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Abstract: The main objective of this paper is to incorporate the technological asymmetries between countries in the formal structure of the so-called Kaldor-Pasinetti model of growth and distribution. We will name such a model as *Kaldor-Pasinetti-Verspagen Growth-Model*. Our basic contribution for the literature of post-Keynesian models of growth and distribution is to redefine Kaldor´s technical progress function to incorporate the technological gap in the determination of the natural rate of growth. Such incorporation will make possible for such class of models to generate uneven development between countries, at least for mature economies, that is, economies where all labor force is employed in the modern or capitalist sector. Since in such models, income distribution is the adjusting variable between natural and warranted rate of growth, one important result of our model is that income distribution between wages and profits is a non-linear function of the level of technological gap: below some threshold level of technological gap, profit-share will be reduced with the reduction of technological gap; above such threshold level, hover, the opposite effect occurs. Another important contribution of this article is to make a general formulation of the saving function, incorporating in the same model the contributions of both Kaldor and Pasinetti. From this general formulation, we can make different closures for the general model, which will allow the analysis of the implications of different assumptions about saving behavior over the income and wealth distribution in the balanced-growth path of mature economies that operate with different levels of technological gap.

Keywords: Uneven Development, Post Keynesian Economics, Technological progress.

JEL-Code: E12; O11; O14.

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1. Introduction

Modern growth theory makes the distinction between the proximate and the ultimate causes of economic development (Maddison, 1988). The proximate causes are those immediately responsible for the object in question; while the ultimate causes are the ones distant in time and are the base causes, that is, the background determinants or the origin of the phenomena. In the context of growth theory, the proximate causes are those directly related to the level of income *per capita*, the existing quantity of physical and human capital, the availability of natural resources, the efficiency in the use of productive resources and the level of technical and scientific knowledge existent at a point in time. On the other hand, the ultimate causes refer to the reasons why countries have distinct availabilities of productive factors and, therefore, different levels on income per capita. Among the ultimate causes are geography, institutions, income distribution and macroeconomic policy regimes (Ros, 2013, p. 15-17).

Regarding the proximate causes, the different theories of economic growth can be separated in two large groups. The first consists of the set of theories developed from the seminal works of Solow (1956), which can be referred to as the *neoclassical approach*[1]. This approach argues that the fundamental limit for long term economic growth is given by supply side constraints. Specifically, these models consider that the long-term real growth is determined by the rate of accumulation of factors of production and by the rate of technical progress. Demand is only relevant to explain the level of capacity utilization but has no impact on the determination of the rate of its expansion. In the long run, Say's law is valid and supply (the availability of factors of production), determines aggregate demand.

In the neoclassical perspective, the supply side factors are the determinants of the long run growth tendency of capitalist economies. Aggregate demand is responsible only for the fluctuations around the long run tendency, which economists call the economic cycle. That is, the essence of the neoclassical approach is that the long run growth tendency is independent of aggregate demand, and only deviations occur during economic cycles.

The second group is formed by a set of theories developed from the extension of Keynes's principle of effective demand, presented in the General Theory of Employment, Interest and Money (Keynes, 1936). The principle of effective demand, which states that the level of

[1] In the neoclassical approach we also include the endogenous growth theories because in these theories the constraints to growth are determined in the supply side, which leaves no role for aggregate demand in determining the pace of economic growth.

employment is determined by aggregate demand, was originally designed for an economy anchored in the Marshallian short run. It was up for Keynes's disciples, more specifically, Roy Harrod, Joan Robinson and Nicholas Kaldor, during the 1950 and 1960's, to extend this principle to the long run, in which the stock of capital, population and production techniques change over time. This generation of authors is referred to as the Post-Keynesian School, or the Cambridge School, as most of these scholars taught at Cambridge University in the United Kingdom.

The models built by these authors contain a common Keynesian idea: investment determine savings. Therefore, entrepreneurs' decisions to invest is fundamental in explaining the long-term economic growth.

The theories developed by the Cambridge School were originally thought to explain the growth of developed or industrialized economies. These economies, on the other hand, have some fundamental characteristics.

The first is that these are *mature economies,* that is, economies that have already completed their industrialization processes in which all the labor force existing in the traditional or subsistence sector was transferred to the modern or capitalist sector. In this situation, labor supply is not perfectly elastic for the capitalist sector as in the initial stages of industrialization. The result is that real wages are not determined by the reproduction costs of labor force. In this sense, long term economic growth is bounded by *the natural rate of growth*, which consists in the sum of the rate of growth of labor force and the rate of growth of labor productivity.

The second fundamental characteristic of mature economies is that they *operate within technological frontier*. Thus, their productive structure incorporates the most advanced, state-of-the-art, production techniques, resulting in goods and services with the highest possible value added per-capita. In this sense, growth of labor productivity results necessarily from technological progress - and from its incorporation in machines and equipments- instead of resulting from the process of imitation or importing of already existing technologies.

Focusing on mature economies and the absence of technological asymmetries in the structure of the growth models developed by the Cambridge School in the 1950 and 1960, rendered the Keynesian inspired growth models incapable of explaining the *uneven economic performance* existing between developed and developing economies.[2]

Indeed, economic growth during the last 200 years was extremely unequal. Different groups of countries experienced large and systemic differences in the rates of growth of labor productivity and income *per capita*.

Table 1 - Average growth rate of gross domestic product per capita of selected countries.

Country	Period	Initial Per Capita GDP (1985 US$)	Final Per Capita GDP (1985 US$)	Average Growth Rate (%)
Japan	1890-1990	842	16,144	3
Brazil	1900-1987	436	3,417	2.39
Canada	1870-1990	1,330	17,070	2.15
Germany	1870-1990	1,223	14,288	2.07
United States	1870-1990	2,244	18,258	1.76
China	1900-1987	401	1,748	1.71
Mexico	1900-1987	649	2,667	1.64
United Kingdom	1870-1990	2,693	13,589	1.36
Argentina	1900-1987	1,284	3,302	1.09
Indonesia	1900-1987	499	1,200	1.01
Pakistan	1900-1987	413	885	0.88
India	1900-1987	378	662	0.65
Bangladesh	1900-1987	349	375	0.08

Source: Barro e Sala-I-Martin (1995). Authors own elaboration.

The differences in income per capita observed reflect, in the first place, the existence of *technological asymmetries* between countries, that is, the fact that countries find themselves in the technology frontier, while others are lagging, and some, far behind. Secondly, these

[2] The neoclassical growth theory, either in the exogenous growth models such as in Solow (1956), or in the endogenous growth models for example in Romer (1990), are incapable of satisfactorily explaining the problem of uneven development precisely for not including asymmetries in technology and productive structure in the formal models. The Neoclassical growth models, by assuming that technology is a public good and that productive structure is irrelevant for economic growth, are incapable of explaining the long-term persistent differences in the rate of growth of income per capita between countries. See Ros (2013) and Oreiro (2016).

differences reflect the existence of asymmetries in the industrialization process, i.e., the existence of industrialized, industrializing, and non-industrialized countries. These asymmetries in the industrialization process led some countries to have an economy specialized in the production and exporting of primary products, while others have a diversified economy- thus, capable of exporting a great variety of manufacture products with an elevated degree of technological intensity.

The main objective of this paper is precisely to incorporate the *technological asymmetries* between countries in the formal structure of the so-called Kaldor-Pasinetti model of growth and distribution. We will name such a model as *Kaldor-Pasinetti-Verspagen Growth and Distribution Model*. Our first contribution for the literature of post-Keynesian models of growth and distribution is to *redefine Kaldor´s technical progress function* (Kaldor, 1957) to incorporate the *technological gap* (See Verspagen, 1993) in the determination of the natural rate of growth. Such incorporation will make possible for such class of models to generate uneven development between countries, at least for mature economies in the sense of Lewis (1954), that is, economies where all labor force is employed in the modern or capitalist sector. Since in such models, income distribution is the adjusting variable between natural and warranted rate of growth, one important result of our model is that income distribution between wages and profits is a non-linear function of the level of technological gap: bellow some threshold level of technological gap, profit-share will be reduced with the reduction of technological gap; above such threshold level, hover, the opposite effect occurs.

Another important contribution of this article is to make a general formulation of the saving function, incorporating in the same model the contributions of both Kaldor (1956) and Pasinetti (1962). From this general formulation, we can make different closures for the models, which will allow the analysis of the implications of different assumptions about saving behavior over income and wealth distribution in the balanced-growth path of mature economies that operate with different levels of technological gap.

This paper is organized in 5 sections, including the introduction. In section 2 will be presented the basic structure of the Kaldor-Pasinetti-Verspagen growth and distribution model. The Kaldorian closure of the model will be presented in section 3, and the Pasinettian closure in section 4. A brief evaluation of the Kaldor-Pasinetti-Verspagen growth and distribution model will be made in section 5.

2. Growth models in mature economies: the Kaldor-Pasinetti-Verspagen growth model[3]

In this article, we analyze the determinants of growth in a *mature economy*, that is, an economy that has already gone through the process of industrialization and all the available labor force in the subsistence sector was transferred to the modern industrial sector. In this situation, labor supply for the capitalist sector is not unlimited, but it is constrained by the long-term population growth. In the short and medium run, labor force can grow at a faster rate than the population growth if changes in labor's working time or in the participation rate. Nevertheless, in the long run both the working time and the participation rate are constant, in such way that the growth of labor supply is only determined by the rate of population growth.

The potential rate of growth in this economy is determined by the so-called *natural rate of growth*, which results from the sum of the growth rate of population and the growth rate of labor productivity. Technical progress is, broadly speaking, embodied in new machinery and capital equipment. Therefore, the growth rate of labor productivity is determined by the rate in which the capital stock per worker is growing, with this relation being expressed by the equation known as the technical progress function. This endogeneity of technical progress allows us to categorize this model as an endogenous growth model.

A mature economy is not necessarily at the technology frontier. Therefore, the technical progress function developed in this article considers the positive and negative effects of the technological gap over the rate of growth of labor productivity.

In the goods markets, we assume that supply is inelastic, thus, the income distribution between wages and profits would be the adjusting variable between the natural rate of growth and the rate of growth compatible with macroeconomic equilibrium between savings and investments, which is called in the literature as the *warranted rate of growth*.

The model presented in this article is developed from the pioneering contributions of Kaldor (1956, 1957) and Pasinetti (1962) for solving the Harrod-Domar dilemma (See Harrod, 1939). Indeed, the fundamental conclusion of the Harrod-Domar growth model is showing that the achievement of a balanced growth path with full employment of the labor force is possible but highly improbable. Therefore, capitalist economies should present an unstable growth path, alternating between periods of accelerated growth rates followed by sharp contractions in the level of economic activity and employment.

[3] This section is based on Oreiro (2016, chapter. 3; 2018, chapter 3)

The incompatibility of these models with the historic experience of developed capitalist economies between 1950 and 1973 led Nicholas Kaldor and Luigi Pasinetti to formulate models in which the long run growth paths are stable and characterized by the full employment of the labor force. However, it required the development of a new theory of income distribution, in which the wage and profit shares are the adjusting variables between the decision to save and to invest.

The importance of this new theory of income distribution was to establish a second mechanism[4] by which investments can determine savings. Indeed, Keynes demonstrated in the General Theory that an exogenous increase in investment would generate an equivalent increase in savings via the multiplier effect. Kaldor and Pasinetti, on the other hand, argued that changes in investment always generate an equivalent increase in savings due to the effects on distribution between wages and profits. In this theory, profits and wages have different marginal propensities to save. Therefore, an increase in the level of investment results in an increase in the profit share of total income, which then causes an increase of total savings due to the higher marginal propensity to save out of profits.

2.1. Production techniques and the natural rate of growth

We will consider an economy producing a single homogeneous good (for instance, wheat) that serves for consumption and investment. The production function is a fixed coefficients type. Regarding labor, we assume that it is a homogenous input - workers have the same skills and qualifications. Fixed capital, on the other hand, is made of machines and equipments produced at different points in time, thus embodying different levels of technical knowledge. In this sense, equipments and machines from different harvests have different levels of productivity. Aggregating different types of capital is a difficult task, for this reason we thus abstract the heterogeneity between capital goods, assuming a homogenous capital stock.

Output, at time *t*, is given by the Leontief production function:

$$Y = min(aL; uvK) \qquad (1)$$

In this setting, factors of production are complementary, with no possibility for substituting inputs due to changes in relative factor prices.

[4] The first mechanism consists of the investment multiplier presented in macroeconomics textbooks.

Where Y is output, $\bar{Y}$ is potential output[5], L is the labor force employed, K capital employed, a is the average labor productivity, $u = \frac{Y}{\bar{Y}}$ is the capacity utilization rate, $v = \frac{\bar{Y}}{K}$ is the potential output-capital relation.

Capital and labor inputs are used efficiently if the following condition is met:

$$Y = aL = uvK \qquad (2)$$

From equation (2), we can derive the amount of labor firms are willing to employ. Indeed, from the first part of equation (2) and solving for L, we find:

$$L = \frac{1}{a} Y \qquad (3)$$

Equation (3) shows that the quantity of labor that firms are willing to employ is proportional to the level production.

Applying the natural log in equation (3) and differentiating the expression with respect to time, we obtain:

$$\hat{L} = \hat{Y} - \hat{a} \qquad (4)$$

Where $\hat{L}$ is the growth rate of employment, $\hat{Y}$ is the growth rate of output and $\hat{a}$ is the rate of growth of labor productivity. In equation (4) employment will increase (decrease) in time if the growth rate of output is higher (lower) than the growth rate of labor productivity.

As we are considering a mature economy, we assume that the growth rate of employment is equal to the growth rate of the labor supply.[6] If η is the growth rate of the labor force, we have:

$$\hat{Y} = \eta + \hat{a} \qquad (5)$$

Equation (5) shows that the rate of output growth is equal to the sum of the rate of growth rate of labor force and the growth rate of labor productivity. The left side of the

[5] Potential output is represented by the maximum amount of output that can be produced when firms are operating with a degree of capacity utilization equal to the desired or normal level. It is important to note that this level is not necessarily full capacity utilization, as firms ought to maintain some degree of idle capacity for strategic reasons.

[6] This doesn't imply that the economy is operating in full employment, but only that the unemployment rate is constant over time. Indeed, the unemployment rate can be expressed by $U = \frac{N-L}{N} = 1 - \frac{L}{N}$, where L is the total labor employed and N is the size of the labor force. If labor force and employment grow at the same rate, U will be constant over time, but at a level which can be higher or lower than full employment scenario.

equation is an upper limit to the growth rate of output. If it deviates to a higher growth rate, employment will grow faster than the supply of labor, in such way that unemployment will converge to zero. In this sense, the economy will face a shortage of workers to sustain this rate of growth- therefore this is an unsustainable rate of growth in the long run. On the other hand, if the economy is growing at a lower rate, employment will increase at a lower rate than the supply of labor, thus unemployment will converge to 100%. Clearly, both trajectories are unsustainable.

For an economy to present a balanced growth rate, i.e., where the rate of employment remains constant over time, it is necessary that the economy grows at a rate given by equation (5). The rate of output growth that allows for a balanced trajectory is given by the natural growth rate g_N in equation (6).

$$g_N = \eta + \hat{a} \tag{6}$$

2.2. Technical progress function

In the conventional or neoclassical growth theory, labor productivity growth can be separated into two parts. First, by an increase in the capital stock per worker, that is, in capital intensity; Secondly, from advances in the so-called state-of-the-art – the level of technical knowledge available in a given point in time. This distinction is possible in an economy which technical progress is disembodied from new machines and capital equipment. However, a large portion of technical progress is embodied in capital goods, thus, it is nearly impossible to distinct the growth of labor productivity attributed only to an increase in capital intensity or to improvements in the state-of-the-art technologies.

Expressing the growth rate of labor productivity as a function of the growth rate of the capital stock per worker, as in Kaldor (1957). We have:

$$\hat{a} = \alpha_0 + \alpha_1 \hat{k} \tag{7}$$

Where $\hat{k}$ is the growth rate of the capital stock per worker.

The technical progress function can be understood in two ways. The first one, an increase in capital intensity, which imply the introduction of more advanced technology, will increase labor productivity. The second way is that most of technical innovations which lead to productivity increases require a higher stock of capital per worker - for instance a more complex equipment or more mechanical power – which means that the most part of technical progress is embodied in new machines and equipment.

The term α_0 in equation (7) represents the share of technical progress that is autonomous in relation to the capital accumulation. This parameter represents the disembodied share of technical progress due to, for example, changes in organization which increases production without the need for additional investment.

The term $\alpha_1\hat{k}$, on the other hand represents the share of technical progress that is embodied in machines and equipments, induced thus by the capital accumulation. The coefficient α_1 represents the sensitivity of the growth rate of labor productivity to changes in the rate of growth of the stock of capital per worker. This coefficient captures the capacity of transforming a flow of new ideas and knowledge into productivity increases via investments.

This coefficient of induction from the productivity growth to the capital accumulation, depends on the technology gap- that is, the distance between the level of technological knowledge of an economy in relation to the technological frontier.[7]

What is the relation between the technology gap and the coefficient of induction of technical progress? At some degree, countries behind the technology frontier can rapidly increase productivity simply by means of imitating and by learning the methods of production employed in countries at the technology frontier. It means that, to a certain extent, presented in the next parts, the growth rate of labor productivity in an economy behind the frontier is a positive function with respect to the same frontier. As imitation involves, at least partially, in purchasing machines and equipment produced in countries which are at the technology frontier, the coefficient of induction of technical progress will depend on the size of the technology gap. In this sense, countries behind the technological

[7] This idea was inspired in Alexander Gerschenskron, which, in his classical work *Economic Backwardness in Historical Perspective,* argues that "Assuming an adequate endowment of usable resources, and assuming that the great blocks to industrialization had been removed, the opportunities inherent in industrialization may be said to vary directly with the backwardness of a country. Industrialization always seemed the more promising the greater the backlog of technological innovations which the backward country could take over from the more advanced country.". It is clear that the advantage of backward countries in industrialization from the possibility of using technological innovation already developed in advanced countries, clearly stating the existence of a technological gap between advanced economies and the ones behind. The use of these new technologies, on the other hand, is given by the possibility of "But all these superficialities tend to blur the basic fact that the contingency of large imports of foreign machinery and of foreign know-how, and the concomitant opportunities for rapid industrialization with the passage of time, increasingly widened the gulf between economic potentialities and economic actualities in backward countries (Gerschenskron, 1962, p.8).

frontier can benefit from positive leakages of knowledge from countries leading in technology.

A remark is that this positive coefficient of induction of technical progress and the technology gap depends on the learning and absorbing capacity in each country. The absorbing capacity, on the other hand, depends on the very distance a country is in relation to the technology frontier. For instance, if the distance is too far, the country will not be capable to benefit from the positive leakages from leading countries. In this case, the induction coefficient of the technical progress is a decreasing function of the technological gap.[8]

Defining $G = \frac{T_N}{T_S}$ as the technology gap,[9] where T_N is the level of knowledge at the technological frontier and T_S the level of technological knowledge of the lagging countries, we suppose, based on Verspagen (1993), that:

$$\alpha_1 = a_2 G e^{-\frac{G}{\delta}} \tag{8}$$

Where δ is a parameter representing the technological learning capacity of the economy in question (also referred as the absorption capacity).

In equation (8) if the technology gap equals one, the coefficient of induction of the technical progress will be constant and equal to $a_2 e^{-\frac{1}{\delta}}$. Differentiating equation (8) with respect to G, we obtain:

$$\frac{\partial \alpha_1}{\partial G} = a_2 e^{-\frac{G}{\delta}} \left(1 - \frac{1}{\delta}(G)^2\right) \tag{9}$$

In equation (9), it is clear that $\frac{\partial \alpha_1}{\partial G}$ is positive only if $\left(1 - \frac{1}{\delta}(G)^2\right) > 0$, that is, if $\delta > G^2$. This means that the coefficient of induction of the technical progress function will only be an increasing function of the technology gap if, and only if, the square of the technology gap is smaller than the parameter representing the absorbing capacity of the economy.

The growth rate of capital stock per worker is:

$$\hat{k} = \hat{K} - n \tag{10}$$

[8] More information available in Verspagen (1993, p. 126-130).
[9] The minimum value for the technological gap is G=1

Where $\widehat{K}$ is the growth rate of the capital stock (henceforth g_K).

Substituting (8) and (10) in (7), we have:

$$\hat{a} = \alpha_0 + \left(a_2 G e^{-\frac{G}{\delta}}\right)(g_k - n) \tag{11}$$

Equation (11) is the technical progress function. We can observe that the growth rate of labor productivity depends on the rate of growth of the capital stock, on the growth rate of labor productivity and the technology gap.

2.3. Capital accumulation and balanced growth

Based on the economic efficiency condition (equation 2), output for a given point in time can be expressed as:

$$Y = uvK \tag{12}$$

Equation (12) expresses output as a function of the existing capital stock, the level of capacity utilization and the potential output-capital relation.

Taking the logarithmic time derivatives, we arrive at the expression of output growth:

$$g_Y = g_u + g_v + g_K \tag{13}$$

Where g_Y is the growth rate of output, g_u the growth rate of level of capacity utilization, and g_v the growth rate of the potential output-capital.

The term g_v in equation (13) is the rate of growth of capital productivity, which depends on the technological progress.[10] When capital productivity is increasing over time, technological progress is thus referred as capital saving. In the case where capital productivity is falling over time, technological progress is referred as capital intensive. Finally, in the case where capital productivity is constant over time, technical progress is thus called neutral.

Empirical evidence presented in Kaldor (1957) point to the stability of the capital-output over time, therefore, for a neutral technical progress.

The term g_u in equation (13) is the rate of change in the level of capacity utilization. In the long run, the only sustainable value for g_u is zero, meaning that the level of capacity

[10] A detailed explanation is available in Bresser-Pereira (1986)

utilization will be constant over the balanced growth trajectory. It does not imply full capacity utilization or that firms are operating with excess over planned capacity. In this case, the trajectory in which u is constant over time is compatible with underutilization of productive capacity.

Assuming that technical progress is neutral and that the economy is in balanced growth trajectory, we have:

$$g_Y = g_K \qquad (14)$$

Where the rate of growth of output is equal to the rate of growth of the capital stock.

In the long run balanced growth path of this economy output must grow at a rate equal to the natural rate, given by the sum of the rate of growth of labor force and the rate of growth of labor productivity. Substituting (11) in (6), we find:

$$g_N = \left(1 - \left(a_2 G e^{-\frac{G}{\delta}}\right)\right) n + \alpha_0 + \left(a_2 G e^{-\frac{G}{\delta}}\right) g_K \qquad (15)$$

Finally, substituting (14) in (15) and solving for g_N, we arrive at:

$$g_N = n + \frac{\alpha_0}{\left(1 - \left(a_2 G e^{-\frac{G}{\delta}}\right)\right)} \qquad (16)$$

Equation (15) is the final expression where the natural rate of growth depends on the rate of growth of the labor force and on the technology gap. From (16), the natural rate of growth is an increasing function of the technology gap if $\delta > G^2$.

2.4. The macroeconomic equilibrium, investment and saving.

We now turn to the demand side of this economy. For simplicity, we assume a closed economy without government activities. The objectives of these assumptions are to make the model as simple as possible. This will allow us to devote our attention to the relations between aggregate supply and demand over a balanced growth trajectory.

A fundamental characteristic of Keynesian growth models is the assumption that planned investment spending is autonomous in relation to planned savings. This is based in an economy which the banking system is sufficiently developed and capable of providing the necessary liquidity for investments to take place[11]. Indeed, increasing investment can be performed prior to increases in saving, enabled by the expansion of credit.

Based on this reasoning, we assume that planned investment (I) is exogenous and given by:

$$I = \bar{I} \tag{17}$$

Total savings (S) is the sum between savings of firms (S_F) and households (S_H). Most Keynesian growth models assume families as homogenous agents. Nevertheless, we can subdivide it into two: capitalists and workers.

Capitalists are households which income derives only from firms' shared profits (Pasinetti, 1962). These are, in Kaldor's (1966) terminology, hereditary barons, with very little relation with the traditional industrial capitalist that simultaneously managed and owned capital. Capitalists here are rentiers, that is, households for which income consists solely of the ownership of capital stock.

On the other hand, workers are households whose income originate from wages and salaries, as well as part of the firms' shared profits. This class includes not only blue-collar workers but also workers directly or indirectly related to management.

According to Pasinetti (1962), capitalists' propensity to save (s_c) is higher than of workers (s_w) propensity to save. The justification of this assumption is not sufficiently convincing. The difference between the propensity to save from both classes seems to be based on the old Ricardian conception according to which wages tend towards the labor force subsistence level. Under these conditions, capital accumulation by workers would be impossible. Therefore, it would be more appropriate to assume workers propensity to save as zero, but this is not what the author assumes. In fact, in his model, workers are capable of accumulating capital, because their propensity to save is larger than zero. Why would then the workers propensity to save be lower than the baron class? Pasinetti never provided a convincing explanation.

A more reasonable assumption is that the propensity to save depends on the type of income (Kaldor, 1966). Specifically, Kaldor (1966) assumes a propensity to save from profits higher than the propensity to save from wages. This difference does not depend on

[11] A comprehensive explanation is provided in Paula (2014, p. 98-120).

the preferences of individuals of each class, but on the very nature of entrepreneurial income. According to the author, in a world where economies of scale prevail, firms are force by competition to either expand or fail, as productivity increases are associated with cost reductions and increases in production over time. In this sense, an elevated coefficient of retained earnings is a necessary condition for the survival of firms in the long run, because retained earnings are an essential source of primary financing to the expansion of firms. Households, on the other hand, are not subject to this same competitive pressure and this is a reason for their slower pace of wealth accumulation.

According to the exposition above, we have the following set of equations:

$$S = S_F + S_H \tag{18}$$

$$S_H = S_W + S_C \tag{19}$$

$$S_W = s_w(W + P_w) \tag{20}$$

$$S_C = s_c P_c \tag{21}$$

$$S_F = P_R \tag{22}$$

$$P_R = \varepsilon P \tag{23}$$

$$P = P_R + P_D \tag{24}$$

$$P_D = P_W + P_C \tag{25}$$

$$P_w = k_w P_D \tag{26}$$

$$P_C = (1 - k_w) P_D \tag{27}$$

Where W are wages, P_w the total profits distributed for the workers, P_c the amount of profits received by the capitalists, P_R the amount of earnings retained by firms, P_D the distributed earnings to capitalists and households, P is the total sum of profits, ε the coefficient of retained profits, k_w the share of the capital stock owned by the workers.

From the savings of firms, substituting (23) in (22), we have:

$$S_F = \varepsilon P = \varepsilon \frac{P}{K} K = \varepsilon r K \tag{28}$$

Where *r* is the profit rate, is given by:

$$r = \frac{P}{K} = \frac{P}{Y}\frac{Y}{\bar{Y}}\frac{\bar{Y}}{K} = huv \qquad (28.1)$$

In Equation (28.1), the profit rate is expressed as the product between the profit share of income (h), the degree of productive capacity utilization (u) and the potential output-capital relation.

Substituting (26) in (20) and remembering that $P_D = (1-\varepsilon)P$, we find:

$$S_W = s_w W + s_W k_w (1-\varepsilon) rK \qquad (29)$$

Finally, substituting (27) in (21):

$$S_C = s_c(1-k_w)(1-\varepsilon)rK \qquad (30)$$

Adding equations (28), (29) and (30), we arrive at the aggregate savings function:

$$S = \{\varepsilon + (1-\varepsilon)[s_w k_w + s_c(1-k_w)]\}rK + s_w W \qquad (31)$$

Equation (31) is the *general expression for planned savings*. Planned savings depends on i) the profit rate and the size of capital stock; ii) total wages; iii) distribution of capital stock between capitalists and workers; iv) the coefficient of retained earnings; v) the propensity to save of capitalists; and vi) the propensity to save of workers.

From equation (31) we can derive four closures of the savings function: the Ricardian, Harrodian, Pasinettian and Kaldorian.

The Ricardian savings function is derived from equation (31) by setting the propensity to save of workers equal to zero and that firms distribute all profits. In this case we have:

$$S = s_c rK \qquad (31.1)$$

The Harrodian savings function, on the other hand, is derived by setting the propensity to save of workers equal to the propensity to save of capitalists ($s_w = s_c = s$), and that firms, as in the previous case, distribute all profits. Thus, we have:

$$S = s\,P + s\,W = s\,Y \qquad (31.2)$$

The Passinetian case is obtained when the propensity to save of capitalists is higher than the propensity to save of workers, and that firms distribute all profits. It can be observed in equation (31.3).

$$S = \{[s_w k_w + s_c(1 - k_w)]\}rK + s_w W \qquad (31.3)$$

Finally, the Kaldorian case refers to a situation where firms retain part of the profits, but the propensity to save of capitalists is equal to the propensity to save of workers. In this case we have:

$$S = \{\varepsilon + (1 - \varepsilon)s_F\}rK + s_F W \qquad (31.4)$$

In this last case, the propensity to save of profits is given by $(s_P = \{\varepsilon + (1 - \varepsilon)s_F\})$, which is larger than the propensity to save of households (s_F).

The equilibrium condition in the goods market is given by:

$$\bar{I} = S \qquad (32)$$

Dividing both sides of equation (32) by K, we have:

$$g_K = \sigma \qquad (33)$$

Where $\sigma = \frac{S}{K}$ is savings as a share of the capital stock.

In other words, the goods market is in equilibrium when the rate which entrepreneurs wish to increase the capital stock is equal to the desired savings as a share of the stock of capital.

3. Balanced growth in the Kaldorian Model

The fundamental characteristic of the growth models presented in this section is that the economy operates with in the level of capacity utilization desired by entrepreneurs over the balanced growth trajectory. It means that the idle capacity of the economy corresponds to the one planned by firms in the economy. We assume then, in this type of model:

$$u = u^n \qquad (34)$$

Where u^n is the normal degree of utilization of the productive capacity.

The reason for this assumption relies on the fact that fluctuations in sales are not perfectly predictable, with peaks at certain times. Therefore, this degree of excess capacity is necessary for firms to meet unanticipated demand peaks.

The level of capacity utilization is equal to the normal or desired level. Therefore, supply and demand adjust is performed not by changes in the variation of the level of capacity utilization, but by changes in the markup, that is, changes in the relation between costs of production and prices. Facing a fall in the expected demand, firms will reduce the markup, keeping the degree of capacity utilization equal to the normal. Similarly, when facing an anticipated increase in demand, profit margins will increase.

Considering labor cost as the only direct cost of production, the profit margin (h), can be expressed as:

$$h = \frac{p - c}{p} = \frac{p - wa_0}{p} = 1 - \frac{w}{p} a_0 \qquad (35)$$

Where p is the goods' price, c unit cost of production, w is the nominal wage rate, $a_0 = \frac{L}{Y}$ is the required unit labor cost, that is, the quantity of labor necessary to produce one unit of output.

Income in this economy is divided between wages and profits:

$$pY = wL + P \qquad (36)$$

Dividing equation (35) by pY, we obtain:

$$1 = \frac{w}{p} a_0 + \frac{P}{pY} \qquad (37)$$

The last term of the right side of the equation (37) is ratio between the number of profits and the monetary income of this economy. This ratio is denominated by the share of profits to income. Thus, the other term of the right side of the equation can only be the wage share.

Substituting (35) in (37), we conclude that:

$$h = \frac{P}{pY} \qquad (38)$$

Equation (38) shows that the profit margin of firms determines, on the aggregate, the profit share. Therefore, changes in the profit margin will result in changes in the profit share (and in the wage share) on the aggregate income.

In the Kaldorian growth model, there are no differences between the propensity to save of different households, but only between households (available income) and firms (profits). As presented in the last section, the distinction between propensities to save does not require the existence of a wealthy class of hereditary barons with high propensity to save. The truth is different propensities to save of firms and households depend on the competitive pressure firms face in a market with the presence of dynamics and static economies of scale.

In this sense, the Kaldorian model consists of the following system of equations:

$$g_N = n + \frac{\alpha_0}{\left(1 - \left(a_2 G e^{-\frac{G}{\delta}}\right)\right)} \tag{16}$$

$$\sigma = \{\varepsilon + (1 - \varepsilon)s_F\}huv + s_F(1 - h)uv \tag{31.1}$$

$$u = u^n \tag{34}$$

$$g_K = g_Y \tag{14}$$

$$g_Y = g_N \tag{14.1}$$

$$g_K = \sigma \tag{33}$$

Assuming, for simplification, that the coefficient of retained earnings is equal to one, the warranted rate of growth is given by:

$$g_w = u^n v(s_F + h(1 - s_F)) \tag{39}$$

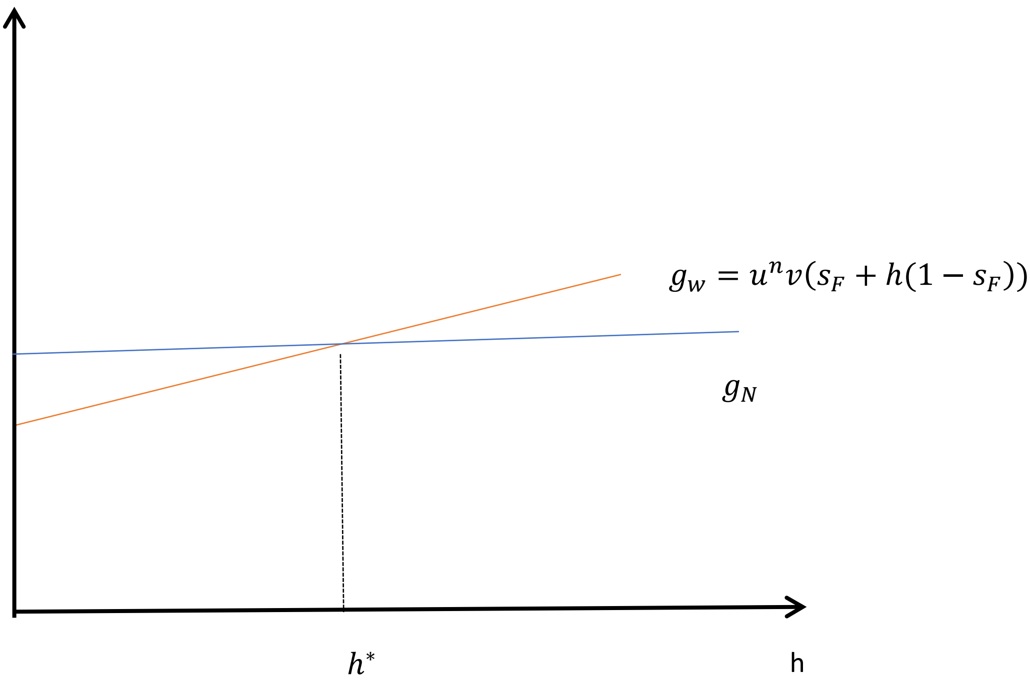
In equation (39), given the normal rate of capacity utilization, the relation of potential output-capital and the propensity to save of households, the warranted rate of growth is an increasing function of the profit share. This results from the fact that the aggregate propensity to save is given by the weighted average of the share of profits, the propensity to save of firms (equal to 1) and the households propensity to save. Therefore, an increase in the share of profits will redistribute income from the sector with a lower propensity to save (households) to the sector with the higher propensity to save (firms). The result is an aggregate increase in the propensity to save and, therefore, in the warranted rate of growth.

In the Kaldorian model, the functional income distribution is the adjustment mechanism between the warranted rate and the natural rate of growth. Substituting (16) in (39) and solving for *h*, we have:

$$h^* = \left(\frac{1}{1-s_F}\right)\left\{\frac{n}{u^n v} + \frac{\alpha_0}{u^n v\left(1-\alpha_2 G e^{-\frac{G}{\delta}}\right)}\right\} - \left(\frac{s_F}{1-s_F}\right) \qquad (40)$$

Equation (40) determines the profit share for which the warranted rate of growth adjusts to the value of the natural rate of growth (figure 1).

FIGURE 1
Adjusting mechanism between the warranted rate of growth and the natural rate of growth



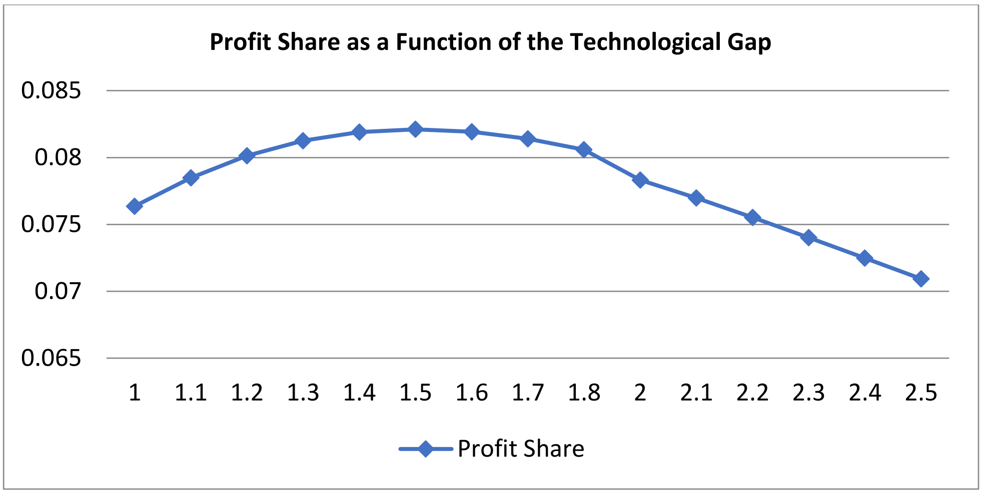
Source: Author's own elaboration

An important result of the Kaldorian model is that the functional distribution between wages and profits is a non-linear function of the technology gap. In this sense, it is possible to show that the profit share is an increasing function of the technology gap until it reaches the limit given by the learning capacity of the economy.

As an exemple, consider an economy which the labor force grows at 1.5% a year. Consider the coefficient α_0 which represents the disembodied part of technological progress equal to 0.015. Also suppose that $\alpha_2 = 0{,}9$ and that the parameter δ which represents the technological learning capacity (absorptive capacity) equals to 1,5. Supposing the normal rate of capacity utilization equals to 0.7, and that the potential output-capital is 0.5, and the propensity to save of households is 0.05.

The development of the profit share as a function of the technological gap is displayed in figure 2.

Figure 2: Share of profits as a function of the technological gap



Source: Authors´ own elaboration

In figure 2, we observe that as the technology gap is reduced, the share of profits initially increases until it reaches a maximum level where the technological gap equals the learning capacity (G= 1,5). From this point the share of profit falls until the technological gap is eliminated. This behavior of the profit share is related to the behavior of the natural rate of growth.

In this sense, for values superior to the learning capacity, the natural rate of growth is decreasing function of the technological gap. Thus, a reduction in the gap will increase the natural rate of growth and the growth rate of the capital stock over a balanced growth trajectory. The equilibrium in the goods market requires an increase in savings as a ratio of capital stock which requires a redistribution of income towards the sector with higher propensity to save- that is, firms. For this reason, the profit share should increase in a way to generate the additional savings required for the acceleration of the growth rate.

4. Balanced Growth in a Pasinettian Model.

Pasinetti's model (1962) it is assumed that the propensity to save from family units is differentiated, and the propensity to save from "capitalists" is greater than the propensity to save workers. The profit retention coefficient is assumed to be zero, so that all savings are made by "households".

In this context, the Pasinettian model consists of the following system of equations:

$$g_N = n + \frac{\alpha_0}{\left(1 - \left(a_2 G e^{-\frac{G}{\delta}}\right)\right)} \quad (16)$$

$$\sigma = \{[s_w k_w + s_c(1 - k_w)]\}r + s_w(1 - h)u^n v \quad (31.3)$$

$$u = u^n \quad (34)$$

$$g_K = g_Y \quad (14)$$

$$g_Y = g_N \ (14.1)$$

$$g_K = \sigma \ (33)$$

From (31.3) we know that the savings of capitalists as a proportion of the capital stock is given by:

$$\frac{S_C}{K} = s_c(1 - k_w)r \quad (41)$$

Dividing the left side of (41) by K_c and making use of the fact that, along the balanced growth trajectory, the capital stock of capitalists must grow at the same rate as the stock of aggregate capital, we get:

$$gk_c = s_c(1 - k_w)r \quad (42)$$

Since $k_c = (1 - k_w)$, we get:

$$r = \frac{g_N}{s_c} \quad (43)$$

The equation (43) is the "Cambridge equation", according to which the rate of profit along the balanced growth path is equal to the ratio between the natural growth rate and the propensity to save from capitalists. Note that in the Pasinetti model this result can be obtained without the restrictive hypothesis that $s_w = 0$

The share of profits in income along the balanced growth path will be given by:

$$h = \frac{g_N}{s_c u^n v} \quad (44)$$

Equations (43) and (44) show us that the greater the propensity to save capitalists, the lower the profit rate and the share of profits in income. This is a result compatible with Kalecki's famous aphorism, according to which "capitalists as a class earn what they spend."

As in previous cases, the share of profits in income will be a non-linear function of the technological gap, given its dependence on the natural rate of growth.

Replacing (33), (34), (14), (14.1) and (43) in (31.3) we obtain the share of the aggregate capital stock that is owned by capitalists. We get:

$$k_c^* = \left(\frac{s_c}{s_c - s_w}\right)\left[\frac{g^n - s_w u_n v}{g^n}\right] \quad (45)$$

For capitalists to exist as a distinct social class from workers, it is necessary that $k_c^* > 0$. A necessary and sufficient condition is that $g - s_w u_n v > 0$, That is, $s_w < \frac{g_n}{u^n v} = s_W^c$. In words: workers' propensity to save must be less than a critical value s_W^c, otherwise the "capitalists" will be eliminated from the system.

In the equation (45) we observed that the share of capitalists in the stock of aggregate capital will be constant along the balanced growth trajectory, that is, the distribution of wealth stock between capitalists and workers is stable in the long term. Since this trajectory is worth the Cambridge equation, so we have to r > g. This follows that r>g it does not imply increasing inequality in the distribution of income and wealth, contrary to what Piketty (2014) states.

5. Evaluation of the Kaldor-Pasinetti-Verspagen growth models

We proceed now to evaluate the ability of the growth models presented previously to explain the stylized facts observed in the historic development experience of capitalist economies. The essential point is to analyze the compatibility of the models presented and the divergence in the rates of growth of income per capita between countries.

In the models presented in this work, the long-term real rate of growth is determined by the natural rate of growth, which consists in the sum of the growth rate of the labor force and the growth rate of labor productivity. Labor productivity, on the other hand, is a function of the technology gap, that is, the distance between the level of technological knowledge of a country in relation to the technology frontier.

For levels of the technology gap below a certain threshold- which is determined by the capacity to absorb new technologies that the country has-, the rate of growth of labor productivity is a decreasing function of the technology gap, that is, countries relatively more distant from the technology frontier will present higher rates of productivity growth.

This result is compatible with the *catching-up* literature, where within certain limits, countries behind in technological development will present conditions to grow faster than those at the technology frontier. It is a result, up to a point, of the positive overflow of technical knowledge and scientific developments.

In the cases where a country is too distant from the technology frontier, i.e., beyond the threshold of the absorptive capacity of the economy, the country will not be able to benefit from the overflow of technological knowledge. In this scenario, the growth rate of labor productivity is less affected by knowledge overflows. Therefore, a large technology gap is associated with a reduction in the growth rate of productivity, and everything else constant, a reduction in the long-term real rate of output growth.

In this sense, the existence of different levels of technology gaps will lead countries to present different rates of natural growth- and, given the rate of population growth, the growth of output and income *per capita*. It thus follows that in this class of growth models, the divergence between the growth rates of income *per capita* results from technology asymmetries between countries.

What are the implications of the models presented in this article for the distribution of income and wealth? Regardless of the specification of the savings function, we saw that along the balanced growth path the functional distribution of income between wages and profits should remain constant. An interesting result is the so-called *Cambridge equation*, according to which in the balanced growth path the profit rate is equal to the ratio between the natural rate of growth and the propensity to save from profits. Being the propensity to save from profits, in general, less than one; it follows that the profit rate should be higher than the natural growth rate along the balanced growth path.

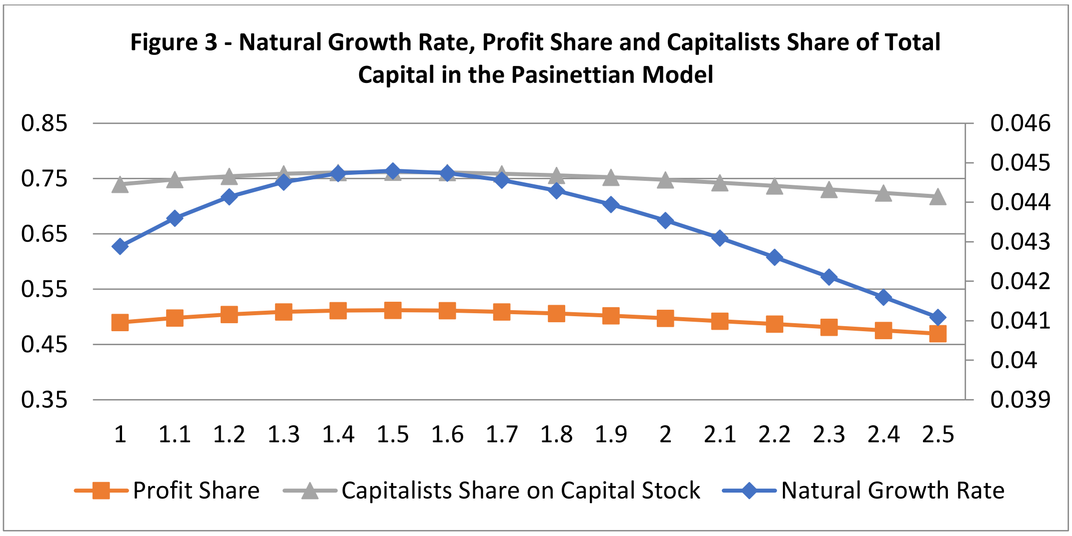
Recently the $r > g$ result has become the focus of the attention of economists and the general public due to Piketty's thesis (2014) that if the rate of return on capital is higher than the rate of growth of the output then there will be a tendency to increase inequality in the distribution of income and wealth over time.

Based on Pasinetti's model, we can see that Piketty's statement is wrong. In fact, the distribution of the wealth stock (capital) between "capitalists" and "workers" tends to remain stable in the long term, despite r > g. This does not mean, however, that the distribution of income and wealth along the balanced growth trajectory is not unequal or even extremely unequal. What Pasinetti's model shows is that capitalist economies do not have an inexorable tendency to increase income and wealth inequality.

The dynamics of income and wealth distribution will depend, fundamentally, on the technological gap. As has been seen before, both the functional distribution of income between wages and profits, and the distribution of capital stock between workers and capitalists depends on the natural rate of growth, which is largely conditioned by the technological gap. Thus, different levels of the technological gap will be compatible with different values for the distribution of income and wealth along the path of balanced growth.

For a better understanding of this point we will take a numerical simulation of the Pasinettian model. Consider savings in which the workforce grows at a rate of 1.5% p.a. The coefficient α_0 which represents the unbodied share of technological progress is equal to 0.015. Also suppose that $\alpha_2 = 0{,}9$ and that the parameter δ which represents the capacity of technological learning is equal to 1.5. Suppose that the normal degree of utilization of productive capacity is equal to 0.7, that the potential-capital output ratio is equal to 0.5. Finally, suppose that the propensity to save of capitalists is equal to 0.20 and that the propensity to save from workers is equal to 0.05.

The values of the natural growth rate, profit sharing in income and the fraction of wealth (capital) that is owned by capitalists can be seen in figure 3 below:



Source: Author´s own elaboration.

As shown in Figure 3, the natural growth rate, the share of profits in income and the fraction of wealth that is owned by capitalists depend non-linearly on the technological gap. For levels of the technological gap lower than absorptive capacity ($G < 1.5$), the share of profits in income and the fraction of wealth that is owned by capitalists tends to be increasing in the technological gap. This means that for $G < 1.5$, the farther a country is from the technological frontier, the greater the inequality in the distribution of income and wealth. It follows, therefore, that an important result of this model is that countries operating on the technological frontier tend to have a more equitable distribution of income and wealth than those behind – but not far behind – that border.

For levels of the technological gap greater than absorptive capacity ($G>1.5$), inequality in the distribution of income and wealth tends to fall as the technological gap widens.

Based on these results, we can conclude that countries that have intermediate values of the technological gap - that is, that are neither very close nor very distant from the technological frontier - tend to present greater inequality in the distribution of income and wealth than other countries.

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