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Expectational and Portfolio-Demand Shifts in a Keynesian Model of Monetary Growth Fluctuations

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Abstract

Abstract: We develop a pair of models to show how non-ad-hoc shifts to expectational variables can be used to model tendencies toward crisis. In the Shackle model, as developed in the book Keynesian Kaleidics (1974), uncertainty can lead to a collapse in the marginal efficiency of investment and a jump in liquidity preference. In the Minsky version of the model, excessive private debt can lead to a financial collapse–again an endogenous breakdown in forces supporting growth. We extend the models to indicate how the dynamics of inflation and distribution affect the dynamics.

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1 Introduction

In the his General Theory, Keynes named three "independent variables" in his basic model, the values of which would determine output and employment the short run:

1. the propensity to consume,
2. a curve relating the marginal efficiency of capital to the interest rate, and
3. the rate of interest itself (Keynes 1936, p. 246).

Further, the determinants of these variables could be divided into givens that could be assumed constant and more changeable quantities that were capable of moving output and employment over business cycle frequencies. The more changeable variables included

> 1. the three fundamental psychological factors, namely the psychological propensity to consume, the psychological attitude toward liquidity and the psychological expectation of future yields from capital assets,
> 2. the wage unit as determined by the bargains reached between employers and employed, and
> 3. the quantity of money as determined by the action of the central bank... (p. 245-247)

These underlying independent variables could not be assumed constant or independent beyond the short run, but they determine output and employment at any given time (Keynes 1936, pp. 248-249). Furthermore, changes in these variables would help to explain business-cycle frequency output and employment fluctuations.

In particular, Keynes, in explaining cyclical fluctuations in output and employment, emphasized the volatility of the psychological attitudes underlying expectations of future yields, which were subject to "sudden and violent" changes because they were based on only very limited knowledge (1936, p. 315). If these changes were particularly large downward moves, they could lead to a phenomenon Keynes called "crisis" in light of "the fact that the substitution of a downward for an upward tendency often takes place suddenly and violently," while upturns were usually not so dramatic (p. 314). G. L. S. Shackle's account of Keynes's theory (1974) emphasized the unpredictable "kaleidic" changeability of the state of long-term expectation and the state of liquidity preference, which Keynes first described in Chapter 12 and 15 of the General Theory (1936, p. 147-164, 194-209).

Keynes emphasized that investment and liquidity preference were less than determinate based solely on past data. Keynes emphasized that his only objection to the investment functions in Kalecki's work was merely that they seemed to entail a false claim of precision, given the possibility of changes in psychological factors underlying investment. Kalecki for his part urged on Keynes the importance of dynamics over time in which changing profits would feed back to the state of long-term expectation. See Asimakopulos (1991, p. 70-76) for a discussion of these issues in expectational dynamics and the respective positions of Keynes, Kalecki, and Shackle. We seek to incorporate strengths of the Keynesian and Kaleckian approaches to investment into the model of this paper.

Debt dynamics and financial crises are crucial in recent macroeconomic history. H. Minsky's theory of financial fragility (as stated for example in Minsky, 1982 [1977], especially p. 62-68) depended upon an erosion of margins of safety and a rise in leverage that tended to occur as time passed following a crisis. Fragility would eventually lead to crisis as agents drastically changed their expectations as to the ability of borrowers to meet their payment commitments. Minsky's theory forms the basis for a second closure of our model, in which expectational shifts play a somewhat different role.

Technically, our model captures Keynesian expectational dynamics in several ways:

1. a nonlinear investment function based on the approach of Kaldor (1940) and followers that leads to endogenous output dynamics,

2. a stochastic approach to generate shifts or continuous movement in expectational variables that has systemic impacts (Hannsgen 2012; Hannsgen 2013; Hannsgen and Young-Taft 2015),

3. the use of (1) stable distributions (Nolan 2015) from which to draw the

sizes of shocks to model variables or (2) stochastic processes that embody (possibly non-Gaussian) stable increments, pathways with jumps, and long memory (Embrechts and Maejima 2002), rather than assuming Gaussian, finite-memory processes.

4. Stock-flow effects (Godley and Lavoie [2007] 2012) in the consumption function of the capitalist sector and in the Minsky closure, the use of probabilistic stock-flow effects — i.e. a probability model in which financial ratios and capacity utilization determine the expected rate of occurrence of shocks to bank balance sheets and the liquidity preference schedule.

A constant wage unit is also among the short-run assumptions that Keynes drops as his mature theoretical statement proceeds (1936, Chapter 19). We do so in the penultimate section of this paper. At the same time, we endogenize our Kaleckian markup. This model extension opens the way to debt-deflation effects, noted as an important possibility in that chapter, and indeed to a nontrivial role for the unit of account and money. Changes in income distribution and price levels have effects on aggregate demand in this extension of the model.

Existing Minsky models fall into several categories: 1) Fragility-variable models following Taylor and OConnell; 2) crunch models in which the interest rate or credit rationing rises or animal spirits or the profit rate declines during a boom phase, but without leading up to a catastrophic collapse (e.g., Nikolaidi 2014); 3) regime-switching models, in which the economy switches between a turbulent and a nonturbulent state (Ferri 1992); 4) models with interacting agents (Di Guilmi and Carvallo, 2015); 5) threshold-effect models, which use Minskys three-way categorization of firms into hedge, speculative, and Ponzi units (Nishi 2012); and 6) models in which leverage, illiquidity ratios, etc. build up during a euphoric boom phase, leading to a sudden crash, as in Hannsgen (2012) and Hannsgen and Young-Taft (2015), which we now seek to develop more completely in a less-bundled form.

Nikolaidi and Stockhammer (2017) offer their own categorization of Minsky models, including Minsky-Kalecki, Godley-Minsky, and Minsky-Kaldor types. Our model seems closest to the Godley-Minsky type, though it does not precisely and uniquely fit into this category.

This paper assumes unstable output-adjustment at equilibrium, drawing from the Kaldorian trade cycle model to generate movement and help bound dynamical pathways. In this regard our model belongs to Nikolaidi's Kaldor-Minsky category. On the other hand, we assume less-than-full capacity utilization, which characterizes the Kalecki-Minsky genre. Moreover, we too borrow ideas from Godley and Lavoies concept of stock-flow-consistent modeling, which would put us in the Minsky-Godley category.

As Hannsgen (2013) labored to point out, Post-Keynesian and other heterodox models of discontinuous dynamic behavior have tended to rely unnecessarily on exogenous shifts or other deterministic devices. Among other studies that have combined formal Minskyan models with Kaldor's trade cycle model is Ryoo (2015). Palley (2011) also combined business cycle–frequency fluctuations with lower-frequency Minskyan dynamics. Dafermos (2015) and Nikolaidi (2014) develop up-to-date SFC Minskyan models. Other formal Minskyan efforts in the broadly post-Keynesian paradigm include work by Charles (2008), Nishi (2012), and Ferri (e.g., 1992). Asada (2004) was an early study using a continuous-time model that featured a variable price level and what structuralists and post-Keynesians now call stock-flow-consistent effects, related in this case to private-sector liabilities. Franke (2014) has recently explored stochastic models as a way of modeling Keynesian economic sentiment dynamics. Ryoo (2013) makes use of an animal spirits variable. Asada (2011) combines neoclassical Keynesian inflationary expectations, New Keynesian monetary policy, a deeply skeptical view of saddle-point jump dynamics, a standard neoclassical Keynesian model of the cycle, and a key role for private-sector debt in demand-determination. Charpe et al. (2011) is a text with a variety of nonlinear models incorporating financial dynamics. Ioannou (2015) developed a two-country model for a scenario in which crossing a threshold of creditworthiness triggers a credit downgrade and a sudden reduction in demand for the sovereign debt of one of the two countries. Rosser, Rosser, and Gallegati (2012) present time-series models of financial series that address features of the data omitted in more standard econometric approaches. The current authors have gradually developed a Poisson-process approach to Minskyan financial instability in the context of models of varying size (Hannsgen 2012, 2013; Hannsgen and Young-Taft 2015). The subsequent sections of this paper cover balance sheet relations in both models (section 2), a set of equations common to both models presented herein (section 3), the Shackle closure (section 4), the Minsky closure (section 5), and an extended model with variable wage unit and markup, respectively (section 6).

2 Balance Sheet Relations in Shackle and Minsky Models

We next cover the balance sheet relations that constitute the structure of the economy in our model. Such structure helps to differentiate an SFC model from more orthodox types of models, such as representative agent models. The Shackle version of the model has three sectors while the Minsky version has five. Both models contain a consolidated public (P) sector that includes the balance sheet of the central bank and a worker (W) sector. The Shackle model has a consolidated capitalist sector that holds assets, controls businesses, and consumes. In contrast, the Minsky model has distinct capitalist sectors for households (KH), banks (F), and nonfinancial firms (K).

In what follows, we normalize variables that are naturally expressed in goods by the capital stock and variables expressed in currency by the goods price times the capital stock. We choose our units so that the stock of capital goods is equal to one. The public sector service is normalized by a constant times the capital stock, with the constant chosen so that p = u when labor costs in the production of the P-sector service are equal to marked-up labor costs in the production of goods, each in their respective sectors. of course the choice of constants of normalization here is somewhat arbitrary.

The balance sheet structure of the Shackle closure is as follows:

TABLE 1 about here

All items in the balance sheet tables are normalized by the stock of capital. The first and second rows show bills b and currency $\mathcal{M}$, which are liabilities of the P sector and assets of the K sector. Capital goods, the tangible asset, are assumed to be held by the K sector only, as shown on the third line. The net worth of each sector is shown on line 4. The K sector's net worth is $1+\ell$, which is equal to government liabilities ℓ plus capital goods. Finally, line 5 shows that the net financial claims of the P sector and the K sector balance, since the W sector has no assets or liabilities. The last entry in that row is zero, indicating that financial assets balance liabilities.

The corresponding table for the Minsky version of the model is somewhat different:

TABLE 2 about here

Shown on line 1, bills b are again liabilities of the P sector, while they are now assets of the new capitalist household (KH) sector. The monetary variable $\mathcal{M}$ is used for reserve deposits that are a P sector liability and an F sector asset, as shown on line 2. Capital goods, on line 3, are as before an asset of the K sector only. Line 4 shows that loans lo are an asset of the F sector and a liability of the K sector, providing all industrial finance, as in a bank-based system. As shown on line 5, Deposits dep are assets for the KH sector and liabilities of the F sector. The net worth of each sector is as follows (see line 6): P sector, -ℓ; K sector 1-lo; W sector 0; KH sector w, or household wealth; and the F sector, 0. The net financial position of each sector on line 7 is the same as line 6 except it omits ownership of capital goods. It sums to zero.

3 The Basic Equations of Both Models

Our basic Kaldor-Kalecki-Steindl fiscal policy model with stock effects model is made up of three differential equations in 3 variables: public production p, capacity utilization u, and government liabilities ℓ. All stock variables are presumed normalized by the capital stock, k. It incorporates a countercyclical policy-setting equation:

$$\dot{p} = -[\alpha_{pp}(p - p_T) + \alpha_{up}(u - u_T)], \tag{1}$$

where p_T and u_T are fixed policy targets with respect to their policy variables, with subscript T suggesting 'target', and α_{pp} and α_{up} constants governing the rate of adjustment. Our notational strategy for fixed parameters except where we are forced to improvise will be to use α with the subscript indicating effect of the first variable relative to the second, so, for example, α_{up} would pertain to effect of u on p. One author compared model properties with alternative fiscal policy functions in Hannsgen (2014). A second adjustment equation shows how output (and sales) over capital u adjusts toward desired capacity utilization (normalized desired sales), u_d.

$$\dot{u} = -\alpha_u(u - u_d), \tag{2}$$

The deficit df and government liabilities, ℓ, are related by the stock-flow equation

$$\dot{\ell} = df - g\ell. \tag{3}$$

which is obtained by The division of $\ell := \mathcal{M} + b$ into holdings of high-powered money, $\mathcal{M}$, and government debt b is determined by the structure of asset demand. The two closures below make different assumptions in this regard. We provide details in sections 4 and 5 below.

In both closures we assume that the public sector sets a target real interest rate $i = \bar{\imath}$, engaging in open-market trades of its two liabilities to ensure that its target is met (Moore 1988; Godley and Lavoie 2012). The variable that adjusts to clear financial asset markets differs across the two closures considered below. We provide details in sections 4 and 5. We explore departures from the endogeneity of state money along with comparisons in settings in which endogeneity makes a difference in Hannsgen and Young-Taft (2017).

Total capacity utilization is accounted for by consumption goods, investment goods, and replacement investment δ, for δ fixed,

$$u = c + g + \delta. \tag{4}$$

The same relation holds for desired quantities, with subscript d suggesting 'desired,'

$$u_d = \delta + c_d + g_d. \tag{5}$$

Worker income is equal to one minus the tax rate times public spending plus the workers' share of private-sector receipts,

$$y_W = (1 - \tau)[p + (1 - s)u], \tag{6}$$

where the private sector income share of the capitalists s is defined as follows

$$s = \frac{m - 1}{m}, \tag{7}$$

for m the gross markup, which we set equal to m-bar, a constant (until a later section),

$$m = \bar{m}. \tag{8}$$

Equation (6) will hold given the pricing equation

$$pr = m\omega \tag{9}$$

where pr is the goods price and ω is the nominal wage. The gross profit rate (profits over the capital stock) is given by

$$\pi = u(pr - \omega)/pr = su \tag{10}$$

Lastly, we add differential equations that allow us to track the capital stock and net investment, g. Net investment adjusts at the same rate as goods production in general

$$\dot{k} = gk, \text{and} \tag{11}$$

$$\dot{g} = -\alpha_u(g - g_d). \tag{12}$$

Additionally, we assume as elsewhere (e.g., Hannsgen 2014) that $\partial(u-u_d)/\partial u > 0$ (Keynes-Kaldor instability) and $\partial u_d/\partial m < 0$ (Kalecki-Steindl stagnationism). Certain restrictions on the fiscal policy function ensured certain types of stability in low-dimensional versions of model of that paper (Hannsgen 2014, 16-17). We do not formally state or assume those here. We seek a reduced (self-contained) set of differential equations, to which we will add stochastic terms. So far, we have three equations that appear in each reduced model: p, u, and g. The Minsky and Shackle models add different variables to this core.

4 Shackle Closure

Here we consider a differential equation for wealth that is specific to this model and introduce a model of desired net investment that can be impacted by the stochastic process for expectations. Our wealth concept for the consumption function includes liquid assets only, namely the sum of high-powered money and government bills,

$$w = b + \mathcal{M}. \tag{13}$$

We note that given our definition of liabilities $\ell = \mathcal{M} + b$, we could likewise state this equation $w = \ell$. We assume that the K-sector, as it accumulates assets, keeps the ratio of these two assets constant, trading bills for money or vice-versa with the P sector if necessary, implying

$$b = \alpha_{\ell b}\ell, \tag{14}$$

for some $0 < \alpha_{\ell b} < 1$. The framework of the model would be consistent with a wide range of alternative assumptions regarding the structure of asset demand. The income of the K sector, on the other hand, includes after-tax interest and after-tax profits net of depreciation,

$$y_K = (1 - \tau)(\pi - \delta + ib). \tag{15}$$

Desired capitalist consumption expenditures are equal to a linear function of K-sector income and K-sector wealth,

$$c_{Kd} = \chi_{y_K} y_K + \chi_w w, \tag{16}$$

for $0 < \{\chi_{yK}, \chi_w\} < 1$. Total desired consumption expenditures are equal to the sum of of the variables in the above equation and the disposable income of the W sector,

$$c_d = c_{Kd} + y_W, \tag{17}$$

as workers are presumed to consume all their disposable income. Capitalist consumption adjusts toward its desired level at a finite rate:

$$\dot{c_K} = -\alpha_u(c_K - c_{Kd}). \tag{18}$$

The stock-flow identity for the public sector P reflects wage payments, interest payments, and tax receipts from the gross income of the other two sectors,

$$df = p + ib - \tau\frac{y_W + y_K}{1-\tau}. \tag{19}$$

This equation says that the sectoral balance df is equal to the sum of (1) public expenditures p, (2) a fixed rate of interest times government debt, ib; and (3) the negative of the tax rate times the sum of wages and unearned income.

The flow-of-funds equation for the K-sector sets the net accumulation of financial assets equal to the financial balance of the sector.

$$\dot{w} = y_K - c_K - s(1-\tau)(g + \delta + c_K) - gw. \tag{20}$$

This equation shows that the rate of accumulation of liquid assets in this sector is equal to after-tax net income minus household consumption, with an adjustment to take out profits on within-sector sales, which do not affect the sector's net acquisition of financial assets. The last term adjusts for changes in the capital stock, which appears in the denominator as a normalizing factor, and can be derived in a matter akin to that conducted with respect to ℓ in Appendix A.1,

Our presentation of Table 1 indicated that the Shackle closure of this section had three sectors: P (public), K (capitalist), and W (worker). Since the balance of the worker sector is zero, the balances of the other two sectors must add up to zero. That is, the rate of accumulation of private sector wealth is equal to the rate at which the public sector issues net liabilities.

$$\dot{w} = \dot{\ell} \tag{21}$$

Hence, one of the two flow-of-funds differential equations can be dropped from the model as redundant. We retain the equation for ℓ in the reduced system, dropping the equation for w. Lastly, we posit the desired net investment function

$$g_d = auton + \kappa(1-\tau)\pi + \varphi\arctan[\gamma(u - u_T)], \tag{22}$$

where the law for the stochastic process auton is given by

$$auton = S \sim \text{self-similar stochastic process.} \tag{23}$$

where g_d is desired net investment, auton is a Keyensian animal spirits term, and arc tan is arc tangent. As in Hannsgen (2014), we have added a nonlinearity of the type proposed by Kaldor (1940) to the investment function used by Dutt (1984) and by Rowthorn (1982), which Dutt (2017) attributes to Steindl (1952). As a first cut, we can assume our self-similar stochastic process for auton is of the type characterized by Levy-stable increments (Mandelbrot 1997; Embrechts and Maejima 2002). That is, the analogous discrete-time process would have shocks drawn from a Levy-stable distribution. Such processes sometimes possess the time series properties of long memory, i.e., one in which dependence on the past never entirely fades out and infinite variance increments. In non-Gaussian cases, such a process generates pathways with discontinuities. Nonetheless, the pathways are right-continuous (Embrechts and Maejima 2002, p. 9). Processes with these properties are well known to capture key time series characteristics of many financial price and total return series–an appropriate property for a term that is to capture animal spirits in a simulation model.

To simulate the Shackle model's dynamics, we need a minimal set of state variables. The deterministic part of the Shackle closure can be reduced to 4 differential equations in 4 variables — p u, ℓ, and g — and the exogenous driving process S.

Equation (1) from the previous section becomes the equation for p in the Shackle system. Equation (2) from the same section serves as the equation for u. To complete the process of solving for a self-contained system of 4 differential equations, we perform the following substitutions. To obtain a differential equation for ℓ one can substitute equation (19) into equation (3) in the previous section. Finally, in order to obtain an equation for g, we substitute the stochastic process (23) into the desired net investment function (22) and then substitute the latter into the g adjustment function (12) from the previous section.

The solutions for other variables follow, once the system is solved. In particular, the pathway for w is implied by the pathway for ℓ and accounting identity (21). Also, the pathway for c is implied by the pathways for u, g, and (4). The pathway for b is implied by equation (14). The pathway for for $\mathcal{M}$ can be determined from (13) and the pathways for b and w. Equations (6) and (15) can now be used to find the pathways for y_W and y_K, given (7) and (8) and the pathways for u and b. The pathway for df can be found using pathways for p, b, y_w and y_K.

5 Minsky Closure

During a simulation interval [0, t], shocks hit the system in our second closure at times

$$\tau_1, \tau_2, \tau_3, ...\tau_N, \tag{24}$$

which are determined by the outcome of simulating a nonhomogeneous Poisson process N(t) (Ross 1997, section 5.4.1). The pathway of a Poisson process over time resembles a step function, with dx(t)/dt=0 almost everywhere and a finite number of right-continuous jumps (Ross 1997). The number of jumps that has occurred in the interval [0, t] is given by the value of the process. The mean arrival rate of jumps depends on a parameter λ, which is the financial fragility variable in the model. For example, if $\lambda = .5$, one expects one jump every 2 = 1/.5 years. We will assume that the value of $\lambda(t)$, known as an intensity function, depends on the ratio of loans to reserves with positive sign and on bank balance sheets and on the difference between capacity utilization's actual and target levels with negative sign,

$$\lambda(t) = f_\lambda\left(\overbrace{u - u_T}^{-}, \overbrace{\frac{lo}{\mathcal{M}}}^{+}\right). \tag{25}$$

The first argument in the function is a measure of the state of the business cycle, and the second is the loans-reserves ratio for the banking system. Considering an interval of time in our process, the number of jumps n in a time interval [t, t+s] N(t+s)-N(t) is Poisson distributed with mean m(t+s)-m(t), where m(t) = integral from zero to t of λ(s). One example of a possible specification for f_λ would be the function

$$\lambda(t) = e^{-coeff1 \cdot \tan[(u-u_T)2\pi - \pi] + coeff2 \frac{lo}{\mathcal{M}}}. \tag{26}$$

Let T be the time elapsed since the most recent crisis. That is,

$$T = t - \tau_{-1}, \tag{27}$$

where t is current date and τ_{-1} is the date of the most recent crisis. Hence, our system of differential equations becomes nonautonomous. Formally, we can include T as a state variable in our system by stating a differential equation for the passage of time,

$$\dot{T} = 1. \tag{28}$$

This transformation allows one to treat the system as it were autonomous for computational purposes. We explain what happens on crisis dates below.

As we saw in Section 2, this closure adds a financial sector F that makes bank loans *lo* and accepts uninsured deposits *dep*. (In the U.S. case, these might for example be in accounts with balances in excess of certain limits at federally chartered depository institutions, or shadow bank instruments. For now, assets and incomes remain in real terms, normalized by the capital stock. The assets of the F-sector include loans to the K sector and high-powered money, which in this model are interpreted as bank reserves accepted by the central bank. Deposits are the F-sector's sole liability. The balance sheet can be characterized by the identity

$$\mathcal{M} + lo = dep. \tag{29}$$

We modify the definition of profits in this section to include interest on business loans as an expense,

$$\pi = su - i_L lo. \tag{30}$$

Our desired investment function in the present section's Minsky closure contains a "memory of crisis" variable, along with the linear profit-rate and nonlinear capacity utilization terms used in our Shackle closure of Section 4 above. (See references there).

$$g_d = \kappa(1-\tau)\pi + \varphi \arctan[\gamma(u - u_T)] - \varrho S_{\tau_{-1}}^{-T}, \tag{31}$$

for

$$S_t \sim \begin{cases} F(\cdot), & \text{for } t \in \{\tau_1, \tau_2, \tau_3, ...\tau_N\}, \\ 0, & \text{otherwise}, \end{cases} \tag{32}$$

for $F(\cdot)$, from which the size of the financial loss is drawn. We assume the measure F possesses infinite variance and is totally skewed (has support that is bounded on one side). Specifically, it has positive support. On crisis dates, the memory-of-crisis term in this function jumps to a negative value that is proportional to the shock. As time passes, this term decays at an exponential rate. This assumption ensures that as Minsky observed, post-crisis eras are characterized by financial caution, which inevitably erodes over time. The income identity for the KH sector is

$$y_{KH} = (1-\tau)(ib + i_{dep}dep + div) \tag{33}$$

Once again, we establish expressions for desired total and KH-sector consumption, this time using KH subscripts for our five-sector model,

$$c_d = c_{KHd} + y_W, \tag{34}$$

$$c_{KHd} = \chi_{y_{KH}} y_{KH} + \chi_w w. \tag{35}$$

The adjustment equation for capitalist household consumption is the same except for different notation

$$\dot{c_{KH}} = -\alpha_u(c_{KH} - c_{KHd}). \tag{36}$$

The equation for w in this model is

$$\dot{w} = y_{KH} - c_{KH} - gw. \tag{37}$$

The sector comprising wealthy households then acquires financial assets at a rate equal to its disposable income minus its consumption spending. In normalized terms, the rate of change of w is slowed or accelerated by the pace of net investment.

The K-sector's financial activity must respect the following flow-of-funds identity:

$$\dot{lo} = (1-\tau)[(\pi-\delta) - s(g+\delta)] - lo \cdot g. \tag{38}$$

This identity states that the F-sector satisfies all residual funding needs through changes in loans, given investment spending. Once again, we have netted out profits from intrasectoral goods purchases. Hence, the effects of lender's risk are encompassed within the equation for net investment g above, where we have included an after-tax profit term partly for this purpose. Once again, we have included a final term that adjusts appropriately for growth in the normalizing variable k.

Capitalist household (KH sector) balance sheets feature wealth made up of bonds and deposits,

$$w = b + dep. \tag{39}$$

Differentiating this identity by time and rearranging,

$$\dot{dep} = \dot{w} - \dot{b}. \tag{40}$$

Here, our money variable is a bank reserves variable. Reserves are determined so as to maintain a balance between assets and liabilities, leading to our equation for high-powered money in the Minsky model,

$$\dot{\mathcal{M}} = \dot{dep} - \dot{lo}. \tag{41}$$

We assume that the public sector provides these non-interest-bearing reserves as a matter of the privileges of the financial sector, as long as the system operates within a legal framework.

Issuance of bills accounts for the remainder of public sector liability issuance after the net rate of issuance of high-powered money has been determined by financial sector demand,

$$\dot{b} = \dot{\ell} - \dot{\mathcal{M}}. \tag{42}$$

The interest rate on deposits stands in a ratio to the interest rate on government securities, which is a form of liquidity preference variable,

$$i_{dep} = \mu i, \tag{43}$$

Liquidity preference depends on (1) the ratio of the most liquid household asset to total assets, (2) a shock-memory term similar to the one in the net investment function, and (3) capacity utilization relative to a reference level,

$$\mu = f\left(\overbrace{\rho}^{-}, \overbrace{\varrho S_{\tau_{-1}}^{-T}}^{+}, \overbrace{u - u_T}^{-}\right), \tag{44}$$

where

$$\rho = \frac{b}{b + dep}. \tag{45}$$

This function resembles a liquidity-preference curve that jumps upward on crisis dates and moves continuously downward at almost all times. The loan rate is a set as a markup on the deposit rate.

$$\imath_L = m_L i_{dep}, \tag{46}$$

with $m_L > 1$. It seems reasonable to assume that expectations are widely shared in the financial world and in business; hence, households' liquidity preference, valuations of bank loans, and deposits are all directly hit by all shocks generated by the Poisson process we have just described. The impact effects effects of each shock can be stated as follows:

$$\Delta lo = -S_t, \tag{47}$$

$$\Delta dep = -\vartheta S_t, \quad \text{and} \tag{48}$$

$$\Delta \mu = \xi S_t. \tag{49}$$

The shock has implications for bank reserve holdings by the bank balance sheet identity, as follows:

$$\Delta \mathcal{M} = -\Delta lo + \Delta dep. \tag{50}$$

The instantaneous effect on KH sector wealth is given by

$$\Delta w = \Delta dep \tag{51}$$

In essence, this equation says that banks are immediately immediately given nonearning reserves in an amount sufficient to bring their net worth back to zero–in essence a bailout in return for accounting recognition of loan losses and allowing funds to be withdrawn at will from large, uninsured accounts. We have obtained the equation by taking first-differences of the relevant identity. The worker (W) and financial (F) sectors have zero financial balances. The balances of the KH, K, and P sectors respectively sum to zero

$$\dot{w} - \dot{lo} - \dot{\ell} = 0 \tag{52}$$

This relation allows us to eliminate the equation for lo leaving 7 differential equations. The Minsky model of this section has independent differential equations for p, u, b, c_{KH}, g, T, and w, including variables common to both closures, along with the crisis dates τ_i and shock realizations S_{τ_i}. The variable T merely keeps track of time elapsed since the most recent crisis and allows one to express a nonautonomous system as if it were autonomous. It will add a zero root. Analytically, one can first consider the dynamics of a reduced system without an equation for T, then consider the impact of putting it back in the system as a

dynamic variable. Standard results state that the local existence of a solution for a nonautonomous system of ordinary differential equations is guaranteed by local continuity (Sánchez 1968, section 5.3), while local asymptotic stability is guaranteed if the linear part of the system is time-homogeneous and its characteristic polynomial is stable (Theorem 5.3.1, p. 100). If the solution referred to in the first result is unique, it is an uninteresting trivial solution.
We summarize the flows of funds in this model in the transactions-flow matrix of Table 4.

6 Alternative Inflation and Distribution Closures

Keynes (1936, 257-271) considered changes in the nominal wage and their possible effects in dislodging the economy from a low-employment equilibrium.We wish to add this essential characteristic of a monetary economy in this section. Also, in the Kaleckian tradition, it seems reasonable to allow changes in the markup variable m that reflect changing market power.

The implications for the model developed so far include the appearance of $s = \frac{(m-1)}{m}$ in the equations for desired production of types of goods. In those equations s appears explicitly. Changes in the wage unit ω and the markup m also impact the real burden of debt and the real value of financial assets. Since ω and m do not appear in the equations for these financial stocks explicitly, they must be slightly revised. For convenience we use the same normalized stock variables. These now represent ratios of nominal amounts to the nominal value of the capital stock, $pr.k$. For example, letting B be bills in nominal terms, b = B/(pr.k). We recall the pricing formula,

$$pr = m\omega. \tag{53}$$

We assume for the sake of simplicity that the technical coefficient for labor productivity is equal to one. We would expect m to remain greater than one, so that gross profitability is positive. Log differentiation yields

$$\frac{\dot{pr}}{pr} = \frac{\dot{m}}{m} + \frac{\dot{\omega}}{\omega}. \tag{54}$$

First, we add differential equations for ω and m,

$$\dot{\omega} = \alpha_\omega (u - u_d)^5 \omega, \quad \text{and} \tag{55}$$

$$\dot{m} = -\alpha_m (u - u_T)^5. \tag{56}$$

Both of these adjustment equations embody slow adjustment to deviations from normal capacity utilization that are not large. Our markup adjustment equation posits countercyclical dynamics based on the notion that the number of competitors in an industry is countercyclical for very large deviations from “normal” u. The wage equation is based on standard ”Phillips curve” logic, in which for example booming demand is likely to result in rising nominal wages—though we use a differing nonlinear specification. Such adjustments are a linchpin of the political-economy approach of, for example, P. Skott.

As mentioned, in a model with changing wages and prices, the flow-of-funds equations in each model must reflect changes in normalized stocks that arise from the effects of changes in the price level. The flow-of-funds equations in each closure becomes:

Basic Model:

$$\dot{\ell} = df - \ell\left(\frac{\dot{\omega}}{\omega} + g + \frac{\dot{m}}{m}\right), \tag{57}$$

Shackle closure:

K sector:

$$\dot{w} = y_k - c_K - s(1-\tau)(g+\delta-c_K) - w\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m} + g\right), \tag{58}$$

Minsky closure:

KH sector:

$$\dot{w} = y_{KH} - c_{KH} - w\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m} + g\right), \tag{59}$$

K sector

$$\dot{lo} = (1-\tau)(\pi-\delta) - s(1-\tau)(g+\delta) - lo\left(g + \frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m}\right). \tag{60}$$

Asset demands in the Shackle model have been assumed interest-inelastic, so no change is needed to incorporate inflation's effects.

7 Conclusion

We have proposed two models that can be simulated using standard techniques. Relative to Hannsgen (2014), we have omitted dynamic effects of unemployment rates and unemployment compensation per se. Of the two fiscal policy functions, we retain only the one that found to be stabilizing in that paper. We have added Shackle and Minsky closures that incorporate exogenous and endogenous expectational dynamics, respectively. The closures allow for the non-independence of shocks across sectors of the economy. We have also added real-asset and debt-burden effects and standard SFC accounting. Finally, we have made the nominal wage into a state variable. In the Shackle closure, we have implemented a differing structure of asset demand that maintains balance between types of financial assets.

In their ongoing work the authors seek to incorporate gendered aspects of the economy, to work toward simulations and analytical approaches to the dynamics, to add nonlinear consumption,to consider a wider range of mechanisms of distributional dynamics, and to include consumer and trading debt.

A Equations List

A.1 Basic Equations

Differential equations:

$$\dot{p} = -[\alpha_{pp}(p - p_T) + \alpha_{up}(u - u_T)], \tag{61}$$

$$\dot{u} = -\alpha_{uu}(u - u_d), \tag{62}$$

$$\dot{\ell} = df - g\ell, \tag{63}$$

$$\dot{k} = gk, \tag{64}$$

$$\dot{g} = -\alpha_g(g - g_d), \tag{65}$$

$$\dot{c_K} = -\alpha_{c_K}(c_K - c_{Kd}), \tag{66}$$

for $0 < u_T < 1$, $p_T > 0$, $b > 0$, $0 < u < 1$, $p > 0$, $k > 0$, and $c_K > 0$. The model can be reduced to a system of 5 differential equations, with the rest of the pathways derivable from algebraic equations (74)-(80) and the 6th differential equation (64). The 5 state variables in that system are p, u, ℓ, g, and c_K. The model requires appropriate initial conditions for all state variables. The initial state vector must obey the second set of inequalities and the static equations below that appear in the appropriate model.

The equation for government liabilities ℓ can be derived as follows. Let capitalized variables represent quantities in goods. All variables in the basic model and closures are normalized by k. For example, $df = \frac{DF}{k}$. Start with

$$\ell = \frac{L}{k}. \tag{67}$$

Log-differentiate to get

$$\frac{\dot{\ell}}{\ell} = \frac{\dot{L}}{L} - \frac{\dot{k}}{k}. \tag{68}$$

The equation for the government (P-sector) deficit in real terms is

$$\dot{L} = DF. \tag{69}$$

Using this expression in the previous equation in growth rates yields

$$\frac{\dot{\ell}}{\ell} = \frac{DF}{L} - \frac{\dot{k}}{k}. \tag{70}$$

Recall the definition

$$g = \frac{\dot{k}}{k}. \tag{71}$$

This changes our growth-rate equation to

$$\frac{\dot{\ell}}{\ell} = \frac{DF}{L} - g. \tag{72}$$

Multiply through by ℓ and recall our notation for normalized variables $x = \frac{X}{k}$ to obtain the result

$$\dot{\ell} = df - g\ell, \tag{73}$$

which is the differential equation for ℓ as stated above.

Algebraic equations:

$$u = c + g + \delta, \tag{74}$$

$$u_d = \delta + c_d + g_d, \tag{75}$$

$$y_W = (1-\tau)[p + (1-s)u], \tag{76}$$

$$s = \frac{m-1}{m}, \tag{77}$$

$$m = \bar{m}, \tag{78}$$

$$i = \bar{\imath}, \tag{79}$$

$$\ell = b + \mathcal{M}. \tag{80}$$

A.2 Additional equations for Shackle Model

$$\pi = su, \tag{81}$$

$$w = \ell, \tag{82}$$

$$b = \alpha_{\ell b}\ell, \tag{83}$$

$0 < \alpha_{\ell b} < 1$,

$$y_K = (1-\tau)(\pi - \delta + ib), \tag{84}$$

$$c_d = c_{Kd} + y_W, \tag{85}$$

$$c_{Kd} = \chi_w w + \chi_{y_K} y_K, \tag{86}$$

$$df = p + ib - \tau\frac{y_W + y_K}{1-\tau}, \tag{87}$$

$$\dot{w} = y_K - c_K - s(1-\tau)(g + \delta + c_K), \tag{88}$$

$$g_d = auton + \kappa(1-\tau)\pi + \varphi \arctan[\gamma(u - u_T)], \tag{89}$$

$$auton = S \ \sim \ \text{a self-similar stochastic process}, \tag{90}$$

A.3 Additional equations for Minsky version

$$g_d = auton + \kappa(1-\tau)\pi + \varphi\arctan[\gamma(u-u_T)] - \varrho S_{\tau_{-1}}^{-T}, \tag{91}$$

$$\pi = su - i_L lo, \tag{92}$$

$$c_d = c_{KH\,d} + y_W, \tag{93}$$

$$c_{KH\,d} = \chi_{y_{KH}} y_{KH} + \chi_w w \tag{94}$$

$$df = p + ib - \tau\Big(\frac{y_W + y_{KH}}{1-\tau} + \pi - \delta\Big), \tag{95}$$

$$div = i_L lo - i_D dep, \tag{96}$$

$$w = b + dep, \tag{97}$$

$$\dot{w} = y_{KH} - c_{KH}, \tag{98}$$

$$c_d = c_{KH\,d} + y_W, \tag{99}$$

$$c_{KH\,d} = \chi_w w + \chi_{Y_{KH}} y_K, \tag{100}$$

$$\dot{\mathcal{M}} = \dot{dep} - \dot{lo}, \tag{101}$$

$$\dot{b} = \dot{\ell} - \dot{\mathcal{M}}, \tag{102}$$

$$\dot{dep} = \dot{w} - \dot{b}, \tag{103}$$

$$\dot{T} = 1, \tag{104}$$

$$y_{KH} = (1-\tau)(ib + div + i_D dep), \tag{105}$$

$$i_D = \mu i, \tag{106}$$

$$i_L^d = m_L i_D, \tag{107}$$

$$i_L = -\alpha_{i_L}(i_L - i_L^d), \tag{108}$$

$$\dot{lo} = (1-\tau)(\pi-\delta) - s(1-\tau)(g+\delta), \tag{109}$$

$$\Delta lo = -S_t, \tag{110}$$

$$\Delta dep = -\vartheta S_t, \tag{111}$$

$$\Delta\mu = \xi S_t, \tag{112}$$

$$lo + \mathcal{M} = dep, \tag{113}$$

$$S_t \sim \begin{cases} F(\cdot), & \text{for } t \in \{\tau_1, \tau_2, \tau_3, ...\tau_N\}, \\ 0, & \text{otherwise}, \end{cases} \tag{114}$$

$$\tau_i \sim \text{Poisson}[\lambda(t)], \tag{115}$$

$$\begin{aligned} \lambda(t) &= f_\lambda\left(\overbrace{u-u_T}^{-}, \overbrace{\frac{lo}{\mathcal{M}}}^{+}\right) \\ &= e^{-coeff1\cdot\tan[(u-u_T)2numpi - numpi] + coeff2\frac{lo}{\mathcal{M}}}, \end{aligned} \tag{116}$$

$$\Delta\mathcal{M} = -\Delta lo + \Delta dep, \tag{117}$$

$$\rho = \frac{b}{b + dep}, \tag{118}$$

$$\mu = f\left(\overbrace{\rho}^{-}, \overbrace{S_{\tau_{-1}}^{-T}}^{+}, \overbrace{u - u_T}^{-}\right), \tag{119}$$

for boundary condition $T = t - \tau_{-1}, m_L > 1, \eta < 1, \mu > 1, \vartheta < 1, P(S_t > 0) = 1$, and $\mu < 1$.

A.4 Altering the model to include a variable wage unit and markup

$$pr = m\omega, \tag{120}$$

$$\dot{\omega} = \alpha_\omega (u - u_d)^5 \omega, \quad \text{and} \tag{121}$$

$$\dot{m} = -\alpha_m (u - u_T)^5. \tag{122}$$

Replace the appropriate equations in sections above with:

Basic Model:

$$\dot{\ell} = df - \ell\left(\frac{\dot{\omega}}{\omega} + g + \frac{\dot{m}}{m}\right), \tag{123}$$

Shackle closure:

K sector:

$$\dot{w} = y_k - c_K - s(1-\tau)(g + \delta - c_K) - w\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m} + g\right) \tag{124}$$

Minsky closure:

KH sector:

$$\dot{w} = y_{KH} - c_{KH} - w\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m} + g\right), \tag{125}$$

K sector:

$$\dot{lo} = (1-\tau)(\pi - \delta) - s(1-\tau)(g + \delta) - lo\left(g + \frac{\dot{\omega}}{\omega} + \frac{\dot{m}}{m}\right). \tag{126}$$

These flow-of-funds equations are derived in a way similar to the example at the end of Appendix A.1.

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