WORKING PAPER 2215

The Impossible Quartet in a Demand Led Growth-Supermultiplier Model for a Small Open Economy

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May 2022

POST-KEYNESIAN ECONOMICS SOCIETY

The Impossible Quartet in a Demand Led Growth-Supermultiplier Model for a Small Open Economy♣

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Abstract: The aim of this paper is to investigate the long run sustainability of a growth path led by multiple non-creating capacity autonomous expenditures in a demand led-supermultiplier model for a small open economy. Using two different models the results show that it is impossible to have in the same model long-term economic growth driven by the non-capacity creating component of domestic demand, exogenous income distribution, long-run balance between productive capacity and aggregate demand and balance of payments equilibrium. Economic viability of the balanced-growth path demands growth to be led by exports, at least for small open economies.

Keywords: Post-Keynesian Economics, Growth and Distribution, Srafian Supermutiplier, Simulation Models

JEL Classification – E12, E37, P10

April 2022

♣ Paper prepared to be delivered at IV Workshop on New Developmentalism to be held at the Center for Studies on New Developmentalism of the São Paulo School of Business Administration, São Paulo, Brazil in the period of 25 to 26 of July, 2019.

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1. Introduction

The basis of the theory of demand led growth was settled by Nicholas Kaldor in a chapter of a book organized by Alain Barrère and published in 1988[1]. For Kaldor the *Principle of Effective Demand*, according to which the level of output can be expressed as a multiple of the level of autonomous demand, can be extended from the short to the long run. The starting point for this extension is the idea that means of production used in a capitalist economy are themselves goods produced within the system. If that is so, the "supply" of means of production should never be considered as a datum independent of the demand for then. In this framework, the fundamental economic problem is not the allocation of a given quantity of resources over the possible alternatives, but the determination of the rate of growth of these resources. If the supply of means of production was not a data for the system, then "(…) under the stimulus of growing demand capacity of all sectors will be expanded through additional investment, there are no long-run limits to growth on account of supply constraints" (Kaldor, 1988, p.157).

In the long run the growth rate of real output is thus determined by the rate of expansion of autonomous demand, *i.e.* the component of effective demand that is "financed out of capital – by borrowing, or by the sale of financial assets (…)" (Ibid, p.153) and so is exogenous to the level and/or the rate of change of economic activity. But what components of demand can be considered exogenous? According to Dejuán (2013) the autonomous demand includes "(a) autonomous consumption by households; (b) residential investment; (c) modernization investment by firms that's transforms the existing capacity, instead of expanding it; (d) real public expenditure; and (d) exports" (Ibid, p.141). It is easy to see that in this framework the most dynamic component of autonomous demand – that is the one with the higher rate of expansion - will set the pace of economic growth in the long run, since the share of all the other components in the composition of autonomous demand will fall to zero.

More recently, Freitas and Serrano (2015a) argued that in "fully adjusted position" the actual rate of capacity utilization must be equal to the "normal" or "desired" rate of capacity utilization, i.e., the rate of capacity utilization that allowed firms to earn "normal" or "long period" profits. To do so it is necessary that economic growth is led by the autonomous component of demand that does not create capacity. For then this

[1] Kaldor, N. (1988). "The role of effective demand in the short run and in the long run" In: Barrère, A. (Org.). The Foundations of Keynesian Analysis. Macmillan: London.

component is the autonomous consumption. They called their approach to the theory of demand led growth as “The Sraffian Supermultiplier” (hereafter SSM)[2] because (i) the combination of the simple Keynesian multiplier mechanism with the mechanism of accelerator for investment demand gives rise to a multiplier of autonomous demand that is higher than the traditional Keynesian multiplier; and (b) “the Sraffian approach is generally considered to be in a fully adjusted or long-period equilibrium situation in which actual and normal capacity are equal” (Dutt, 2018, p.2).

The SSM approach[3] was (weakly) criticized by Dutt (2018) and (strongly) by Nikiforos (2018). According to Dutt (2018) a “fully adjusted position” is compatible with other sources of autonomous demand growth like government expenditures, exports, worker consumption and investment driven by technological change. This means the SSM approach is not a general closure for demand-led growth models; but only one of the possible closures[4].

Nikiforos´s criticism is a little bit stronger. For him, the SSM approach had two main weakness. The first one is the assumption that normal level of capacity utilization is exogenous and thus independent of demand. The problem with this assumption is that it implies that “the role of demand vanishes, and the model becomes classical in the long run (…) “(Nikiforos, 2018, p.9)[5]. Once scale effects are taken into consideration, however, the normal level of capacity utilization becomes an endogenous variable, and a higher demand leads to a higher normal rate. In this setting, the “long-run state of the economy becomes path dependent” (Ibid, p.10) and the system does not converge to an exogenous and predetermined center of gravity anymore.

The second criticism regards to the stock-flow implications of a debt-financed autonomous expenditure[6]. In the SSM any debt-financed expenditure is considered to be

[2] The name supermultiplier comes from Harrod-Hicks dynamic export multiplier/supermultiplier (Dutt, 2018, p.2).

[3] This approach was also independently developed by Bortis (1997) and Dejuán (2005, 2013).

[4] One alternative closure is the Kaldor-Pasinetti closure, also called neo-keynesian closure, where income distribution is the adjustment variable that assures actual capacity utilization to be equal the normal level. See Dutt (2018) for alternative closures of growth and distribution models.

[5] The same criticism is made by Dávila-Fernandes, Oreiro and Punzo (2018) to the utilization of the SSM approach by Lavoie (2016, 2017), as an attempt to answer the criticism made by Skott (2010; 2016) to the neo-kaleckian growth models.

[6] To say that we will take the stock-flow implications of dynamics of government expenditures and exports does not mean that we are proposing a full SFC model. Our aim is just to see what the implications for international reserves and public debt of different rates of growth for exports and government expenditures are. This exercise is not a trivial one since the dynamics of the stocks of reserves and/or public debt can define if a growth path is sustainable or not in the long run.

autonomous. Debt financing generates an intrinsic dynamic for private or public debt and thus for debt-to-income ratios. Although these ratios should stabilize at some level in the long run; the growing financial fragility of balance sheets during the transition to the steady state may force families or even the government to reduce the rate of growth of their expenditures. Thus, autonomous expenditure stop being autonomous (Ibid, p.14), because expenditure decisions have become endogenous; to stabilize the debt-to-income ratios at some desired level. This line of criticism was partially endorsed by Brochier and Macedo e Silva (2018) for whom SSM approach "still do not properly account for the interactions between financial stocks and flows" (p.2)[7].

In the case of exports, however, this problem could not arise. Indeed, there is no limit to the continuous accumulation of a net financial position abroad because of a current account surplus due to a high growth rate of exports. As soon as the economy at hand continues to be a small open economy, its exports can be considered an autonomous expenditure and hence growth can be export-led. For economies like United States, Germany or China, however, growth of exports can´t be considered an exogenous variable due to the feedback effects of their growth rates over the growth rate of the rest of the world, and hence over the growth rate of their exports (Nikiforos, 2018, p.16).

What will happen with the SSM approach if we consider a small open economy with two sources of autonomous demand growth, one for domestic demand (government expenditures) and another for foreign demand (exports)? This question was firstly addressed by Bortis (1997). In the SSM model developed by Bortis, there are two sources of autonomous demand growth: government expenditures and exports. If government expenditures and exports grow at the same rate; then trend output and productive capacity will grow at the same rate of autonomous demand and trade account as well as government budget will be at balance (Ibid, p.155). But complications arise if exports do not grow at the same trend as government expenditures. If the growth rate of government expenditures is higher than the growth rate of exports; then chronic trade deficits and also

[7] Brochier and Macedo e Silva (2018) tried to overcome this limitation of the SSM approach by means of a simple *Stock-Flow Consistent Model* that preserves the essentials of this approach, "namely, the autonomous expenditure component, induced business investment and the Harrodian investment behavior through which firms react to the discrepancies between actual and desired utilization rates" (p.2). The problem with their solution is that it makes consumption expenditure a complete endogenous variable, except in the very short-run when household wealth – which is the basis for financing autonomous consumption expenditure - can be taken as a pre-determined variable. The accumulation of financial wealth through savings by households makes the autonomous component of consumption demand to increase by the endogenous workings of the system, making consumption expenditures a pure endogenous variable to the model. This result seems to be very far from the "essentials" of the SSM approach.

chronic government deficits will arise. The dynamic path of government debt and external debt may be unsustainable if steady-state values of the government debt-to-income ratio and external debt-to-income ratio are higher than some "normal" or "desired" level.

These considerations lead us to the conclusion that exports can be the only true component of autonomous demand, as already emphasized by Thirwall (2002); and also, to the conclusion that it is *impossible to have in the same model long-term economic growth driven by the non-capacity creating component of domestic demand, exogenous income distribution, long-run balance between productive capacity and aggregate demand and balance of payments equilibrium*. We name this result as the *impossible quartet* of the demand-led growth-supermultiplier model. This means steady growth can only be possible if it is of export-led type (and only for small open economies), as emphasized by the *developmental economics school of economic thought*, which is the theoretical basis of the growth strategy known as *new-developmentalism* (Bresser-Pereira, Oreiro and Marconi, 2015; Oreiro, 2018).

The objective of the present paper is to develop the argument presented above in a formal model of demand led growth-supermultiplier for a small open economy with two sources of autonomous demand (exports and government expenditures), taking into consideration the stock-flow implications of the dynamics of government and exports expenditures[8]. These implications were not formally addressed by Bortis (1997) and constitute a novel contribution for the literature of demand-led growth. As we will see through the paper, once we consider the stock-flow implications of autonomous demand growth in a small open economy; balanced growth path requires government expenditures trend growth to be determined by growth rate of exports. If government expenditures increase at a faster rate than exports than steady growth may be impossible due to the violation of the balance of payments constraint.

The paper proceeds as follows. Section 2 presents the structure of the demand-led growth-supermultiplier model for a small open economy with two sources of growth for autonomous expenditures, but without considering the stock-flow implications of the dynamics of public debt and net foreign asset position. **This is the demand led-growth-supermultiplier model type-T**.

[8] The general aim of this paper is not to use the SFC approach as presented by Godley and Lavoie (2007). We only care about consistency with public debt stocks and foreign exchange reserves. In this sense, some simplifications must be assumed for the simplicity of model resolution.

Section 3 presents the structure of the demand-led growth-supermultiplier model that incorporates public debt and the dynamics of the public debt to GDP ratio and foreign exchange reserves to GDP ratio. **This is the demand led-growth-supermultiplier model type-S.**

The configuration of steady-state equilibrium and the stability analysis of both models is analyzed in section 4. The **model type-T will be stable** for small values of marginal propensity to invest [with not too much acceleration as pointed out by Skott, Santos and Oreiro (2021); Franke (2021)]; otherwise, the steady-state equilibrium would be unstable. Regarding the steady-state configuration of the Type-S model it will be shown that in the case where $g_x < g_g$ - that is when the growth rate of exports is lower than the growth rate of government expenditures - **the steady-state level of the ratio of foreign exchange reserves to GDP is clearly negative**; which means that a balance growth path for the endogenous variables does not exist, since the growth of autonomous demand generates an unsustainable trade deficit that can´t be financed by reserves de-accumulation. A balanced growth path requires $g_x \geq g_g$.

The stability analysis of the Type-S Model is made in section 5 by employing simulation and numerical methods. The results of the simulations shown that if $g_x < g_g$; than the ratio of foreign exchange reserves to GDP ratio converges to a negative value, although the steady-state equilibrium is stable in the sense of Liapunov. If $g_x > g_g$; then the public debt to GDP ratio converges to a negative value, so the government sector becomes a net creditor of the private sector. Although this situation does not represent an economically impossible situation, it is an extremely unlike situation. The only possible and reasonable balanced growth path for the Type-S model is the one for which $g_x \geq g_g$.

As a conclusion, the analysis carried over the paper establishes what can be defined as the *Impossible Quartet of the Demand-Led Growth Supermultiplier Models:* It is impossible to have at the same model long-term economic growth driven by the non-capacity creating component of domestic demand, exogenous income distribution, long-run balance between productive capacity and aggregate demand and balance of payments equilibrium.

2. The Structure of the Demand Led-Growth-Supermultiplier Model Type-T

Let us consider a small open economy that produces a homogeneous output, which is an imperfect substitute for goods produced abroad. The availability of goods in the domestic market is given by the sum between domestic production and the actual value of imports. The aggregate demand for goods and services, in turn, can be decomposed in two parts. A first part, which we will call D, is constituted by those components of demand that are induced by the level of economic activity. In the economy in consideration the induced demand will consist of the sum between consumption and investment expenditures. The second part, which we will call A, is constituted by autonomous expenditures, that is by those components of aggregate demand that are largely independent of the level of economic activity. As stated earlier, the autonomous demand is composed of the sum between government spending and exports.

The goods market equilibrium condition is given by the following expression:

$$Y + \theta M = D + A \tag{1}$$

Where: Y is the level of real output; θM is the real value of imports; $\theta = \frac{EP^*}{P}$ is the level of real exchange rate; E is the level of nominal exchange rate; P^* is the price of imported goods nominated in foreign currency; P is the price of domestic goods nominated in domestic currency; M is the quantity of imports.

Without loss of generality, we will assume the validity of purchasing power parity, so that $\theta = 1$.

The demand for consumption is originated entirely from wages, that is, the propensity to consume from the profits is supposed to be equal to zero. The government charges an income tax rate equal τ on working income, while capital gains are exempt from taxation. In this way, the consumption demand is given by the following expression.

$$C = c_w.(1-\tau).(1-\pi).Y \tag{2}$$

where: c_w is the propensity of consume out of wages; π is the profit share; C is real consumption demand.

Following Freitas and Serrano (2015a), we will suppose that aggregate investment (*I*) is entirely done by private sector, being induced by the level of economic activity, as we can see in the equation bellow:

$$I = h.Y \tag{3}$$

Where: h is the average/marginal propensity to invest.

Autonomous demand is given by:

$$A = \bar{G} + \bar{X} \tag{4}$$

Where: $\bar{G}$ is the real government expenditures, $\bar{X}$ is the quantity of exports.

Finally, let us assume that the quantity of imports is entirely determined by the level of economic activity, as we can see in the following equation:

$$M = m.Y \tag{5}$$

Where: m is the marginal propensity to import.

Substituting equations (2)-(5) in (1) and solving for the level of economic activity we get:

$$Y = \sigma.A \tag{6}$$

Where[9]: $\sigma = \frac{1}{s+m-h}$ is the Harrod-Hicks Supermultiplier (HHS) of autonomous expenditures; $s = 1 - c_w(1-\tau).(1-\pi)$;

Taking total derivative in (6), we have:

$$dY = \dot{\sigma}.(\bar{G} + \bar{X}) + \sigma.(d\bar{G} + d\bar{X}) \tag{6a}$$

Dividing both side of (6a) by Y, and after some manipulation, we get the following equation:

$$g_Y = \frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x \tag{7}$$

[9] In the following we will assume that wage (profits)-share in income is exogenous to the model, being determined at the microeconomic level from the rate of *mark-up* fixed by firms over unit cost of production, in order to determine the sales price of their products. For more details see Oreiro (2016, chapter 5).

Where: $g_Y = \frac{dY}{Y}$ is the growth rate of real output; $\alpha = \frac{\bar{G}}{A}$ is the share of government expenditures in domestic demand; $g_\sigma = \frac{\dot{h}}{s+m-h}$ is the growth rate of HHS; $g_g = \frac{dG}{G}$ is the growth rate of government expenditures; g_x is the growth rate of exports.

Equation (7) above shows that the growth rate of real output is the weighted average of the growth rate of government expenditures and the growth rate of exports plus HHS growth rate.

For the growth path given by (7) to be sustainable in the long run is necessary for the growth rate of productive capacity to adjust itself to the growth rate of autonomous demand. The growth rate of capital stock is given by:

$$g_K = \frac{h}{v}.u - \delta \qquad (8)$$

Where: g_K is the growth rate of capital stock; $v = \frac{K}{Y_p}$ is the capital (K)/ potential output (Y_p)[10] ratio; $u = \frac{Y}{Y_p}$ is the level of capacity utilization and δ is the rate of depreciation of capital stock.

The rate of change of capacity utilization is given by [11]:

$$\dot{u} = u.\left(g_y - g_K\right) \qquad (9)$$

Substituting (7) and (8) in (9) we get:

$$\dot{u} = u.\left[\frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x + \delta - \frac{h}{v}.u\right] \qquad (10)$$

Following Freitas and Serrano (2015a, p.266), we will suppose that the adjustments of marginal propensity to invest are made in continuous time rather than by "jumps"; being compatible with the so-called *flexible accelerator model* for induced investment[12]. Thus, the marginal propensity to invest changes according to the equation below:

[10] Potential output is defined as the level of output achieved when firms as operating with a level of capacity utilization that is equal to the normal long-run value. So, we have $Y_p = \frac{K}{v}$.

[11] This differential equation can be obtained by taking logs at the definition of capacity utilization $u = \frac{Y}{K}$, and taking time derivatives of the resulting expression.

[12] The introduction of the flexible accelerator in the realm of the SSM approach is due to Dejuán (2013).

$$\dot{h} = h.\mu.(u - u_n) \tag{11}$$

Where μ is a parameter that measures the growth rate of the marginal propensity to invest to the deviation of the actual to the normal level of capacity utilization.

Finally, the rate of change of the share of government expenditures in autonomous demand is given by:

$$\dot{\alpha} = \alpha.(1 - \alpha).\left(g_g - g_x\right) \tag{12}$$

3. The Structure of Demand Led-Growth-Supermultiplier Model Type-S.

The model presented in the last section disregards two relationships between stocks and flows that arise internally in the model and are important for model dynamics. The first is related to the situation in which the rate of growth of autonomous government spending is higher than growth rate of exports (another autonomous component of demand). In this situation, we must see the dynamics of two stocks. The first is the ratio of public debt over GDP. For this path to be sustainable, it must converge to some steady state positive value[13]. The second variable whose dynamics deserves attention is the ratio of foreign exchange reserves over GDP. As well described in the literature[14], deficits in public budget are usually accompanied by trade deficits. In our model, we need to check for the three scenarios whether the ratio of exchange reserves over GDP converges to a positive value at steady state[15]. If this does not happen, we have an economically unsustainable growth regime.

The second case that needs to be verified is one in which the rate of growth of exports exceeds the growth rate of the autonomous component of public expenditure. In this case, we will have to monitor the path of the same previous variables (public debt

[13] In the case of the ratio reaching a negative value, economically the government would be a net creditor of other sectors in the economy. In other words, the government would have all its wealth allocated in liabilities of other sectors. This is an atypical and unlikely situation in the capitalist dynamics although it is possible to be obtained in pure mathematical terms.

[14] See Aristovnik and Djuric (2010); Bluedorn and Leigh (2011).

[15] We are considering a small open economy in which *capital mobility is equal to zero*, which means that balance of payments deficits must be financed by foreign exchange reserves de-accumulation up to the point where reserves are exhausted. On the other hand, there is no limit to the accumulation of foreign exchange reserves, which means that the ratio of foreign exchange reserves to GDP can reach any positive value. For small open economies *external imbalances are asymmetrical*: trade deficits can´t be financed forever, but trade surplus can be sustained for an indefinite period of time.

over GDP and foreign reserves over GDP) and verify if there is convergence to economically plausible values.

Thus, equation (16) bellow gives the dynamic of the ratio of public debt to GDP. Assuming that $\dot{B} = (G - T) + i.B$[16], where G is the public expenditures, T the taxation, i interest rate, B the stock of public bonds and $\dot{B}$ is the rate of change of the stock of public debt. Replacing $B/Y = b$, taking the total derivative and after some algebraic manipulation[17], we have:

$$\dot{b} = \alpha.(s + m - h) - \tau.(1 - \pi) + \left[i - \left(\frac{h.\mu.(u - u_n)}{s+m-h} + \alpha.g_g + (1 - \alpha).g_x\right)\right].b \quad (13)$$

The equation (18) gives us the following short-run relations:

$$\partial\dot{b}/\partial b = i - \left(\frac{h.\mu.(u - u_n)}{s+m-h} + \alpha.g_g + (1 - \alpha).g_x\right); \quad (13a)$$

$$\partial\dot{b}\ /\ \partial h = -\alpha + [\mu.(u - u_n).b.(s + m)]/(s + m - h)^2; \quad (13b)$$

$$\partial\dot{b}/\partial\alpha = s + m - h - \left(g_g - g_x\right).b \quad (13c)$$

$$\partial\dot{b}/\partial u = (h.\mu.b)/(s + m - h) \quad (13d)$$

Equation (13a) shows that if $i > \frac{h.\mu.(u-u_n)}{s+m-h} + \alpha.g_g + (1 - \alpha).g_x$, the rate of change of the bonds/GDP will be positive. Equation (13b) shows that derivative of the change of bonds/GDP in respect to propensity to invest will be zero or negative, since $[\mu.(u - u_n).b.(s + m)]/(s + m - h)^2 - \alpha \leq 0$. Equation (13c) shows that the partial derivative of the change of bonds/GDP in relation to α will only be positive if $s + m > h + \left(g_g - g_x\right).b$. In other words, in the short-run, there will only be an increase in normalized debt stock as a result of an increase in the share of autonomous government expenditure on autonomous demand; if the sum of the propensity to save and the propensity to import are greater than the sum of the investment share on output plus the difference between the growth rate of government expenditures and the growth rate of exports times the actual value of the ratio Bonds/GDP. Equation (13d) show that

[16] We are implicitly assuming that the monetary sterilization of the changes in the international reserves are being done by means of changes of voluntary deposits of commercial banks in the central banks (as it occurs in the European Central Bank), not by transactions of bonds in the interbank market.
[17] The complete mathematical steps are in the appendix.

derivative of the change of bonds/GDP in respect to capacity utilization is positive since $b > 0$ and $s + m > h$.

Following, we now present the dynamics of foreign exchange reserves. Based on the assumption that there is no capital mobility, and the only form of external financing is through accumulation (or de-accumulation) of foreign exchange reserves, we have the following identity $\dot{R} = X - \theta.M$. Where X represent exports, M imports, θ the real exchange rate, $\dot{R}$ represents foreign reserves variation. Assuming the PPP, we have $\theta = 1$. After some algebraic manipulation[18] we find equation (17).

$$\dot{r} = (1-\alpha).(s+m-h) - m - \left[\frac{h.\mu.(u-u_n)}{s+m-h} + \alpha.g_g + (1-\alpha).g_x\right].r \qquad (14)$$

Equation (14) gives us the following short-run relations:

$$\partial\dot{r}/\partial r = -\left[\frac{h.\mu.(u-u_n)}{s+m-h} + \alpha.g_g + (1-\alpha).g_x\right] \qquad (14a)$$

$$\partial\dot{r}/\partial u = (h.\mu.r)/(s+m-h) \qquad (14b)$$

$$\partial\dot{r}/\partial h = (\dot{h}.r).(s+m-h)^{-2} - (1-\alpha) \qquad (14c)$$

$$\partial\dot{r}/\partial\alpha = h - s - m - (g_g - g_x).r \qquad (14d)$$

The previous equations (14a), (14b), (14c) and (14d) allow us to infer the following relations: The rate of change of foreign exchange reserves must be negative in the short run. A raise in the investment share will produce a fall in the variation of exchange reserves over GDP ratio if $1 > \alpha \geq 0$. However, if $\alpha = 1$, the effect will vanish. Finally, an increase in the government expenditures share on autonomous demand will only impact positively the change in the ratio of exchange reserves over GDP if $h > s + m + (g_g - g_x).r$, *i.e.* the investment share must be greater than the sum of the propensity to save plus propensity to import plus the differential of growth rates of the different components of autonomous demand times the actual ratio of foreign external reserves over GDP. This specific case arises when $g_x > g_g$. In this condition, we have that $h + \Big(\underbrace{g_x - g_g}_{>0}\Big).r > s - m$. If $g_x = g_g$, we just have that $h > s - m$.

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[18] The complete mathematical steps are in the appendix.

4. The Steady State Equilibrium and Stability

4.1. The Steady State Equilibrium for the Type-T Model

The existence of a steady state for the economy at hand requires $\dot{\alpha} = \dot{h} = \dot{u} = 0$. Given the set of parameters, we must here investigate two possible scenarios. **The first one is $g_x > g_g$; the second is $g_g > g_x$.**

Table 1 – Steady State Values for Type-T Model

Steady-State Equilibrium (fixed point)	**Scenario 1:** $g_x > g_g$	**Scenario 2:** $g_g > g_x$
h^*	$= (g_x + \delta).\dfrac{v}{u_n}$	$= (g_g + \delta).\dfrac{v}{u_n}$
u^*	$u^* = u_n$	$= u_n$
α^*	$= 0$	$= 1$

4.2. The Stability Analysis of Type-T Model in Steady State

In the Scenario 1, where $g_x > g_g$ and $(u^*, h^*, \alpha^*) = [u_n, v.(g_x + \delta)/u_n, 0]$, we have the following Jacobian matrix:

$$\begin{bmatrix} \dot{u} \\ \dot{h} \\ \dot{\alpha} \end{bmatrix} = \underbrace{\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & 0 & 0 \\ 0 & 0 & J_{33} \end{bmatrix}}_{\mathbf{J}^*} . \begin{bmatrix} u - u^* \\ h - h^* \\ \alpha - \alpha^* \end{bmatrix} \quad (18)$$

And the following signals: $J_{12} < 0$; $J_{13} > 0$ and $J_{33} < 0$.

Proposition: Since $\mu > 0$, the fixed point of model T is locally stable if the bound $\mu < s + m - (g_x + \delta)$ and $s + m - (g_x + \delta) > 0$ are satisfied.

Observe that if the condition $\mu < s + m - (g_x + \delta)$ and $s + m - (g_x + \delta) > 0$ holds, so, necessarily we have $J_{11} < 0$.

Using again the Routh-Hurwitz conditions, local stability requires:

- $\text{Tr J} = J_{11} + J_{33} < 0$. So, as $J_{11}, J_{33} < 0$, the Tr J must be negative.
- $\det \text{J} - c_2 . \text{Tr J} = J_{12}.J_{21}.\text{J}_{33} - (J_{11}.J_{33} - J_{12}.J_{21}).(J_{11} + J_{33}) > 0$. The first part, $J_{12}.J_{21}.J_{33}$, must be positive. The second part, c_2, must be positive either. The trace, as mentioned before is negative. So, the second condition is satisfied without any additional constrains.
- If we have $s + m - (g_x + \delta) < 0$, we don't have boundaries for μ and J_{11} must be negative.

In scenario 2, where $g_g > g_x$ and $(u^*, h^*, \alpha^*) = \left[u_n, \left(g_g + \delta\right).v/u_n, 1\right]$, we have the following Jacobian matrix:

$$\begin{bmatrix} \dot{u} \\ \dot{h} \\ \dot{\alpha} \end{bmatrix} = \underbrace{\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & 0 & 0 \\ 0 & 0 & J_{33} \end{bmatrix}}_{\text{J}^*} . \begin{bmatrix} u - u^* \\ h - h^* \\ \alpha - \alpha^* \end{bmatrix} \tag{22}$$

And the following signals: $J_{12} < 0$; $J_{13} > 0$ and $J_{33} > 0$.

Proposition: Since $\mu > 0$, the fixed point of model T is locally stable if the bound $\mu < s + m - \left(g_g + \delta\right)$ and $s + m - \left(g_g + \delta\right) > 0$ are satisfied.

Observe that if the condition $\mu < s + m - \left(g_g + \delta\right)$ and $s + m - \left(g_g + \delta\right) > 0$ holds, so, necessarily we have $J_{11} < 0$.

Using again the Routh-Hurwitz conditions, local stability requires:

- $\text{Tr J} = J_{11} + J_{33} < 0$. So, as $J_{11} < 0$ and $J_{33} > 0$, the Tr J should be negative, for small values of $g_g - g_x$.
- $\det \text{J} - c_2 . \text{Tr J} = J_{12}.J_{21}.\text{J}_{33} - (J_{11}.J_{33} - J_{12}.J_{21}).(J_{11} + J_{33}) > 0$. The first part, $J_{12}.J_{21}.\text{J}_{33}$, must be negative. The trace, as mentioned before is negative. So, the second part, c_2, must be positive to satisfy the second condition. Since $J_{11} < 0$, this will always be the case.
-

4.3.The Steady State Equilibrium for the Type-S Model

The existence of a steady state for the model presented in section 3 require $\dot{\alpha} = \dot{b} = \dot{h} = \dot{u} = \dot{r} = 0$. Given the set of parameters, we must here investigate two possible scenarios. The first one is $g_g > g_x$ and the second is $g_g < g_x$. Regarding the variables α^*, h^*, u^*, the steady state values previously founded remain. To summarize, we show table 2 below, with b^* and r^* for all two scenarios.

Table 2 – Steady State Values for Type-S Model

Scenarios	b^*	r^*
$g_g > g_x$	$= \frac{v.(g_g + \delta) + u_n.[\tau.(1-\pi) - s - m]}{u_n.(i - g_g)}$	$= -m/g_g$
$g_x > g_g$	$= \frac{\tau.(1-\pi)}{i - g_x}$	$= \frac{s.u_n - v.(g_x + \delta)}{u_n.g_x}$

For both scenarios, we found for b^* possibilities to found positive and negative values. It is important to note that in second scenario, as $\pi < 1$, necessarily we need to have $i > g_x$ to find a positive equilibrium value. About r^*, in scenario one we can have both positive and negative values. In first scenario, we have a negative value, which means that it is an economically unsustainable steady state.

4.4.The Stability Analysis of the Type-S Model in Steady State

In this section we show the local stability analysis for the 4 x 4 system[19]. In the first one, we linearize the dynamic system around the internal equilibrium point and after that we use the Routh-Hurwitz conditions for analyze the local stability. The second method is through numerical analysis.

$$\begin{bmatrix}\dot{u}\\ \dot{h}\\ \dot{b}\\ \dot{r}\end{bmatrix} = \underbrace{\begin{bmatrix}J_{11} & J_{12} & 0 & 0\\ J_{21} & 0 & 0 & 0\\ J_{31} & J_{32} & J_{33} & 0\\ J_{41} & 0 & 0 & J_{44}\end{bmatrix}}_{\mathbf{J}^*} \cdot \begin{bmatrix}u-u^*\\ h-h^*\\ b-b^*\\ r-r^*\end{bmatrix} \tag{26}$$

The characteristic equation is:

$$\lambda^4 + b_1.\lambda^3 + b_2.\lambda^2 + b_3.\lambda + b_4 = 0 \tag{27}$$

Following the Asada and Yoshida (2003), we can represent the coefficients as:

$$b_1 = -tr\mathbf{J}^* = -J_{11} - J_{33} - J_{44} \tag{28}$$

b_2 is the sum of all principal minors of second order of $\mathbf{J}^*$.

$$b_2 = \begin{vmatrix}J_{11} & J_{12}\\ J_{21} & 0\end{vmatrix} + \begin{vmatrix}J_{11} & 0\\ J_{31} & J_{33}\end{vmatrix} + \begin{vmatrix}J_{11} & 0\\ J_{41} & J_{44}\end{vmatrix} + \begin{vmatrix}0 & 0\\ J_{32} & J_{33}\end{vmatrix} + \begin{vmatrix}0 & 0\\ 0 & J_{44}\end{vmatrix} + \begin{vmatrix}J_{33} & 0\\ 0 & J_{44}\end{vmatrix} \tag{29a}$$

$$b_2 = -J_{21}.J_{12} + J_{11}.J_{33} + J_{11}.J_{44} + J_{33}.J_{44} \tag{29b}$$

b_3 is the minus sum of all principal minors of third order of $\mathbf{J}^*$.

$$b_3 = -\begin{vmatrix}0 & 0 & 0\\ J_{32} & J_{33} & 0\\ 0 & 0 & J_{44}\end{vmatrix} - \begin{vmatrix}J_{11} & 0 & 0\\ J_{31} & J_{33} & 0\\ J_{41} & 0 & J_{44}\end{vmatrix} - \begin{vmatrix}J_{11} & J_{12} & 0\\ J_{21} & 0 & 0\\ J_{41} & 0 & J_{44}\end{vmatrix} - \begin{vmatrix}J_{11} & J_{12} & 0\\ J_{21} & 0 & 0\\ J_{31} & J_{32} & J_{33}\end{vmatrix} \tag{30a}$$

$$b_3 = J_{21}.J_{12}.(J_{44} + J_{33}) - J_{11}.J_{33}.J_{44} \tag{30b}$$

$$b_4 = \det\mathbf{J}^* \tag{31a}$$

$$b_4 = -J_{44}.J_{21}.J_{12}.J_{33} > 0 \tag{31b}$$

To find local stability in the neighborhood of the fixed point, the coefficients need to satisfy the following Routh-Hurwitz conditions:

$$b_1, b_2, b_3, b_4 > 0 \qquad \text{and} \qquad b_1.b_2.b_3 - b_1^2.b_4 - b_3^2 > 0 \tag{32}$$

[19] Since $\dot{\alpha}$ depends only on α and parameters, we can reduce the analysis of 5-D ODE system into a 4-D ODE, and use α as a fixed point. For the sake of simplicity, now we assume $v = 1/u_n = 1$.

On the following steps, we split the two cases, namely $g_g > g_x$ (case 1) and $g_x > g_g$ (case 2).

Scenario 1: When $g_x > g_g$

The next table 4 shows the analytic form and the signal of the Jacobian elements valued at the fixed point.

Table 3 – Jacobian Elements for Model-S in Case 1 - $g_x > g_g$ – Model S.

Jacobian Element	Analytic Form	Signal
$J_{11} = \left.\frac{\partial \dot{u}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-(g_x+\delta).\left(1-\frac{\mu}{s+m-g_x-\delta}\right)$	>0 or <0
$J_{12} = \left.\frac{\partial \dot{u}}{\partial h}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-u_n^2$	<0
$J_{21} = \left.\frac{\partial \dot{h}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$\mu.(g_x+\delta)$	>0
$J_{31} = \left.\frac{\partial \dot{b}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-\frac{(g_x+\delta).\mu.\tau.(1-\pi)}{(i-g_x).(s+m-g_x-\delta)}$	>0 or <0
$J_{33} = \left.\frac{\partial \dot{b}}{\partial b}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$i-g_x$	>0 or <0
$J_{41} = \left.\frac{\partial \dot{r}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$\frac{(g_x+\delta).\mu.[s-(g_x+\delta)]}{g_x.[s+m-(g_x+\delta)]}$	>0 or <0
$J_{44} = \left.\frac{\partial \dot{r}}{\partial r}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-g_x$	<0

The difference between cases 1 and 2 in terms of null Jacobian elements is the position J_{32}. However, we still have the same equations mentioned early (33a-33d). The similar Routh-Hurwitz conditions to be satisfied implies $i - g_x < 0$, that is the usual condition in models that involve public debt dynamics. We performed a numerical simulation[20] and we find again there is no possibility of finding $J_{11} > 0$. In other words, in all simulation

[20] In the next section, we present the way that we performed numerical simulations.

the model must satisfy $\mu < s + m - (g_x + \delta)$. Since $\mu > 0$, when $s + m - (g_x + \delta) > 0$, only small values of μ, namely μ smaller than $s + m - (g_g + \delta)$ could satisfy the condition. If we have $s + m - (g_x + \delta) < 0$, all values of $\mu > 0$ could generate $J_{11} < 0$. In short, analyzing the jacobian elements in case 2, we found the following signals for the local stability analysis: $J_{11}, J_{12}, J_{33}, J_{44} < 0$ and $J_{21} > 0$. The last condition, namely $b_1.b_2.b_3 - b_1^2.b_4 - b_3^2 > 0$, will be presented in the numerical simulation section[21].

Scenario 2: When $g_g > g_x$

The next table 4 shows the analytic form and the signal of the Jacobian elements valued at the fixed point.

Table 4 – Jacobian Elements for Model-S in Case 2 - $g_g > g_x$ – Model S.

Jacobian Element	Analytic Form	Signal
$J_{11} = \left.\frac{\partial \dot{u}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-(g_g + \delta).\left(1 - \frac{\mu}{s + m - g_g - \delta}\right)$	$+,0,-$
$J_{12} = \left.\frac{\partial \dot{u}}{\partial h}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	-1	$-$
$J_{21} = \left.\frac{\partial \dot{h}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$\mu.(g_g + \delta)$	$+$
$J_{31} = \left.\frac{\partial \dot{b}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-\frac{\mu.(g_g + \delta).[\tau.(1-\pi) - (s + m - g_g - \delta)]}{(i - g_g).(s + m - g_g - \delta)}$	$+,0,-$
$J_{32} = \left.\frac{\partial \dot{b}}{\partial h}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	-1	$-$
$J_{33} = \left.\frac{\partial \dot{b}}{\partial b}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$i - g_g$	$+,0,-$
$J_{41} = \left.\frac{\partial \dot{r}}{\partial u}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-\frac{\mu.\left(1 + \frac{\delta}{g_g}\right).m}{s + m - g_g - \delta}$	$+,0,-$
$J_{44} = \left.\frac{\partial \dot{r}}{\partial r}\right\|_{(u^*,\alpha^*,h^*,b^*,r^*)}$	$-g_g$	$-$

[21] We also performed simulations using this condition to verify the signals of J_{11} in the stable models.

Using the Routh-Hurwitz conditions mentioned early (equation 31b), the sufficient conditions to find a local stable equilibrium are:

$$b_1 = -tr\mathbf{J}^* = -J_{11} - J_{33} - J_{44} > 0 \quad (33a)$$

$$b_2 = -J_{21}.J_{12} + J_{11}.(J_{33} + J_{44}) + J_{33}.J_{44} > 0 \quad (33b)$$

$$b_3 = J_{21}.J_{12}.(J_{44} + J_{33}) - J_{11}.J_{33}.J_{44} > 0 \quad (33c)$$

$$b_4 = -J_{44}.J_{21}.J_{12}.J_{33} > 0 \quad (33d)$$

Thus, to satisfies the previous four conditions, the model needs: $J_{33} < 0$, which means, $i - g_g < 0$. This is a usual condition in models that involve public debt dynamics. We performed a numerical simulation[22] and we find there is no possibility of finding $J_{11} > 0$. In other words, in all simulation the model must satisfy $\mu < s + m - \left(g_g + \delta\right)$. Since $\mu > 0$, when $s + m - \left(g_g + \delta\right) > 0$, only small values of μ, namely μ smaller than $s + m - \left(g_g + \delta\right)$ could satisfy the condition. If we have $s + m - \left(g_g + \delta\right) < 0$, all values of $\mu > 0$ could generate $J_{11} < 0$. In short, analyzing the Jacobian elements in case 1, we found the following signals for the local stability analysis: $J_{11}, J_{12}, J_{33}, J_{44} < 0$ and $J_{21} > 0$. The last condition, namely $b_1.b_2.b_3 - b_1^2.b_4 - b_3^2 > 0$, will be presented in the numerical simulation section[23].

5. Numerical Simulations

In this section we present three numerical simulations involving models S and T. The first numerical essay uses the Monte Carlo simulation method to generate random parameters with uniform distribution within the interval [0,1]. They were generated in the order of 10e7. Using the Routh-Hurwitz criteria for a 4D system, given by equation (32), we filtered the results that simultaneously met the five conditions for both case 1 $\left(g_x > g_g\right)$ and case 2 $\left(g_g > g_x\right)$. Subsequently, we use the nonparametric Kernel density function for two dimensions to quantify the point's density on the generated

[22] In the next section, we present the way that we performed numerical simulations.
[23] We also performed simulations using this condition to verify the signals of J_{11} in the stable models.

surface. The importance of this type of exercise is to carry out a mapping of the parameters that guarantee dynamic stability for the model (which implies mathematically satisfying the criteria), although it does not necessarily make economic sense[24]. In this way, it is possible to evaluate under which conditions the supermultiplier remains stable in the neighborhood of the steady state.

Figure 1 –Surface plus 2D Kernel Density (Case 1 - $g_x > g_g$)

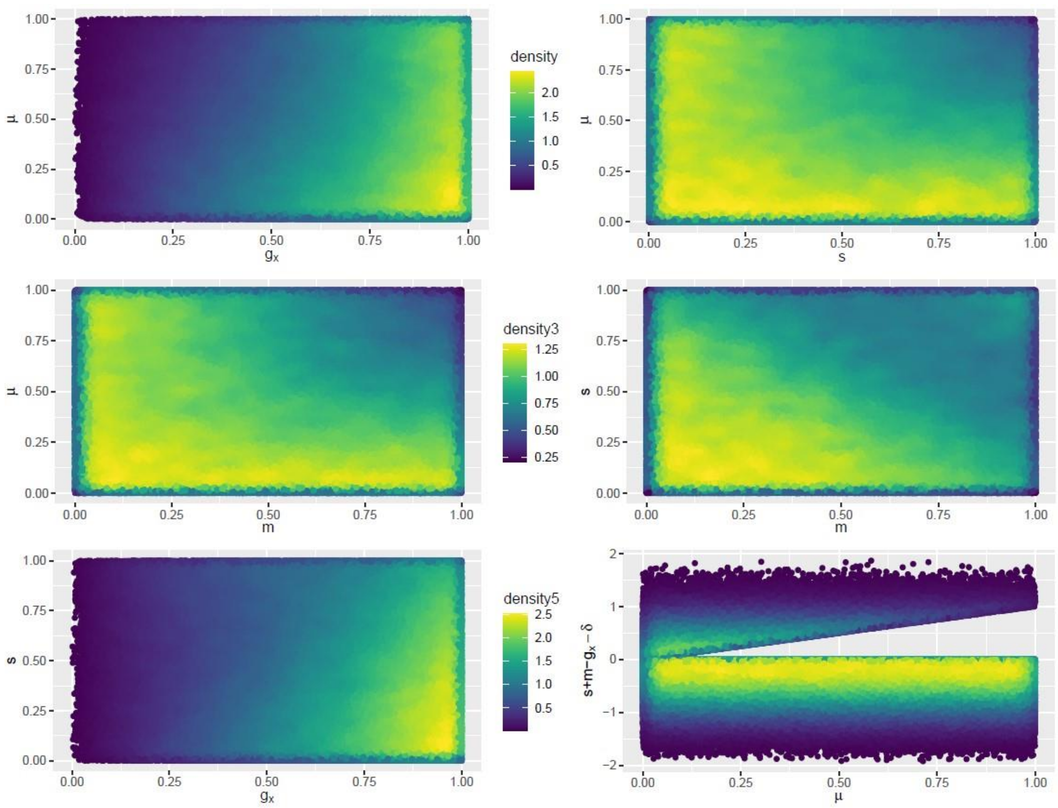


Figure 1 shows six quadrants with the surface of points generated for the parameters $g_x, \delta, \mu, m, s, i$. Lighter colors show increasing density in the region while darker colors show decreasing density. The first quadrant on the left shows that under the entire generated surface it is possible to find a stability relationship for the supermultiplier parameter against the growth rate of the autonomous component of external demand. In terms of probability, it is more likely to find relationships of high growth rate of the demand component with a low value of μ. In this numerical exercise, we did not find the

[24] A simple example is that the simulation makes it possible to generate high growth rates of the autonomous components of demand without them suffering any type of constrain. Thus, the parameter may be mathematically possible, although economically unsustainable.

problem reported by Skott *et al* (2020) and Franke (2021) of excess acceleration (at least, mathematically) for both case 1 and case 2. However, using a combination point of a high autonomous component of demand (g_x or g_g) with a high value of μ and large differences between g_x and g_g (or g_g and g_x), the time path shows an explosive dynamic before reaching the steady state [25]. In other words, the problem of excess acceleration was found in the transition between fixed points[26,27].

In the second left quadrant, we find more sparse density relationships. There was only low probability in the relationships between high μ and high m. In the third left quadrant, we find a higher probability of high value for the autonomous component of demand with increasing density values as the value of s decreases.

In the first and second quadrant to the right, we have more sparse density. However, the lowest density in the first quadrant was obtained for high values of μ and high values of s. For the second quadrant, the lowest density is for high values of s and high values of m.

Finally, we have the last right quadrant that evaluates the values of μ and $s + m - g_g - \delta$. As the graph itself points out, if it is in the positive quadrant, the condition $\mu < s + m - g_g - \delta$ needs to be satisfied for stability behavior remain. If it is in the negative quadrant, the condition is not necessary.

The second simulation is an offshoot of the first. We use a numerical algorithm like the first to generate random parameters. The steps of the algorithm are as follows:

Second Numerical Simulation - Algorithm steps

1. Looping from the first to n^{th}.
2. Generation of Random Parameters with Uniform Distribution.
3. If $g_g = g_x$, use analytic expressions of $u^*, h^*, \alpha^*, b^*, r^*$ [Table 1 and 2] to determine the fixed points.

[25] Solved by numerical integration via the 4th Order Runge-Kutta.
[26] It is important to mention that the model dynamics is high non-linear. The local stability analysis is performed by linearizing the dynamic near the fixed points, which makes this result plausible.
[27] The reader can perform their own simulations of the model using the following link: https://juliofcsantos.shinyapps.io/Sistem_S/

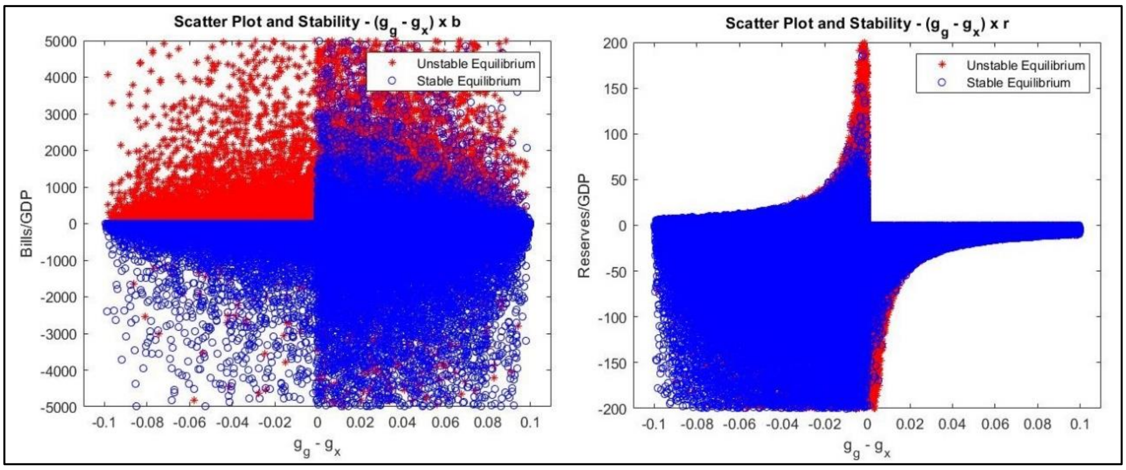
4. If $g_g > g_x$, use analytic expressions of $u^*, h^*, \alpha^*, b^*, r^*$ [Table 1 and 2] to determine the fixed points.
5. If $g_g < g_x$, use analytic expressions of $u^*, h^*, \alpha^*, b^*, r^*$ [Table 1 and 2] to determine the fixed points.
6. Use the fixed points and parameters generated for the calculation of the Jacobian matrix.
7. Calculate the eigenvalues of the Jacobian matrix.
8. If the real part of the eigenvalues 1,2,3,4 and 5 are simultaneously less than zero, set Boolean variable to be equal to 1. Otherwise equal to zero.
9. Repeat the previous steps (closing the looping window).
10. Show the scatter plot with fixed points, growth rates, and stability boolean variable.

Thus, we did according to the description of the previous steps, and we used the intervals values for generation of random parameters with uniform distribution. The values appear in table 5 below.

Table 5 – Parameters and Values used in the Local Stability Analysis

Parameter	Interval	Parameter	Interval
μ	[0;1]	s	[0;1]
u_n	[0;1]	m	[0;1]
g_g	[0;0.1]	i	[0;0.1]
g_x	[0;0.1]	τ	[0;1]
δ	[0;0.1]	π	[0;1]
v	2.5	α_0	0.5

Figure 2 – The stable and unstable fixed points



Finally, the results of this numerical analysis are shown in figure 2 above. We performed over 10^7 sets of random parameter generation steps and found the following results: (a) When $g_g > g_x$, there are no steady state values where $r^* > 0$; (b) When $g_x > g_g$, there are no stable steady-state values for $b^* > 0$; (c) The only stable equilibrium that makes economic sense is found in scenario 1, which $g_g \leq g_x$.

With respect to the third simulation, the model was calibrated using table 6 parameters, in which the initial condition is $g_g = g_x$. We give a permanent increase shock in the value of g_x and observe the new convergence to the new steady-state values. Among the set of parameters chosen, preference was given to those that would generate stability at equilibrium points. In scenario 2, we also started from the base scenario and the shock was an increase in g_g. In scenario 3, we used the same conditions and shock of scenario 1 and changed the propensity to invest from 0.30 to 0.42. The idea is to show two stable scenarios and one unstable [reporting the problem of too much acceleration]. We checked the trajectory for fixed points at the end. Below is the table with the parameters used.

Table 6 – Parameters and Values used in the time path simulation

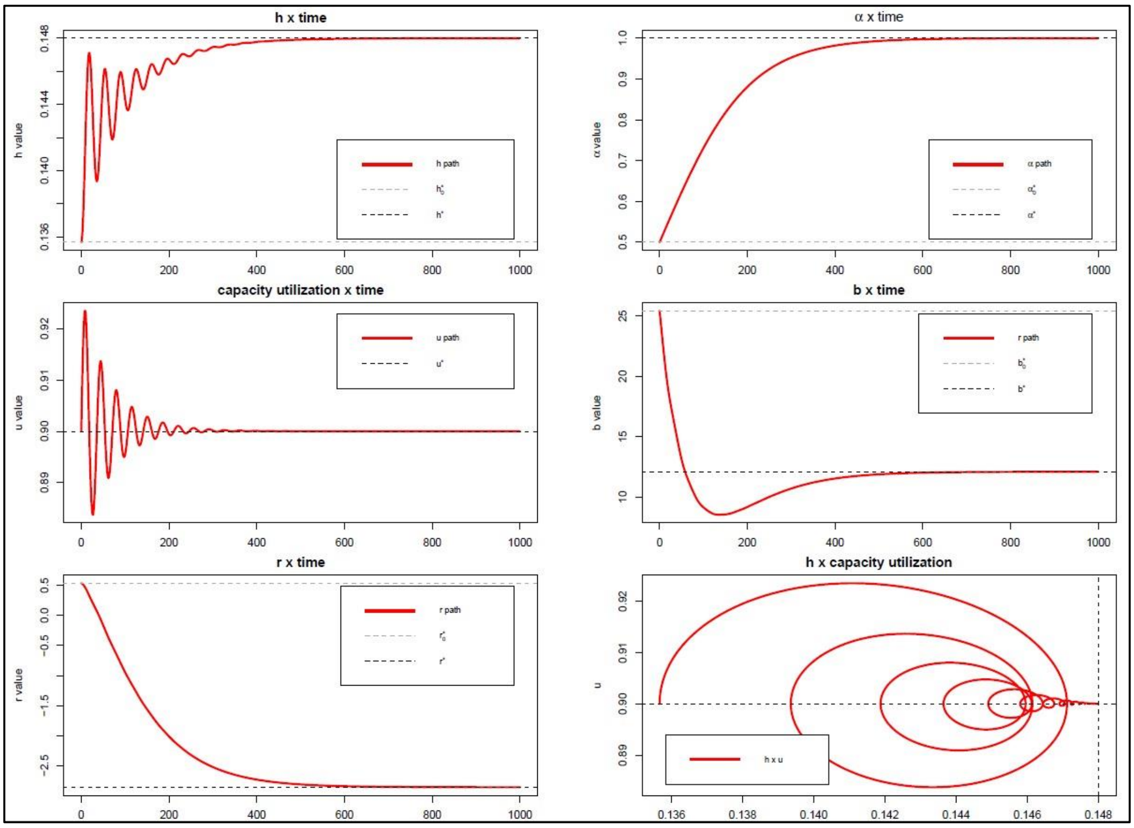
Parameter	Value	Parameter	Value
μ	0.30 and 0.42	s	0.40
u_n	0.90	m	0.20
g_g	0.07	i	0.05
g_x	0.06	τ	0.30
δ	0.05	π	0.30
v	1.11	α_0	0.50

In the sequence we present the simulation results divided by scenarios. Figure 3 shows the temporal path of the five variables of the system and the phase $u \times h$. We can observe that all of them converge to the steady state defined by equations in table 1 and 2.

It can be noticed that in figure 3, all variables converge to their stationary states and find stability in their neighborhood of steady state. Regarding scenario one, it is important to highlight that the r^* balance is positive, which is economically possible. This is the case where there is a stock of accumulated reserves.

Still on the interpretation of the results, we have that b^* reaches a negative value. This case arises because the pace of growth in public spending grows below the rate of GDP growth. For this reason, the model has a gradual drop in the public debt/GDP ratio until it reaches a fixed point. Although it is an economically weird situation, it is possible and in this case the government would be a creditor[28] in the economy.

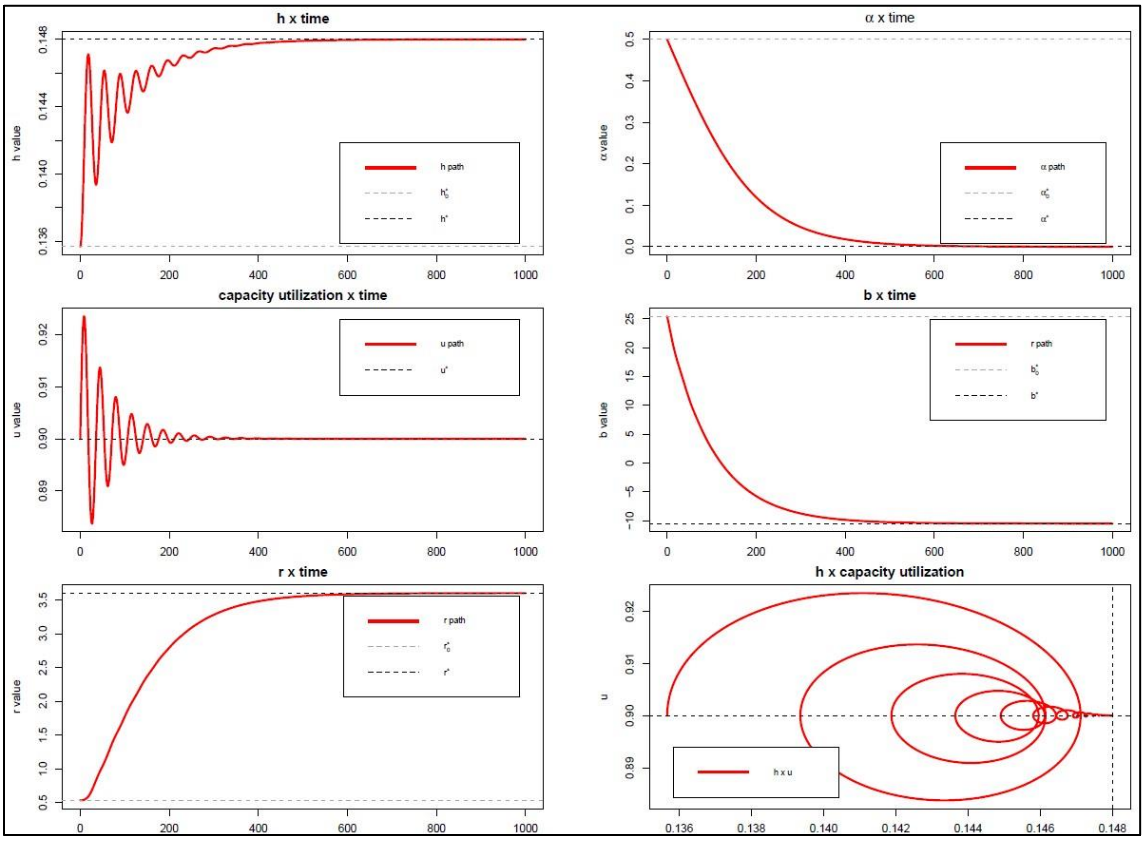
Figure 3 – Time Path for the five state variables and the steady state (from scenario 1 to scenario 2)



[28] Possible because in this condition the government would have positive net worth, allocating all its wealth in other private assets.

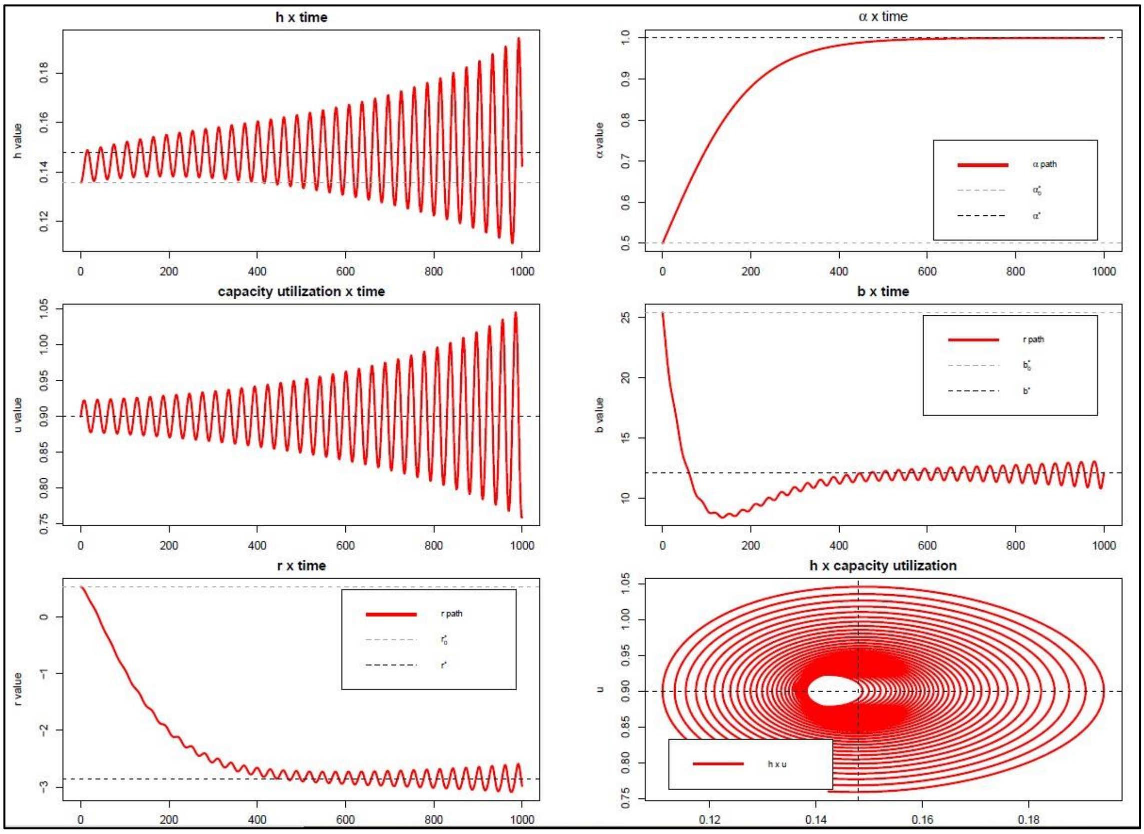
Now let's look at the results in figure 4. The variables u and h converged to a positive value. The α converges to 1, which represents the share domain of the autonomous component of public spending over the other autonomous components, that is, exports. **As shown in the steady-state calculation, the value of exchange reserves over GDP converges to a negative value**. This result is economically unsustainable since before the economy can reach such a situation, there will be a foreign exchange crisis. In this way, this growth regime is unsustainable.

Figure 4 – Time Path for the five state variables and the steady state (from scenario 1 to scenario 3)



Finally, we present the last simulation which is similar to the first one, however we modify the parameter of marginal propensity to invest, μ. The idea is to show that the configuration of our model presents the same problem identified by Franke (2021) and Skott, Santos and Oreiro (2021) of instability emerging in models with the Supermultiplier with “too much” acceleration. What the figure 5 shows us is that the convergence speed in models with the supermultiplier needs to be very slow for the stability holds. As the result obtained by Skott, Santos and Oreiro (2021), stability requires completely unreasonable periods for convergence (around 400 years), which clashes with the modern capitalist experience and calls into doubt the validity of the use of supermultiplier as an element of stabilization of the Harodian forces in a long-term model.

Figure 5 – Time Path for the five state variables and the steady state (from scenario 1 to scenario 2 with too much acceleration)



6- Final Remarks

The results of this paper involving both the analytical part and the numerical analysis indicate that there is an impossibility for domestic demand-led regimes. If growth is led by the autonomous components of domestic demand, whether it is autonomous consumption, autonomous public spending, inherited wealth (which may not be an endogenous variable in the short run) or any other component, will bring the economy to an economically impossible steady-state equilibrium. Thus, the only growth regime that is sustainable in the long run is the one where exports are the engine of growth of autonomous demand. This result had been exhaustively reported both in the *balance of* *payments constrained growth* (McCombie and Thirwall, 1997) and in the *developmental macroeconomics* literature (Bresser-Pereira, Oreiro and Marconi, 2015).

Moreover, the results presented in the article are also consistent with the recent work of Nah & Lavoie (2017). The theoretical survival of the demand-led growth-supermultiplier models is only possible in export-led growth models. Maybe this is the reason that explain why SSM are usually presented in a closed economy framework.

It is important to stress that the results obtained in this article are not trivial ones. One can get a wrong interpretation of our argument saying that what we just make a trivial statement: "if there is a balance of payments constraint than growth will be constrained by it". Our argument is not fullish as that. What we are saying is that for growth to be sustainable in the long run it must be export-led, otherwise the economy will face a balance of payments crisis in a finite period. The argument of triviality may be based in the idea that autonomous domestic demand grows in the long run at a lower rate than exports, in which case the long-run growth rate is always lower than the one allowed by the external constraint. In this case, growth will be led by domestic demand, but it will be a foolish wasting of growth opportunities. In the SSM output is not labor constrained so growth rate of output is always lower than the natural growth rate (if this rate can be defined at all). The only constraint is the balance of payments constraint, but why any reasonable policy maker will prefer to grow at say 3% *p.y* if the balance of payments allowed a growth of 4% *p.y***?** This reasoning makes no sense at all because it makes no sense not exploit opportunities for growth acceleration if there is no cost to do that

Let us make the argument in another way. Consider an economy in which income elasticity of imports is higher than one. Consider also that capital account is closed (as we done in this article) so a trade deficit can only be financed temporarily by loss of international reserves. Then the maximum growth rate compatible with balance of payments equilibrium will be equal to the ratio of exports growth and income elasticity of imports. Since income elasticity of imports is higher than one, this necessarily means that growth of exports had to be higher than the growth rate of domestic output, which means that the ratio of exports to GDP will increase over time. This is precisely what we define as an export-led growth. Moreover, if domestic autonomous demand (capitalist consumption or government expenditures) grows at a rate lower than exports, then the ratio of domestic demand in total autonomous demand will converge to zero in the long term and the only source of autonomous demand will be exports. If domestic autonomous demand (capitalist consumption or government expenditures) grows at a rate grows at a rate higher than exports than output growth will be higher than the one compatible with balance of payments equilibrium and economy will start to have increasing trade deficits that will result in a continuous reduction of international reserves. When reserves reach zero level – or even before that point – the policy makers will have to take actions to reduce the growth rate of domestic autonomous demand, for example increasing interest rates, raising taxes, or reducing the growth rate of government expenditure. In this scenario growth will be no longer led by autonomous domestic demand.

In other words, in the long run the actual growth rate will be always equal to the balance of payments constraint growth rate, a result that had a lot of empirical evidence to support it (Thirwall, 2013). In the short to medium run, the economy can grow at a rate that is higher or lower than the balance of payments constraint due to several reasons, but this result is not valid for the long run.

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Mathematical Appendix

1. **The dynamics of Bills/GDP ratio:**

$\dot{B} = (G - T) + i.B$

Where G is the public expenditures, T the taxation, i interest rate, B the bills (stock) and $\dot{B}$ is the bills time variation.

$\frac{B}{Y} = b$ [b is the Bills/GDP]

$\dot{b} = \frac{\dot{B}.Y - \dot{Y}.B}{Y^2}$

$\dot{b} = \frac{[(G-T)+i.B].Y - \dot{Y}.B}{Y^2} = \frac{[(G-T)+i.B]}{Y} - g_y.b$

$\dot{b} = \frac{[(G-T)+i.B]}{Y} - g_y.b$

$\dot{b} = \frac{G}{Y} - \frac{T}{Y} + i.b - g_y.b$

$\dot{b} = \frac{G}{A}.\frac{A}{Y} - \frac{\tau.(1-\pi).Y}{Y} + i.b - g_y.b$

$\dot{b} = \frac{\alpha}{\sigma} - \tau.(1-\pi) + (i - g_y).b$

Since $\sigma = \frac{1}{s+m-h}$, we have:

$\dot{b} = \alpha.(s+m-h) - \tau.(1-\pi) + (i - g_y).b$

Since $g_y = \frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x$, we have:

$\dot{b} = \alpha.(s+m-h) - \tau.(1-\pi) + \left[i - \left(\frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x\right)\right].b$

2. **The dynamics of Foreign Reserves/GDP ratio:**

$\frac{\dot{R}}{Y} = \frac{X}{Y} - \theta.\frac{M}{Y}$

Where Y is the GDP, X exports, M imports, θ the real exchange rate, $\dot{R}$ foreign reserves (time variation).

Considering $\theta = 1$, we have:

$r = \frac{R}{Y}$ [Foreign Reserves/GDP]

$\dot{r} = \frac{\dot{R}.Y - \dot{Y}.R}{Y^2}$

$\dot{r} = \frac{(X-M)}{Y} - g_y.r$

$\dot{r} = \frac{(1-\alpha)}{\sigma} - m - g_y.r$

Since $\sigma = \frac{1}{s+m-h}$, we have:

$\dot{r} = (1-\alpha).(s+m-h) - m - g_y.r$

Since $g_y = \frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x$, we have:

$\dot{r} = (1-\alpha).(s+m-h) - m - \left[\frac{\dot{h}}{s+m-h} + \alpha.g_g + (1-\alpha).g_x\right].r$