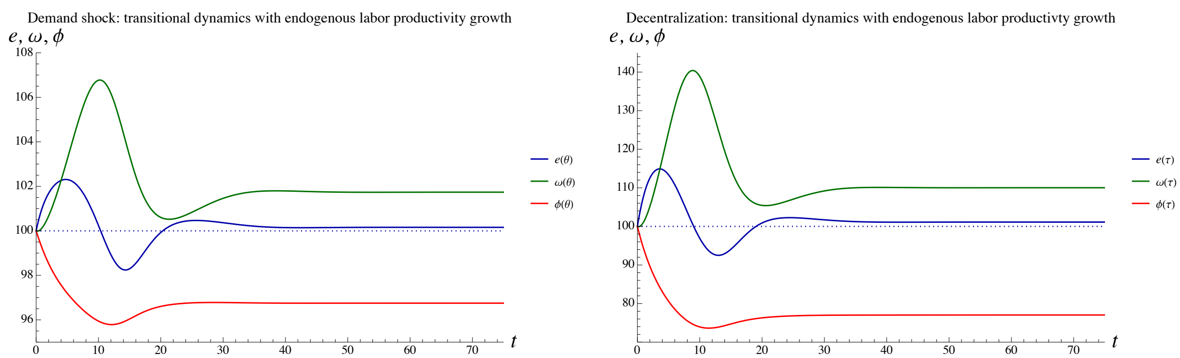


Figure 7: A demand shock (left) and of a fiscal policy shock (right) with endogenous labor productivity growth. See Appendix D for parameter values.

Moreover, and importantly, the profit rate along a balanced growth path will be higher at full utilization than in equilibrium, since

$$r = \rho^c + \alpha[u(\omega_{ss}; \bar{\theta})] + n \tag{33}$$

$$r^* = \rho^c + \alpha[u^*(\omega^*_{ss}; \bar{\theta})] + n \tag{34}$$

with the full-utilization profit rate r^* being greater than the equilibrium profit rate r given that $u^* > u$ as per equation (22). Thus, up to the initial amount of capital stock, an economy operating at full capacity will earn higher profit income than one operating at equilibrium utilization. As already anticipated above, the political economy of this result is even the more striking than the baseline model. Indeed, the model rationalizes Kalecki (1943)'s insight that although "profits would be higher under a regime of full employment than they are on the average under *laissez-faire*" capitalists nonetheless resist this regime because "the social position of the boss would be undermined" and "'discipline in the factories' and 'political stability' are more appreciated than profits by business leaders" (p. 3). Figure 8 provides a visual illustration of the argument through a simulation that displays the full path (of the natural log, given they are exponential functions) of profits corresponding to the general solution of the dynamical system at equilibrium and full utilization rate respectively, starting from the same initial condition on capital stock.[18] As the figure makes it clear, the full utilization path of profits starts above the equilibrium path, then is briefly dominated but completely overtakes the equilibrium path within just over 15 periods.

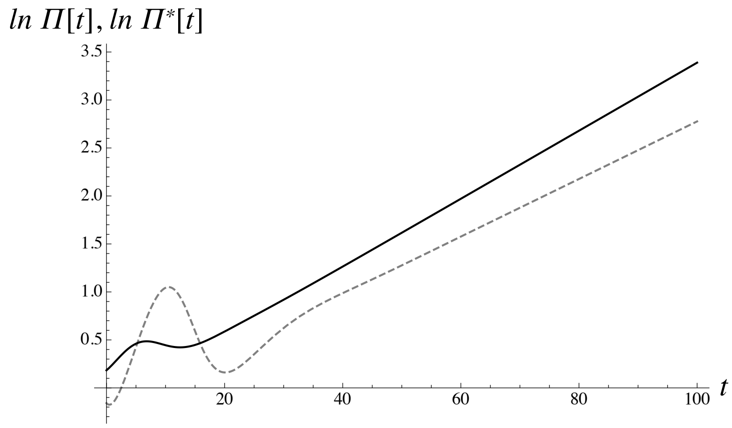


Figure 8: Log maximized profits at equilibrium (gray) vs. full (black) utilization. The equilibrium path plots $\ln \Pi(t) = \ln[(1-\beta)(1-\omega(t))u(\omega(t);\bar{\theta})K(0)\exp\{g(t)t\}]$, where $g(t)$ obeys equation (12), while the efficient path is obtained by replacing the equilibrium values with their full-utilization counterpart. In both cases, $K(0) = 10$. See Appendix D for parameter values.

7 Conclusion

In this paper, we studied a dynamic micro-to-macro model of growth and distribution at the intersection of contemporary work in classical political economy and post-Keynesian economics. The intersection amounts to introducing a role for aggregate demand as an externality in an otherwise standard cyclical growth model featuring workers' saving and the distribution of wealth (Michl, 2009; Foley et al., 2019, Ch. 17). Additionally, by incorporating aggregate demand as an externality, our model provides a potential bridge between the non-mainstream literature and recent mainstream work on hysteresis and animal spirits (Engler and Tervala, 2018; Farmer and Platonov, 2019; Cerra, Fatas, and Saxena, 2021). The resulting framework, similarly to earlier work (Petach and Tavani, 2019; Tavani and Petach, 2021), produces equilibrium underutilization —a coordination failure— coexisting with capital accumulation. However, and importantly, we showed that underutilization has regressive effects not only the functional distribution of income but also the distribution of wealth. Similar to Cooper and John (1988), we

[18]The parameter calibration is the same as in Appendix D, with the following difference: labor productivity growth is now a linear function of utilization such that $\alpha = \alpha_1 u$ and the parameter α_1 is calibrated to return a long-run growth rate of 2% at the steady state along the equilibrium path.

highlighted the coordinating role of allocation policy, with an emphasis on its distributional effects. In particular, we showed that a fiscal package that decentralizes the full utilization rate —the rate of utilization attainable if firms were able to internalize the aggregate demand externalities— will have progressive distributional effects not only on the labor share, but also on the distribution of wealth. We motivated our contribution with the retreat of the State from allocation policy during the Neoliberal era: our analysis suggests that market economies, left on their own, are likely to be prone to stagnation and inequality, and that fiscal allocation policy that achieves full utilization plays the important secondary role of taming the concentration of wealth and improving the workers' distributional position.

We suggested that, despite the possibility of achieving higher income and a more labor-friendly distribution of both income and wealth, the fact that both the wage share and the workers' wealth share increase through fiscal policy that push the economy toward full capacity may prevent the owners of capital stock from signing off on such policies, despite earning the same or even higher total profit income than in equilibrium. As such, our argument can be seen as offering, to paraphrase Kalecki (1943), insights on the "political aspects of full utilization."

Finally, we looked at the effects of aggregate demand shocks, or equivalently shocks to Keynesian "animal spirits," and showed that they have permanent distributional effects of the same sign on both the long-run wage share and the workers' wealth share. In the baseline model, the long-run employment rate is tied up to an exogenous labor productivity growth rate: as a consequence, allocation policy and demand shocks have no long-run effect on either growth or employment. However, persistent effects are found in simple extensions allowing for employment or growth hysteresis. Given the lackluster economic recovery following the Great Recession, our hope is that this stylized model can be useful as an organizing framework for thinking through the persistency of shocks to economic activity—such as the Covid-19 pandemic—and the importance of countervailing economic measures.

Acknowledgements

We thank Tom Michl, Codrina Rada, Rudi von Arnim, Roberto Veneziani, and participants at seminars at the New School and University of Massachusetts Boston for helpful comments and discussions. Addressing comments and criticism from the Editor and two anonymous referees greatly improved the paper. All errors are ours.

A Dynamic Optimization

A.1 Capitalists

The capitalist's optimization program is as follows. With logarithmic preferences over consumption streams and discount rate ρ^c, the capitalist household chooses $c^c(t)$ to maximize the present-discounted value $\int_s^\infty \exp\{-\rho^c(t-s)\} \ln c^c(t) dt$ subject to the resource constraint $\dot{K}^c(t) = r(t)K^c(t) - c^c(t)$, where $r(t)$ is the profit rate, equal to $(1-\beta)(1-\omega(t))u(t)$ at time t, and given to the household. The problem requires to specify initial and terminal conditions, which are standard and omitted. Forming the current-value Hamiltonian and using the first-order conditions for an optimal control leads to the Euler equation (7) in the text.

A.2 Workers

In solving the workers' consumption and saving problem, crucial assumptions are: that neither workers nor capitalist households are responsible for the utilization choice, so they take $u(t)$ as a given at all times; and that each worker household is negligible enough not to be able to internalize its influence either on the economy-wide wealth or income distribution. Hence, the typical worker household also takes $\phi(t)$ and $\omega(t)$ as a given at all times. It solves the following problem:

$$\begin{aligned}
\text{Given} \quad & \{u(t), \omega(t), \phi(t)\} \;\; \forall t, \\
\max_{\{c^w(t)\}_{t\in[s,\infty)}} \quad & \int_s^\infty \exp\{-\rho^w(t-s)\} \ln c^w(t) dt \\
\text{s. t.} \quad & \dot{K}^w(t) = \frac{u(t)}{1-\phi(t)} \left[\omega + (1-\beta)(1-\phi(t))(1-\omega(t))\right] K^w(t) \\
& K^w(s) \equiv K^w_s > 0, \text{ given} \\
& \lim_{t\to\infty} \exp\{-\rho(t-s)\} K^w(t) \geq 0
\end{aligned} \tag{35}$$

This problem involves a strictly concave objective function to be maximized over a convex set. Thus, with co-state variable $\mu^w(t)$, the standard first-order conditions on the associated current-value Hamiltonian

$$\mathcal{H} = \ln c + \mu^w \left\{ \frac{u}{1-\phi} \left[\omega + (1-\beta)(1-\phi)(1-\omega)\right] \right\} K^w$$

will be necessary and sufficient for an optimal control. They are:

$$c^{-1} = \mu^w \tag{36}$$

$$\rho \mu^w - \dot{\mu}^w = \mu^w \left\{ \frac{u}{1-\phi} \left[\omega + (1-\beta)(1-\phi)(1-\omega) \right] \right\} \quad (37)$$

$$\lim_{t \to \infty} \exp\{-\rho^w t\} \mu^w(t) K^w(t) = 0 \quad (38)$$

To obtain the Euler equation for consumption, differentiate (36) with respect to time and use (36) and (37) to get:

$$\frac{\dot{c}^w}{c^w} = \frac{u}{1-\phi} \left[\omega + (1-\beta)(1-\phi)(1-\omega) \right] - \rho^w$$

Imposing a balanced growth path where consumption and capital stock grow at the same rate gives equation (8). As it is standard under log utility, what ensures that workers' consumption and capital stock grow at the same rate is that workers' consume a constant fraction of their end-of-period wealth, the fraction being equal to the discount rate ρ^w.

B Proofs

Proposition 1: Both results in the first claim can be proven directly, by comparing the equilibrium solutions with the socially-coordinated solution. Start with the wealth share: dividing equation (26) by (20) side by side we obtain

$$\frac{1-\phi_{ss}^*}{1-\phi_{ss}} = \left(\frac{1}{1-\gamma} \right)^{\frac{\beta}{1-\gamma}} > 1$$

which shows that the workers' wealth share at full utilization is higher than in equilibrium. For the wage share, the comparison between equation (24) and (18) implies that

$$\frac{1-\omega_{ss}^*}{1-\omega_{ss}} = (1-\gamma)^{\frac{\beta}{1-\gamma}} < 1$$

which proves that the profit (wage) share at full utilization is lower (higher) than in equilibrium.

As for the effect of demand shocks, it is enough to see that $\partial(1-\omega)/\partial\bar{\theta} <)$, which implies that the profit share decreases following a shock to $\bar{\theta}$, and that $\partial(1-\phi)/\partial\bar{\theta} > 0$, which implies that the workers' wealth share responds positively to a demand shock.

Proposition 2: The first two claim have already been proven in Tavani and Petach (2021). As for the third, it is enough to notice that, for $\sigma = \gamma$, the economy achieves the full-utilization wealth share and wage share, so that the result in Proposition 1 applies.

C Local Stability Analysis

Linearization of the dynamical system formed by (15), (14), and (13) around the two-class steady state with $\phi_{ss} \in (0,1)$ given by (16) gives a Jacobian matrix with the following sign structure:

$$J(e_{ss},\omega_{ss},\phi_{ss}) = \begin{bmatrix} \begin{matrix} J_{11} \\ (-) \end{matrix} & \begin{matrix} J_{12} \\ (-) \end{matrix} & \begin{matrix} J_{13} \\ (+) \end{matrix} \\ \begin{matrix} J_{21} \\ (+) \end{matrix} & 0 & 0 \\ 0 & \begin{matrix} J_{32} \\ (\pm) \end{matrix} & \begin{matrix} J_{33} \\ (-) \end{matrix} \end{bmatrix}$$

given that:

$$\begin{array}{llll} J_{11} & = \frac{\partial \dot{e}}{\partial e}|_{ss} & = -\dfrac{\beta}{1-\beta-\gamma}\dfrac{\omega_{ss}}{1-\omega_{ss}}\lambda e_{ss} & < 0 \\ J_{22} & = \frac{\partial \dot{e}}{\partial \omega}|_{ss} & = (1-\beta)u_\omega e_{ss} & < 0 \\ J_{13} & = \frac{\partial \dot{e}}{\partial \phi}|_{ss} & = (\rho^w - \rho^c)e_{ss} & > 0 \\ J_{21} & = \frac{\partial \dot{\omega}}{\partial e}|_{ss} & = \lambda \omega_{ss} & > 0 \\ J_{32} & = \frac{\partial \dot{\phi}}{\partial \omega}|_{ss} & -\phi_{ss}[u(\omega_{ss}) + \omega_{ss}u_\omega] & \gtreqless 0 \\ J_{33} & = \frac{\partial \dot{\phi}}{\partial \phi}|_{ss} & = -(\rho^w - \rho^c)\phi_{ss} & < 0 \\ J_{22} & = J_{23} & = J_{31} & = 0 \end{array}$$

The only entry in the matrix that is in principle ambiguous in sign is J_{32}, given that it reduces to

$$-\phi_{ss}\bar{\theta}(1-\omega_{ss})^{\frac{\beta}{1-\beta-\gamma}}\left(1 - \frac{\omega_{ss}}{1-\omega_{ss}}\frac{\beta}{1-\beta-\gamma}\right)$$

and can be positive or negative depending on parameter values for β, γ. In order to check for local stability, we need to evaluate the Routh-Hurwitz conditions, namely:

- $TrJ_{ss} < 0$, which is clearly satisfied, given that $J_{11} + J_{33} < 0$.
- $DetJ_{ss} < 0$. We have that:

$$\begin{aligned} DetJ_{ss} &= J_{21}(J_{13}J_{32} - J_{12}J_{33}) \\ &= \phi_{ss}\omega_{ss}(\rho^w - \rho^c)\lambda e_{ss}[u_{\omega,ss}(1-\omega_{ss}) - u(\omega_{ss})] \\ &= \phi_{ss}\omega_{ss}(\rho^w - \rho^c)\lambda e_{ss}\left[u(\omega_{ss})\left(\tfrac{\beta}{1-\beta-\gamma} - 1\right)\right] \end{aligned}$$

  whose sign depends on whether the last term in the multiplication is positive or negative. A necessary and sufficient condition for negativity of the determinant is that $\frac{\beta}{1-\gamma-\beta} < 1$, or $\beta < (1-\gamma)/2$.

- $\sum_{j=1}^{3} PmJ_j > 0$, where Pm_j is the principal minor obtained removing row j and column j from the whole matrix. This sum is equal to $-J_{12}J_{21} + J_{11}J_{33} > 0$ as required.

- $-\sum_{j=1}^{3} PmJ_j + \frac{DetJ_{ss}}{TrJ_{ss}} > 0$. This condition boils down to:

$$-J_{11}J_{33} + \frac{J_{21}}{J_{11} + J_{33}}(J_{11}J_{12} + J_{13}J_{32}) < 0$$

  The first term is unambiguously positive. Since the ratio $J_{21}/(J_{11}+J_{33})$ is *negative*—the numerator is positive while the denominator is the trace of the Jacobian, which we just shown to be negative—a sufficient condition for this fourth requirement to be satisfied is that the term in parentheses $J_{11}J_{12} + J_{13}J_{32}$ be *positive* instead. Now, $J_{11}J_{12}$ is certainly positive, given it is the product of two negative numbers. If we can identify a sufficient condition such that the second addendum $J_{13}J_{32} > 0$ as well, we are done.[19] We have that:

$$J_{13}J_{32} = \phi_{ss}(\rho^w - \rho^c)e_{ss}\left[u(\omega_{ss}) + \omega_{ss}u_{\omega,ss}\right]$$

  whose sign depends on the sign of:

$$u(\omega_{ss}) + \omega_{ss}u_{\omega,ss} \quad = \quad \bar{\theta}(1-\omega_{ss})^{\frac{\beta}{1-\beta-\gamma}}\left[1 - \tfrac{\omega_{ss}}{1-\omega_{ss}}\tfrac{\beta}{1-\beta-\gamma}\right]$$

  which in turn is positive if the term in brackets is positive. This will be the case provided that $\beta < (1-\omega_{ss})(1-\gamma)$. Even though this involves the value of an endogenous variable, so long as $1-\omega_{ss} < 1/2$, which is certainly satisfied for high-income economies such as the United States, the condition for the determinant to be negative identified above is sufficient for this stability requirement to be satisfied.

We conclude that $\beta < (1-\gamma)/2$ is sufficient for local stability. Note finally that, although the steady state is locally stable, the stability analysis above does not rule out the occurrence of (damped) oscillations around the steady state. These oscillations actually occur in the simulations presented in the text.

D Parameter Calibration

In order to calibrate the parameters of the model, we use the following strategy. For the user cost parameters β, γ we take the point estimates provided in Petach and Tavani (2019), which satisfy the restriction required for local stability, namely $2\beta < 1-\gamma$. For the capitalist discount rate ρ^c, we set a value of 5% which is standard in the literature. The growth rate of

[19] Checking necessary conditions is more complicated.

labor productivity α and the population growth rate n are set at 2% and 1% respectively, also standard. For the slope parameter of the Phillips curve λ, we use the naïve estimates provided in intermediate macro textbooks such as Blanchard (2017), namely .52. We are then left with three parameters to calibrate internally. As for the Phillips curve intercept ξ, we solve for the value required to return $e_{ss} = 94\%$, which is in line with a long-run unemployment rate of 6%, in equation (17). The solution is $\xi = .4688$. To calibrate the workers' discount rate, we solve for the value required to obtain a capitalist wealth share of 40%, in line with the current estimates of the top 1% wealth share in Saez and Zucman (2016). Finally, we internally calibrate $\bar{\theta} = .450156$ which, given the other parameters, returns a wage share of .6, conforming with the values provided in Figure 2. Table 1 summarizes the baseline parameter calibration.

Parameter	Moment to match	Source
β	0.2	Petach and Tavani (2019)
γ	0.45	Petach and Tavani (2019)
α	0.02	Standard
α_1	$\alpha(u_{ss}) = .02$	Calibrated internally
n	.01	Standard
$\bar{\theta}$	$\omega_{ss} = .6$	Calibrated internally
ρ^c	.05	Standard
ρ^w	$\phi_{ss} = .4$	Saez and Zucman (2016)
λ	.52	Blanchard (2017)
ξ	$e_{ss} = .94$	Calibrated internally

Table 1: Parameter calibration.

References

Allain, O. "Tackling the Instability of Growth: A Kaleckian-Harrodian Model with an Autonomous Expenditure Component." Camb. J. Econ. 39 (5), 1351-1371.

Appelbaum, B. 2019. *The Economist's Hour: False Profits, Free Markets, and the Fracture of Society*. New York: Little, Brown and Company.

Blanchard, O.J., 1985. Debt, Deficits and Finite Horizons, J. Pol. Econ. 93, 223-247.

Blanchard, O.J., 2017. Macroeconomics, 7th Edition. Pearson.

Carter, Z., 2020. *The Price of Peace: Money, Democracy, and the Life of John Maynard Keynes*. Random House, New York, NY.

Cerra, V., Fatas, A., and Saxena, S. 2021. Hysteresis and Business Cycles. J. Econ. Lit. forthcoming.

Cooper, R., and John, R. 1988. “Coordinating Coordination Failures in Keynesian Models.” Q. J. Econ. 103 (3), 441-463.

Ederer, S., and Rehm, M. 2020a. Will Wealth Become More Concentrated in Europe? Evidence from a Calibrated Post-Keynesian Model. Camb. J. Econ. 44 (1), 55-72.

Ederer, S., and Rehm, M. 2020b. “Making Sense of Piketty’s ‘Fundamental Laws’ in a Post-Keynesian Framework: the Transitional Dynamics of Wealth Inequality.” Rev. Keynes. Econ. 8 (2), 195-219.

Engler, P., and Tervala, J. 2018. Hysteresis and Fiscal Policy. J. Econ. Dyn. Control. 93, 39-53.

Farmer, R., and Platonov, K. 2019. Animal Spirits in a Monetary Model. Europ. Econ. Rev. 115, 60-77.

Fatas A (2019) Fiscal policy, potential output and the shifting goalposts. Paper presented at the Euro 20 academic conference organized by the IMF and Bank of Ireland. Available at: https://faculty.insead.edu/fatas/euro20.pdf.

Fazzari, S., Ferri, P., and Variato, A. 2020. “Demand-led Growth and Accommodating Supply. Camb. J. Econ. 44 (3), 583-605.

Foley, D.K., 2016. Keynesian microeconomics of output and labor markets. In: Bernard, L., Nyambuu (Eds.), Dynamic Modeling, Empirical Macroeconomics, and Finance: Essays in Honor of Willi Semmler. Springer, pp. 183-194.

Foley, D.K., Michl, T.R, and Tavani, D. 2019. Growth and Distribution, Second Edition Harvard University Press, Cambridge, MA.

Franke, R., 2020. An Attempt at at a Reconciliation of the Sraffian and Kaleckian View on Desired Utilization. Europ. J. Econ. Economic Policies: Intervention 17 (1), 61-77.

Gahn, S., and Gonzalez, A. 2019. On the ‘Utilisation Controversy’: A Comment. Camb. J. Econ. 44 (3), 703-707.

Gahn, S. 2020. Is There a Declining Trend in Capacity Utilization in the US Economy? A Technical Note. Rev. Pol. Econ. https://doi.org/10.1080/09538259.2020.1769906.

Girardi, D., and Pariboni, R. 2019. Normal Utilization as the Adjusting Variable in Neo-Kaleckian Growth Models: A Critique. Metroeconomica 70 (2), 341-358.

Goodwin, R. 1967. A Growth Cycle. in *Socialism, Capitalism, and Economic Growth*. Cambridge University Press, Cambridge UK.

Greenwood, J., Hercowitz, Z., and Huffman, G., 1988. Investment, Capacity Utilization, and the Real Business Cycle. Am. Econ. Rev. 78 (3), 402-417.

Harris, D.J., 1978. *Capital Accumulation and Income Distribution*. Stanford University Press.

Jones, C., 1995. R&D-based Models of Economic Growth. J. Pol. Econ. 103 (4), 692-733.

Julius, A.J., 2005. Steady-State Growth and Distribution with an Endogenous Direction of Technical Change. Metroeconomica 56 (1), 101-125.

Kaldor, N., 1957. A Model of Economic Growth. Econ. J. 67(268), 591-624.

Kalecki, M., 1943 Political Aspects of Full Employment. Pol. Q., 14 (4), 322-330.

Kydland, F., and Prescott, E. 1982. Time to Build and Aggregate Fluctuations. Econometrica 50 (6), 1345-1370.

Lucas, R. 1977. Understanding Business Cycles. *Carnegie-Rochester Conference Series on Public Policy*, 5, 7-29.

MacLean, N. 2017. Democracy in Chains: The Deep History of the Radical Right's Stealth Plan for America. Penguin Random House, New York.

Marglin, S., 1984. Growth, Distribution, and Prices Harvard University Press, Cambridge MA.

Michl, T.R., 2009. Capitalists, Workers, and Fiscal Policy. Harvard University Press, Cambridge MA.

Nikiforos, M., and Foley, D. 2012. Distribution and Capacity Utilization: Conceptual Issues and Empirical Evidence. Metroeconomica 63 (1), 200-229.

Nikiforos, M. 2016. On the 'Utilisation Controversy': A Theoretical and Empirical Discussion of the Kaleckian Model of Growth and Distribution. Camb. J. Econ. 40 (2), 437-467.

Nikiforos, M. 2018. Some Comments on the Sraffian Supermultiplier Approach to Growth and Distribution. J. Post Keynes. Econ. 41 (4), 659-675.

Nikiforos, M. 2019. On the 'Utilisation Controversy': a Rejoinder and Some Comments. Camb. J. Econ. 44 (3), 709-722.

Palley, T., 1990. Applied Fix-Price Macro Models: a Reconsideration. Atl. Econ. J. 18 (2), 1-16.

Pasinetti, L., 1962. Rate of profit and income distribution in relation to the rate of economic growth. Rev. of Econ. Stud. 29 (4), 267-279.

Petach, L. 2020. Distribution and Capacity Utilization in the United States: Evidence from State-level Data. Rev. of Keynes. Econ. 8 (2), 240-267.

Petach, L., Tavani, D. 2019. No One is Alone: Strategic Complementarities, Capacity Utilization, Growth, and Distribution. Structural Change and Econ. Dyn. 50, 203-215.

Petach, L., Tavani, D. 2020. Income Shares, Secular Stagnation, and the Long-Run Distribution of Wealth. Metroeconomica 71 (1), 235-255.

Petach, L., Tavani, D. 2021. Consumption Externalities and Growth: Theory and Evidence for the United States. J. Econ. Behav. Org. 183, 976-997.

Piketty, T., Saez, E., 2003. Income Inequality in the United States, 1913-1998. Q. J. Econ. 113 (1), 1-39.

Piketty, T., Saez, E., Zucman, G., 2018. Distributional National Accounts: Methods and Estimates for the United States. Q. J. Econ. 133 (2): 553-609.

Rada, C., Kiefer, D. 2015. Profit Maximising Goes Global: The Race to the Bottom. Camb. J. Econ. 39 (5), 1333-1350.

Saez, E. & Zucman, G. (2016) Wealth inequality in the United States since 1913: Evidence from capitalized income tax data. Q. J. Econ. 131 (2), 519-578.

Samuelson, P. & Modigliani, F. (1966) The Pasinetti paradox in neoclassical and more general models. Rev. Econ. Stud. 33 (4), 269–301.

Shah A., Desai M. (1981) Growth cycles with induced technical change. Econ. J. 91, 1006-1010.

Setterfield, M. 2019. Long-Run Variation in Capacity Utilization in the Presence of a Fixed Normal Rate. Camb. J. Econ., 43 (2), 443-463.

Slobodian, Q. Globalists: The End of Empire and the Birth of Neoliberalism. Harvard University Press, Cambridge MA.

Stiglitz, J., Tucker, T., Zucman, G. 2020. The Starving State. Foreign Affairs, January-February 2020.

Stockhammer, E., 2008. Is the NAIRU Theory a Monetarist, New Keynesian, Post-Keynesian or Marxist Theory? Metroeconomica 59 (3), 479-510.

Tavani, D., and Petach, L., 2021. Firm Beliefs and Long-Run Demand Effects in a Labor-Constrained Model of Growth and Distribution. J. Evol. Econ. 31(2), 355-377.

Taylor, L., Foley, D. K., Rezai, A., 2019. Demand Drives Growth All the Way: Goodwin, Kaldor, Pasinetti and the Steady State. Camb. J. Econ. 43 (5), 1333-1352.

van der Ploeg R (1985) Classical growth cycles. Metroeconomica 37 (2), 221-230.

Yaari, M. E., 1965. Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. Rev. Econ. Stud. 32 (2), 137-150.

Zamparelli, L., 2015. Induced Innovation, Endogenous Technical Change and Income Distribution in a Labor-Constrained Model of Classical Growth. Metroeconomica 66 (2), 243-262.

Zamparelli, L. 2016. Wealth Distribution, Elasticity of Substitution and Piketty: An ‘Anti-Dual’ Pasinetti Economy. Metroeconomica 68 (4), 927-946.