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Household credit-financed consumption and the debt service ratio: tackling endogenous autonomous demand in the Supermultiplier model

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Credit-financed household consumption and the debt service ratio: tackling endogenous autonomous demand in the Supermultiplier model

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Abstract

We develop a Supermultiplier model where debt-financed household autonomous consumption drives growth. However, instead of taking autonomous consumption growth as exogenous, we assume that households' debt service ratio partially determines it. We define a consumption function that allows for (i) households' demand for credit to depend on the burden interest payments have on their income; and (ii) credit conditions to affect the pace of household expenditures. The model has two equilibria, with the steady state one (two) combining a lower (higher) household debt ratio with a higher (lower) growth rate. However, only one equilibrium (steady state one) is locally stable for the chosen set of parameters. The system converges to this first steady state when departing from outside of equilibrium. Real and financial variables affect the steady state growth path in the model. The wage share has a positive effect, while the interest rate has a negative effect on growth.

Keywords: demand-led growth, Supermultiplier, household debt, consumption, endogenous autonomous demand

JEL classification codes: B50, C61, E11, G51, O41

1 Introduction

It is now widely documented that there has been a secular deterioration of income distribution in the US and other advanced economies (Atkinson et al., 2011; Piketty & Saez, 2003; van Treeck, 2014).

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It has also been argued that as income inequality worsened throughout the 20th century, household indebtedness increased to avoid consumption stagnation, especially in Anglo-Saxon economies (Kumhof et al., 2015; Kumhof et al., 2012; Rajan, 2010; van Treeck, 2014). As a consequence of these observations, more recently, the issue of household debt dynamics has been emphasized in both mainstream and heterodox macroeconomics.

In the heterodox literature, several demand-led growth models incorporate household debt dynamics (e.g., Dutt, 2005, 2006; Fagundes, 2017; Hein, 2012a; Pariboni, 2016; Setterfield and Kim, 2017, 2020 among others). Some of these works employ the stock-flow consistent (SFC) approach (e.g., Byrialsen and Raza, 2020; Caverzasi and Godin, 2015; van Treeck, 2009, among others). Meanwhile, the mainstream approach mainly explains some of these patterns through a general equilibrium framework (e.g., Mian et al., 2017, 2021).

Most of the heterodox literature on credit-financed household consumption can be divided into the neo-Kaleckian approach (see Dutt, 2005, 2006; Hein, 2012a; Setterfield and Kim, 2017, 2020) and, more recently, the Supermultiplier approach (see Avritzer, 2021; Fagundes, 2017; Mandarino et al., 2020; Pariboni, 2016). While in the usual neo-Kaleckian framework, credit-driven consumption is endogenous and induced by income, this is not the case in the Supermultiplier approach, which assumes (the pace of) credit-financed consumption of workers to be autonomous and exogenously given.

In general, Supermultiplier models assume that the non-capacity creating autonomous components of demand, such as government expenditures, household consumption out of credit, or capitalists consumption, that lead growth in the long run, grow at an exogenously given rate (Allain, 2015; Freitas & Serrano, 2015; Lavoie, 2016).[1]

However, a few exceptions formally deal with "semi-autonomous" (Fiebiger & Lavoie, 2017) or endogenous autonomous expenditures. In Brochier and Macedo e Silva (2019) and Brochier (2020), household consumption out of wealth is the semi-autonomous component of demand that makes the economy's growth rate endogenous to the model. Ferri and Tramontana (2020) consider that the pace of durable consumption is semi-autonomous since its growth rate depends on an exogenous component and partially on the unemployment rate. Caminati and Sordi (2019) and Nomaler et al. (2021) also build a Supermultiplier model where autonomous expenditures grow endogenously. They both assume the pace of R&D investment and part of consumption to be autonomous.[2] Fiebiger (2021) also proposes a Supermultiplier model where government expenditures are semi-autonomous. At last, Allain (2022) also analyses the dynamics of (two) semi-autonomous demand components in a Supermultiplier model.

[1] To be fair, there are growth models in the neo-Kaleckian tradition that assume consumption and (or) government expenditures to be autonomous, such as Godley and Lavoie (2007). The same goes for the growth models in the neo-Kaleckian approach that follow the supermuliplier mechanism such as Allain (2015) and Lavoie (2016).

[2] In both, the pace of R&D investment is partially endogenous, while the pace of autonomous consumption (capitalist consumption that grows in line with productivity in Caminati and Sordi (2019); and workers' consumption out of wealth in Nomaler et al. (2021)) is fully endogenous.

We contribute to this literature by explicitly providing an initial closure for a semi-autonomous growth rate. We assume credit-financed consumption can be understood as semi-autonomous[3], tackling potential determinants of the pace of household credit demand for consumption. By doing that, we integrate the post-Keynesian literature concerned with household credit demand determinants (Dutt, 2006; Hein, 2012b; Setterfield & Kim, 2016) and the Supermultiplier literature, which deals with autonomous household consumption as one of the potential growth drivers (Freitas & Serrano, 2015; Lavoie, 2016; Pariboni, 2016).

The objectives of this exercise are threefold: (i) to analyze the stability and equilibrium properties of a semi-autonomous growth Supermultiplier model; (ii) to provide a reasonable channel through which real and financial variables may affect the long run growth path; and (iii) to explore how these variables affect household debt ratio, the growth rate, and firms' propensity to invest. To achieve these goals, we derive the steady state solution for a semi-autonomous growth model where credit-financed consumption drives economic growth.

More precisely, following Dutt (2006) and van Treeck (2009), we assume that the debt burden negatively affects households' demand for consumer credit. As a result, the credit-financed consumption growth rate becomes a semi-autonomous component of demand, negatively impacted by the interest rate and positively impacted by the wage share.[4] Once we solve the model for the steady state growth path and find two equilibria, of which only one is stable, we demonstrate that income distribution positively affects growth. In contrast, financial variables, such as the interest rate or households' sensitivity to their debt burden, have a negative effect.

Beyond this introduction, the paper is divided into four sections as follows. Section 2 presents the model, the short run equilibrium, the analytical steady state solution and discusses the model's local stability conditions. Section 3 presents a numerical illustration of the model in which we analyze both steady state solutions and their stability using a chosen set of parameters' values. Section 4 discusses the relevant derivatives of the steady state equilibrium that was found to be stable. Section 5 concludes.

2 A credit driven household consumption model

The model deals with a pure credit closed economy with no inflation and without a government composed of three institutional sectors: households, banks, and firms. Following the post-Keynesian literature (see Dutt, 2005, 2006; Hein, 2012b; Setterfield and Kim, 2016, 2017, 2020 among others), we further divide the household sector into two groups - workers and rentiers. Rentier households buy all the equities (e) issued by firms and hold the rest of their wealth as deposits (M_r) at banks. Worker households take on loans (L_w) to consume. Since workers do not consume all their after-

[3] See, for instance, the pieces of evidence provided by Fasianos and Lydon (2022) and Fiebiger and Lavoie (2017).

[4] See Fasianos and Lydon (2022) and Stockhammer and Wildauer (2018) for some empirical work on this.

interest wage income, they accumulate wealth as deposits (M_w) at banks (the only asset they hold). Banks give out loans to worker households and take on household deposits. Firms produce a homogeneous good that is used both for consumption (C) and investment (I) purposes. They also issue equities to households to finance the part of investment not covered by retained earnings.

Tables 1 and 2 detail the balance sheet and the transactions and flow of funds of the institutional sectors, respectively.

Table 1: Balance sheet matrix

	Households		Firms	Banks	Total
	Workers	Renters			
Deposits	$+M_w$	$+M_r$		$-M$	0
Equities		$+ep_e$	$-ep_e$		0
Loans	$-L_w$			$+L_w$	0
Firm's Capital			K		K
Total	V_w	V_r	V_f	0	K

Table 2: Transaction flow matrix

	Households		Firms		Banks		Total
	Workers	Rentiers	Current	Capital	Current	Capital	
Consumption	$-C_w$	$-C_r$	$+C$				0
Firms' Investment			$+I$	$-I$			0
Wages	$+W$		$-W$				0
Firm's Profits		$+FD$	$-(FD+FU)$	$+FU$			0
Bank's Profits		$+FB$			$-FB$		0
Interest on Loans	$-i_l L_w$				$+i_l L_w$		0
Interest on Deposits	$+i_m M_w$	$+i_m M_r$			$-i_m M$		0
Change in Loans	$+\dot{L}_w$					$-\dot{L}_w$	0
Change in Deposits	$-\dot{M}_w$	$-\dot{M}_r$				$+\dot{M}$	0
Change in Equity issues		$-p_e\dot{e}$		$+p_e\dot{e}$			0
Total	0	0	0	0	0	0	0

In what follows, we describe the behavior of all aggregate demand components, focusing on credit-financed workers' consumption.[5]

2.1 Aggregate demand behavior

2.1.1 Workers' consumption

Following the Supermultiplier literature, as in Fagundes (2017) and Pariboni (2016), we assume that workers take on new loans ($\dot{L}_w$) to finance autonomous consumption (Z) (equation 1). We

[5] See Appendix A for the model's full set of equations.

also assume that this autonomous component of demand grows at a rate g_z (equation 2). For simplification purposes, we abstract from household debt amortization.[6]

$$\dot{L}_w = Z \tag{1}$$

$$Z = Z_0 e^{g_z t} \tag{2}$$

However, instead of taking the growth of debt-financed consumption as exogenously given, we suppose that the pace of household consumption out of credit is partially endogenous to the model or semi-autonomous, as coined by Fiebiger and Lavoie (2017).[7] More precisely, we assume it depends positively on autonomous factors (φ_0) and negatively on the debt service to workers' income ratio (ds):

$$g_z = \varphi_0 - \varphi_1 ds \tag{3}$$

Where φ_1 depicts how sensitive household credit demand for consumption is to the debt service and the credit conditions as represented by the interest rate on loans. The debt service ratio is given by equation 4.

$$ds = \frac{i_l L_w}{W} = \frac{i_l L_w}{wY} \tag{4}$$

Where L_w is workers' accumulated debt, i_l is the interest rate on workers' loans, Y is total income, and w is the wage share.[8] Substituting 4 into 3, we arrive at equation 5:

$$g_z = \varphi_0 - \frac{\varphi_1 i_l l_w}{w} \tag{5}$$

Where $l_w = \frac{L_w}{Y}$ is the total loans to output ratio.

One may wonder why the pace of credit-financed consumption would react to the burden of debt. The reasoning of previous work on credit-financed consumption may justify this specification for the growth rate of autonomous household consumption. Bhaduri et al. (2006) and van Treeck (2009) among others, claim that households' burden of debt, as it reduces their creditworthiness, may reduce their ability to take out loans. Besides, it may deteriorate the credit conditions (Isaac & Kim, 2013). Dutt (2006) also shows evidence that the debtors take out fewer loans, the higher their indebtedness. More recently, Fasianos and Lydon (2022) show that indebted households reduce consumption when their income falls. And Stockhammer and Wildauer (2018) also provide some evidence for low interest rates boosting household credit.

[6]For Supermultiplier models that take household debt amortization into account see Fagundes (2017), Pariboni (2016), and Pedrosa et al. (2022) .

[7]For more on the discussion of semi-autonomous or endogenous autonomous expenditures see Allain (2021), Brochier and Macedo e Silva (2019), Fazzari et al. (2020), and Fiebiger and Lavoie (2017).

[8]Income distribution is exogenous in the model so that $w = 1 - \pi$. Where π is the profit share. This is compatible with neo-Kaleckian and Sraffian income distribution theories (Brochier & Freitas, 2022). For conflict theory of inflation extended Supermultiplier models that deal with endogenous income distribution, see Brochier (2020) and Nah and Lavoie (2019).

Therefore, we consider that debt servicing (ds) may, on average, affect the pace of new credit demand for consumption purposes (g_z). As a result, the semi-autonomous demand component becomes a function of financial variables – the interest rate and the debt ratio – and real variables – the wage share.

As for the debt burden, we write it as a ratio of interest payments on loans to wages (instead of disposable income) in a rough approximation to the Minskyan typology applied to households,[9]where interests on loans represent workers' cash commitments and wages their cash flow. As highlighted by Setterfield and Kim (2020), for indebted households, the limit of Ponzi finance (as applied to firms) is not feasible since households could not use all their income for debt servicing and still provide for their subsistence. Therefore, we assume that, on average, the debt service ratio is positive but lower than one ($0 < ds < 1$).

Besides credit-financed autonomous consumption, workers also consume a fraction (α_1) of their disposable income (Y_{d_w}) (equation 6):

$$C_w = \alpha_1 Y_{dw} + Z \tag{6}$$

Where workers' disposable income is given by the sum of their earnings from wages plus the difference between what they receive as interest on deposits ($i_m M_w$) and pay as interest on loans ($i_l L_w$) (equation 7):

$$Y_{dw} = W + i_m M_w - i_l L_w \tag{7}$$

In this version of the model, we will assume that deposits earn no interest ($i_m = 0$) for simplification purposes. Nevertheless, this is a realistic assumption, interpreting these deposits as demand deposits. That means workers' disposable income will amount to wages net of interest payments on loans. It also means that only rentiers earn financial income, as deposits are the only asset of workers.

2.1.2 Rentiers' consumption

We assume that rentier households earn financial income accruing from firms' distributed profits (FD), banks' profits (FB), and interest payments on deposits (equation 8). Since we initially assume that banks pay no interest on deposits, rentiers' disposable income will amount to total distributed profits.

$$Y_{dr} = FD + FB + i_m M_r \tag{8}$$

We assume that firms retain a fraction (s_f) of their total profit (πY) and distribute the rest of it to rentiers (equation 9).

$$FD = (1 - s_f)\pi Y \tag{9}$$

[9]See Cynamon and Fazzari (2008) on this.

In turn, banks profit from the interest differential of their assets and liabilities (equation 10) and distribute all of it to rentiers.

$$FB = i_l L_w - i_m M \tag{10}$$

In this version ($i_m = 0$), banks' profits will be equivalent to interests charged on loans ($i_l L_w$).

At last, rentier households consume a fraction of their disposable financial income (equation 11).

$$C_r = \alpha_2 Y_{dr} \tag{11}$$

2.1.3 Firms' investment

As for firms' investment, we assume it to be endogenously determined by current income (Y) and firms' marginal propensity to invest (h)(equation 12). We also suppose, as in Freitas and Serrano (2015), that firms adjust their investment behavior to the discrepancies between the actual (u) and the normal (u_n) capacity utilization rate (equation 13). This provides a conditional solution for the problem of Harrodian instability as highlighted by Allain (2015). [10]

$$I = hY = \dot{K} \tag{12}$$

$$\dot{h} = h\gamma(u - u_n) \tag{13}$$

It follows from the assumptions in equations 12 and 13 that the rate of capital accumulation in this economy will be a function of the marginal propensity to invest and the rate of capacity utilization, both divided by the capital-output ratio, v (equation 14):

$$g_k = \frac{hu}{v} \tag{14}$$

As should be clear from equations 12 and 14, we also abstract from capital depreciation.

2.2 Short-run equilibrium

By substituting the behavioral equations 6, 11, and 12 into the identity of supply and demand (equation 15) and solving it for output, we obtain the short run goods' market equilibrium level of income (equation 16).

$$Y = C_r + C_w + I \tag{15}$$

$$Y = \frac{Z - [(\alpha_1 - \alpha_2)i_l]L_w}{(s - h)} \tag{16}$$

[10] See Gahn (2021), Hein (2014), Hein et al. (2012), Setterfield and Avritzer (2020), and Skott (2010) for a review on this debate. See also Gahn (2021) for an empirical study that finds evidence of a transitory effect on the capacity utilization rate after an aggregate demand expansion episode.

In equation 16 $-(\alpha_1 - \alpha_2)i_l$ represents the net effect on aggregate demand (and income) of the interest payments on loans; and $s = 1 - \alpha_1 w - \alpha_2(1 - s_f)\pi$ is the marginal propensity to save.[11]

Dividing both sides of equation 16 by the full capacity output level ($Y_{fc} = K/v$), we then find the short-run capacity utilization rate (u):

$$u = \frac{v\{[g_l - (\alpha_1 - \alpha_2)i_l]l_{k_w}\}}{(s - h)} \tag{17}$$

Where $g_l = \frac{\dot{L}_w}{L_w} = \frac{Z}{L_w}$ is the loans growth rate; and $l_{k_w} = \frac{L_w}{K}$ is the worker's loans to capital ratio. We must emphasize that since both l_{k_w} and g_l have entered the equation that describes the short run capacity utilization rate, they will both play a fundamental role in defining the long run steady state dynamics of the model. [12].

Assuming the Keynesian stability condition holds in the model ($s > h$), for having a positive capacity utilization rate, the numerator of equation 17 has to be positive. Since $\alpha_1 > \alpha_2$, this will be the case when $g_l > (\alpha_1 - \alpha_2)i_l$, that is, when the positive effect of autonomous consumption (new credit) exceeds the negative effect interest payments on loans have on aggregate demand (as they reduce workers' disposable income and, therefore, induced consumption).[13] Having presented the short run capacity utilization rate and the key variables for analyzing the model, in section 2.3, we present the main results of the steady state solution.

2.3 Steady state solution and dynamics

We defined the model and its critical behavioral assumptions in the previous section. Since the debt to income ratio, l_w (see equation 5), partially determines the credit-financed workers' consumption growth rate, we can write the key variables and ratios of the model as functions of l_w. Taking into account that in the steady state growth path, we must have $g_k = g_y = g_l = g_z$ and $u = u_n$, the steady state solution of the model is given by the following equation:

$$(l_w^*)^2 + bl_w^* + c = 0 \tag{18}$$

Where $b = \frac{v}{u_n} + \frac{w}{\varphi_1 i_l}[i_l(\alpha_1 - \alpha_2) - \varphi_0]$ and $c = \frac{w}{\varphi_1 i_l}[1 - \alpha_1 w - \alpha_2(1 - s_f)\pi - \frac{v}{u_n}\varphi_0]$. As detailed in Appendix C, the derivation of equation 18 is obtained by replacing the steady state equation for the autonomous consumption to income ratio, derived from equation 16, as well as the steady state result for the marginal propensity to invest, h, defined in equations 12 - 13 in the original definition of steady state growth ($g_l^* = g_z = \varphi_0 - \frac{\varphi_1 i_l l_w^*}{w}$). From equation 18, we know that the steady state solution of the model is defined by two roots for the loans to income ratio. In other words, the

[11]The details of this derivation can be found in Appendix B.
[12]See section 2.3 for the steady state dynamics analysis.
[13]See Brochier and Freitas (2022) on this.

model has two possible long run equilibria fully determined by the parameters describing its internal dynamics. The immediate consequence of having credit-financed consumption as the autonomous, yet endogenous, component of demand that drives growth in a Supermultiplier model is twofold: (i) the steady state value of l_w is fully described by a combination of the model's core parameters; (ii) the values for the debt to income ratio will fundamentally define the steady state solutions, and its dynamics will explain all the other relevant variables. More precisely, for each l_w^* the full steady state solution of the model will be described by:

$$g^* = g_z^* = \varphi_0 - \frac{\varphi_1 i_l l_w^*}{w} \tag{19}$$

$$h^* = \frac{v g_z^*}{u_n} = \frac{v}{u_n}\left(\varphi_0 - \frac{\varphi_1 i_l l_w^*}{w}\right) \tag{20}$$

Since we have arrived at a two-equilibria steady state solution, we must now look at the stability of the dynamical system to determine which equilibrium is stable. The system describing the dynamics of the model is composed of three differential equations (21). We start with the fundamental dynamic equation of Supermultiplier models as described in equation 13, replace the short run capacity utilization rate (equation 17), obtaining the first equation of the dynamical system (21). Since g_l and l_{k_w} appear in this first equation, we must also derive their dynamic behavior. In order to do so, we depart from the very definition of the two variables and replace equation 14 (the rate of capital accumulation) in the equation for $\dot{l}_{k_w}$ and 5 (the rate of growth of autonomous consumption) in the equation for $\dot{g}_l$.[14]

$$\begin{array}{lllll} \dot{h} & = & h\gamma(u-u_n) & = & h\gamma\left[\frac{v(g_l-(\alpha_1-\alpha_2)i_l)l_{k_w}}{(s-h)} - u_n\right] \\ \dot{l}_{k_w} & = & l_{k_w}(g_l - g_K) & = & l_{k_w}\left[g_l - \frac{h(g_l-(\alpha_1-\alpha_2)i_l)l_{k_w}}{(s-h)}\right] \\ \dot{g}_l & = & g_l(g_z - g_l) & = & g_l\left[\varphi_0 - \varphi_1 \frac{i_l}{w}\frac{(s-h)}{(g_l-(\alpha_1-\alpha_2)i_l)} - g_l\right] \end{array} \tag{21}$$

The dynamics of the marginal propensity to invest and the loans to capital ratio depend on the same three variables: firms' propensity to invest, h; household debt growth rate, g_l; and household debt to capital ratio, l_{k_w}. As for the dynamics of household debt growth rate, it will only be affected by itself and firms' propensity to invest.

The analysis of the dynamical system described in 21 should allow us to understand whether or not the model is dynamically stable, i.e., converges to any of the steady state solutions, or unstable, if it does not converge to any of the two steady-state solutions described in equation 18.

We adopt two complementary procedures to address the stability of this non-linear three-dimensional system. The first is to find the Routh-Hurwitz determinants of the Jacobian matrix of the system. The second is to look at the dynamical system's behavior through a numerical simulation exercise, including the values of the Routh-Hurwitz determinants for each steady state.

[14] The details of these derivations can be found in Appendix D.

For the first approach, we look at the Jacobian matrix to study the asymptotical stability of the two equilibrium solutions found in the previous section, using a linear approximation (Gandolfo, 2010, p.385). [15]

It is worth noticing that the complexity of the model's analytical solution thwarts an economic interpretation of these stability conditions, except for the first Routh-Hurwitz condition, which easily translates into a negative trace of the original Jacobian matrix evaluated at the steady state position. [16] It is nonetheless important to mention that a few parallels can be drawn with some of the results found in Hein and Woodgate (2021). First, just as they have found an upper and lower limit for g_z^* under a Supermultiplier model, we have found upper and lower limits on the autonomous demand growth rate. Secondly, we notice from the conditions described that the stability of the steady-state growth rate will depend on the Harrodian instability parameter (γ) and the interest rate. This result is also similar to what is found in Hein and Woodgate (2021).[17]

3 Steady state analysis: numerical exercise

The numerical exercise proposed in this section aims at investigating (i) whether the system converges to one of the two equilibria of the model; (ii) whether the two equilibria - or one or none – are locally stable for a reasonable set of parameters by presenting the numerical values of the Routh-Hurwitz determinant of both of them.

Table 3 presents the values used in the model's calibration, which were calculated based on relevant intervals for the US economy. The latter presents a good case for illustration of the model's properties since it has been widely argued – see Barba and Pivetti (2009) and Cynamon and Fazzari (2008), for example – that credit-financed consumption has played an important role in boosting US recent economic performance but also contributed to its financial instability. More recently, Góes and Deleidi (2021) have also provided empirical evidence for the multiplier effect of autonomous consumption on US economic growth. In table 4, we present the steady state results for all relevant variables assuming the parameters' values shown in table 3.

[15]The details of the derivation of the Routh-Hurwitz determinants and the derivations for the Jacobian evaluated at the steady state can be found in Appendices E and F, respectively. See also Hein and Woodgate (2021), Nikolaidi (2014), and Spinola (2020) for other applications of this.

[16]The details of the derivation of the trace of the Jacobian is left to Appendix G.

[17]See Morlin (2022) for an analysis on the upper limit external (financial) constraints can impose on the growth rate in a Supermultiplier model and on how this limit may be avoided by managing autonomous government expenditures. See also Pedrosa et al. (2022) for an analysis of how the different institutional sectors' liabilities may affect the range of values for g_z compatible with a stable, steady state growth path.

Table 3: Calibration of the model

Parameter	Value
α_1	0.9
α_2	0.6
u_n	0.75
i_l	0.02
π	0.3
v	2
φ_0	0.075
φ_1	0.05
s_f	0.4
γ	0.02

At this point, a few things are worth pointing out in the table 3. First, all of the values used in this part of the paper have been chosen within a range of economically significant values calibrated to produce steady state results that are also economically meaningful. More precisely, the chosen values were controlled for a steady growth rate of the economy and household debt to income ratio that would make economic sense for the US economy, respectively $0 < g^* < 0.1$ and $0 < l_w^* < 1$.

One can also notice that most of the values we have found fall within a range that is under what is commonly adopted in the literature.[18] One exception worth mentioning is the value for the capital-output ratio, which is high but still falls within a range that makes sense economically.[19] It is important to mention this because v was one of the buffer variables for calibrating the model's steady state.

Secondly, some of the parameters of the consumption and savings functions were taken from the current literature on demand-led growth models. Following Setterfield and Kim (2020), we used the consumption and savings parameters suggested by other authors.[20] The exceptions were φ_0, which was chosen so as to guarantee $g_z > 0$, and α_2, which was calibrated, once again, to guarantee that steady state results made economic sense.

Finally, we have chosen a value for γ that guarantees steady state stability. As has been pointed out by a few authors (Allain, 2015; Fazzari et al., 2020; Ferri & Tramontana, 2020; Freitas & Serrano, 2015; Hein & Woodgate, 2021; Lavoie, 2016), the Supermultiplier model and the modified neo-Kaleckian model need firms' reaction (through the propensity to invest or trend growth rate of sales)

[18] See for instance Setterfield and David Avritzer (2020), Haluska et al. (2021) and Gahn (2020) and their estimations for the normal rate of capacity utilization.

[19] Some recent estimates show the capital-output ratio ranging from 0,8 to 1,5. See Fazzari et al. (2020) and Franke (2017).

[20] van Treeck (2009) suggests $b_1 = 0.05$ for one case of his analysis. The parameter b_1 is analogous to φ_1 here. Bunting (1998) also suggests a α_1 close to what we use in the simulations

to the gap between the actual and normal capacity utilization rates (γ here) to be low enough to prevent Harrodian instability. The results of this numerical exercise are presented in subsection 3.1.[21]

3.1 Results of the numerical exercise

Table 4: Steady State Results

Variables	Steady state 1	Steady state 2
l_w^*	0.971752	44.6616
g^*	0.0736118	0.0111977
$l_{k_w}^*$	0.364407	16.7481
h^*	0.196298	0.0298606
Local stability analysis (R-H Determinants)		
$\Delta_1(J^*)$	> 0	< 0
$\Delta_2(J^*)$	> 0	> 0
$\Delta_3(J^*)$	> 0	< 0

In table 4, we observe that the steady state equilibrium solution for l_w^* (18) results in two positive real roots. More precisely, there is one steady state position characterized by a lower debt ratio (l_w^*) but a higher rate of growth of the economy (g^*) (steady state one), and the other one by a high debt ratio and lower growth rate (steady state two). Steady state one arrives at more reasonable results for the combined growth rate and debt ratio values. Table 4 still reports steady state values for the marginal propensity to invest (h^*) and the debt to capital ratio ($l_{k_w}^*$). Finally, table 4 shows that steady state one is the only one with local stability besides making economic sense. The Routh Hurwitz determinants of the two steady state positions reported at the bottom of table 4 attest this.

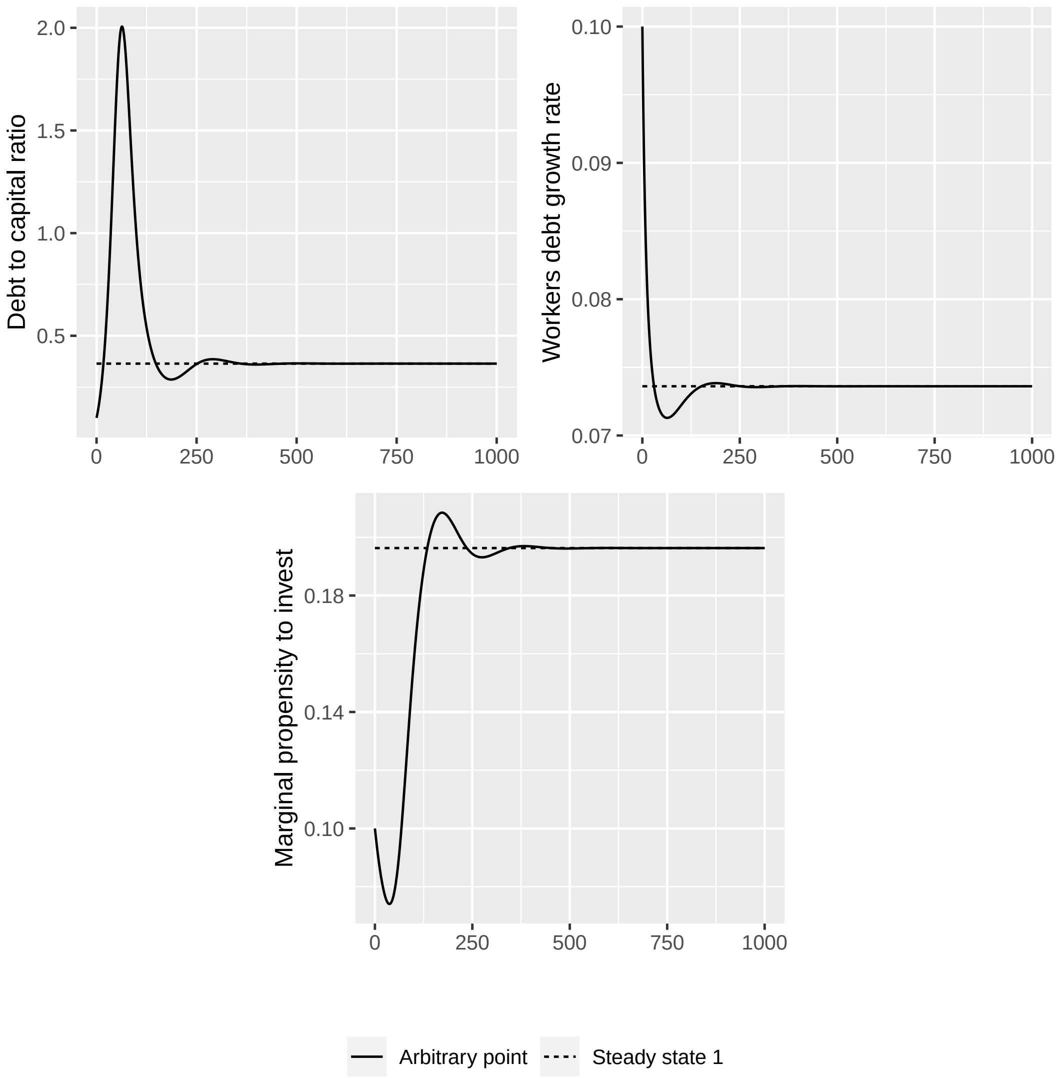
3.2 Reassessing the steady state stability of the model

Taking the features of steady state one into account, we further investigate its stability in this section. Figure 1 shows what happens to the dynamical system when we start at an arbitrary point outside the steady state equilibrium. The graphs present the simulation of the dynamical system described in 21 starting at two different points: i) steady state one as described in table 4; ii) an arbitrary point given by $(h, l_{k_w}, g_l) = (0.1, 0.1, 0.1)$. The top-right graph shows the results for the debt growth rate

[21]The codes of the numerical exercise are available upon request. We used *Wolfram Mathematica* for the solution and stability conditions. For the simulation of the steady state dynamics and the shocks in the parameters, we used the packages *sfcr* and *deSolve* developed for *R*.

g_l, the top-left for the debt to income ratio l_{k_w}, and the bottom graph for the marginal propensity to invest.

Figure 1: Steady state stability



Debt to capital ratio, growth of workers debt and marginal propensity to invest results for 1000 iterations of Euler's approximation of the dynamical system (21) starting at steady state one (dashed line) and an arbitrary point (full line).

The full black lines on the graphs represent what happens to the variables after 1000 iterations of the dynamical starting from the arbitrary point. Meanwhile, the dashed lines represent what happens to the variables after 1000 iterations of the same dynamical system, but now starting from the stable steady state position (steady state one). As we can see, for all variables the dynamics seem to converge to the steady state position in which the debt growth rate is around 0.074 (7.4%), and the debt to capital ratio is around 0.36 (36%) – the values estimated for steady state one.

This illustration exercise confirms steady state one as the stable position of the dynamical system described in 21. Besides, the numerical analysis has also allowed us to exclude steady state two as economically feasible (for this numerical set of parameters) since its value for the household debt ratio is way too far from the values this ratio assumes in reality. That said, in section 4, we further explore steady state one by looking at the numerical derivatives of this steady state solution with respect to the main parameters of the model: the wage share, the interest rate, the exogenous autonomous consumption and workers' sensitivity to their debt burden.

4 Effects of the parameters on the stable steady state position

There remains to talk about the main economic features of the steady state growth path. For the reasons stated in section 3, here we focus on steady state one, addressing the effects of the core parameters on this steady state. We numerically solve (and analyze) the derivatives for the steady state parameter values of table 3.

Table 5: Effects of the parameters on the steady state growth path

Parameters	Variables		
	l_w^*	g^*	h^*
φ_0	−	+	+
φ_1	−	−	−
w	−	+	+
i_l	+	−	−

Notes: the plus and minus signs represent the signs of the numerical partial derivatives of the main variables of the model in the state steady growth path (steady state one) with respect to the parameters presented in the first column;
The values for the parameters employed to calculate the derivatives are the same presented in table 3.

The main variables of interest are the long run household debt to income ratio (l_w^*), growth rate (g^*), and firms' propensity to invest (h^*). Therefore, we focus on the key parameters' effects on these variables. In turn, the key parameters are the exogenous component of household autonomous consumption growth rate (φ_0), the sensitivity of household autonomous consumption growth rate to the debt service ratio (φ_1), the wage share (w), and the interest rate (i_l). In table 5, we present the signs of the derivatives for the equilibrium values of the household debt ratio (second column), growth rate (third column), and firms' propensity to invest (fourth column) with respect to the key four parameters (first column).

As expected in the Supermultiplier model, the exogenous component of the autonomous consumption growth rate has a negative effect on the household debt ratio in the long run. This happens since, cet. par., a faster pace of autonomous consumption will increase capacity utilization in the short run, triggering firms' reaction to adjust capacity to demand. As capital and output grow temporarily faster than household credit demand, the household debt ratio will decrease in the long run. Therefore, we notice that the paradox of debt, in the long run, is a result of the model, as in canonical versions of the Supermultiplier model that assume there is only one source of non-capacity creating autonomous expenditures (Allain, 2015; Brochier & Freitas, 2022; Freitas & Serrano, 2015; Lavoie, 2016).[22]. However, we are aware that this is not a necessary result of the Supermultiplier closure, as when there is more than one source of autonomous injections, a faster pace of autonomous spending in one institutional sector will be associated with a higher debt ratio for that respective sector (Freitas & Christianes, 2020; Pedrosa et al., 2022).

We notice that increasing households' sensitivity to credit conditions contributes to a higher household debt ratio in the long run. This happens since an increase in households' sensitivity to credit conditions would harm the pace of households' credit demand and, therefore, the activity level. As the slower consumption out of credit contributes to reducing the capacity utilization rate and firms adjust their capacity utilization to the (presumably permanent) lower demand, the output growth rate will fall to a larger extent than the initial fall in the autonomous demand growth rate. Therefore, a higher (lower) sensitivity parameter will lead to a higher (lower) household debt ratio in the long run.

An increase in the wage share reduces the household debt ratio in the long run, as it alleviates households' debt burden and further stimulates demand through induced consumption. At last, a higher interest rate[23] on loans will lead to a higher household debt ratio in the long run as it increases households' debt burden and puts a drag on aggregate demand since the net effect on households' consumption will be negative as long as the propensity to consume out of wages is higher than the one out of financial income.

Regarding the long run growth rate, the exogenous component of the credit-financed consumption growth rate will positively affect growth since it represents precisely the autonomous factors that may fasten the pace of household credit demand. About the sensitivity of growth to credit conditions, the higher the households' sensitivity to the average debt burden, the lower the long run growth rate. As for the wage share, it has a mild positive effect on the growth rate. It stimulates induced consumption and, therefore, may lead firms to perceive the higher pace of demand as permanent, temporarily fastening the pace of accumulation. It also directly reduces workers' debt burden by increasing their share of total income. A higher interest rate unequivocally reduces the growth rate in the long run as it increases the debt burden for any household leverage ratio but also contributes to a higher household debt ratio in the long run.

[22]For more on this see Brochier and Freitas (2022).

[23]It is worth mentioning that as Hein and Woodgate (2021) pointed out, there is a limit to the interest rate increase without the model losing stability. In the case of this model, this limit is related to how large is the exogenous component of the consumption function, φ_0, and how small is workers' sensitivity to their debt burden, φ_1.

In what concerns firms' investment behavior, a faster pace of credit-financed consumption arising from autonomous and exogenous demand would lead to a higher propensity to invest in the long run. This happens because firms will adjust their capacity utilization to the desired one, which requires a permanently higher investment rate as the trend growth rate is also higher. In turn, a higher sensitivity of autonomous consumption growth to credit conditions would have the opposite effect on firms' propensity to invest: it reduces the pace of credit-financed consumption and the output growth rate, and as firms react to the lower trend of demand, the propensity to invest decreases. A higher wage share reduces worker households' debt burden and contributes to a faster pace of credit-financed consumption in the long run. Once again, this requires a higher propensity to invest as firms try to keep the capacity utilization around the desired level. At last, the interest rate will negatively affect firms' propensity to invest since it reduces the growth rate of credit-financed consumption, and, thus, firms adjust their investment behavior downwards, keeping capacity utilization compatible with the lower expected growth rate of sales.

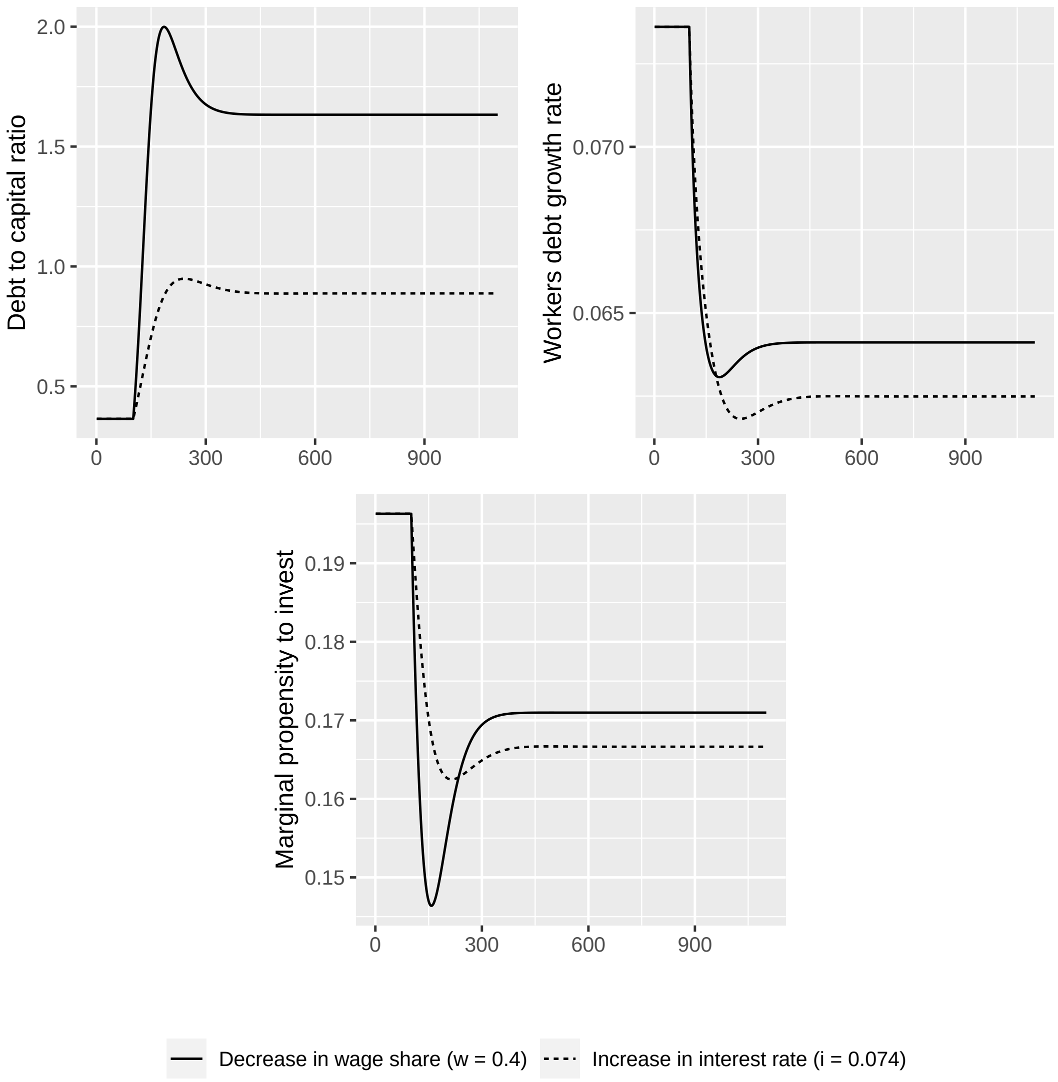
At last, figure 2 illustrates the effects of decreasing the wage share (solid line) and increasing the interest rate (dashed line) on the debt to capital ratio (top-left), on the workers' debt growth rate (top-right) and on the marginal propensity to invest (bottom).

In line with the derivatives of table 5, the graphs in figure 2 show that a decrease in the wage share (from 0.7 to 0.4) increases the debt to capital ratio and decreases the workers' debt growth rate and firms' propensity to invest, which are the same qualitative effects that an increase in the interest rate (from 0.02 to 0.074) will have on these variables. These results are the direct consequence of assuming a credit-financed consumption function as described by equations (1) - (4). These graphs also illustrate that the dynamical system described in equation 21 will converge to new steady state positions when parameters are changed within reasonable ranges.

The partial derivatives and the shocks to the wage share and interest rate highlight some of the fundamental features of the model. First, changes in the wage share affect the economy's growth rate in the long run. Contrariwise to what has been argued in recent criticism of the Supermultiplier (Nikiforos, 2018), income distribution may have permanent growth effects, affecting the trend and not only the average growth rate. This was already pointed out in Brochier and Macedo e Silva (2019). This is the case when the growth rate of autonomous expenditures is assumed to be partially endogenous to the model or "semi-autonomous".

Secondly, financial variables affect real spending decisions in the long run. This is captured in the model by the debt service impact on the pace of worker households' credit demand for consumption. A worsening in credit conditions, as represented by an increase in the interest rate and stronger reaction of credit demand for consumption to the burden of debt, could lower demand and growth initially through consumption and, in a second moment, induce firms to cut back on investment as they expect a sluggish level of activity, reducing their investment ratio.

Figure 2: Effects of interest rate and wage share shocks



5 Conclusion

Supermultiplier growth models usually present the autonomous demand component that drives growth as exogenously determined. Most certainly for simplification purposes. That said, exploring the determinants of autonomous demand within the model seems relevant to understand how they affect growth. With that in mind, we presented a Supermultiplier growth model where credit-financed household consumption is the autonomous demand component that drives growth. However, instead of assuming an exogenous growth rate for this component, we consider it to be partially determined by the financial burden that loans may impose on household consumption.

We have solved the model analytically and found two steady state equilibria. From the steady state solution, we derived an important feature of the model: in the steady state, all of the relevant variables - the growth rate and the marginal propensity to invest - become a function of the debt to income ratio. Therefore, understanding what happens to this ratio is fundamental to understanding the long run dynamics of the model.

Due to the complex nature of the model's analytical solution, further clarification of the model's properties – such as the stability conditions and the economic interpretation of the two equilibria – required a numerical illustration. From the numerical exercise, we found that the steady state solution with higher growth and lower debt ratio (steady state one) is compatible with local stability. Besides seemingly less stable, the lower growth-high debt ratio steady state (steady state two) also presents weaker economic meaning for the chosen set of parameters, as the household debt ratio is far beyond what is observable in real economies.

We also looked at the relevant derivatives of the higher growth-lower debt ratio steady state (steady state one). We emphasize two results of this final illustration: (i) income distribution may have a permanent effect on the long run growth rate, as an increase in the wage share reduces the debt to income ratio, increasing the economy's growth rate; (ii) financial variables – such as the interest rate and the sensitivity of workers to their debt burden – affect steady state variables. More precisely, an increase in interest rates or households' sensitivity to their debt burden decreases the growth rate and the investment to output ratio in the long run.

These results show that the Supermultiplier model can account for the permanent effects of both real and financial variables on the long run growth rate. This was made possible by combining an autonomous demand-led growth model where credit-financed household consumption drives growth but is partially explained by households' reaction to their debt burden. This simple model produced interesting insights and helped clarify important issues due to the modeling of semi-autonomous expenditures.

However, we must keep in mind its very high level of abstraction. Representing real economies' experiences meaningfully would require a much higher complexity of real and financial interactions. As a next step in that direction, we intend to incorporate other institutional sectors, such as the government, their spending behavior, and financial counterparts, in future research.

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Appendix A - Defining Equations

Worker households' consumption function

$$C_w = \alpha_1 Y_{dw} + Z \tag{6}$$

$$Y_{dw} = W + i_m M_w - i_l L_w \tag{7}$$

$$\dot{L}_w = Z \tag{1}$$

$$Z = Z_0 e^{g_z t} \tag{2}$$

$$g_z = \varphi_0 - \varphi_1 ds \tag{3}$$

$$V_w = M_w - L_w \tag{22}$$

Rentier households' consumption function

$$C_r = \alpha_2 Y_{dr} \tag{11}$$

$$Y_{dr} = FD + FB + i_m M_r \tag{18}$$

$$S_r = Y_{dr} - C_r \tag{23}$$

$$V_r = S_r + e\dot{p}_e \tag{24}$$

$$p_e = \frac{\lambda V_r}{e} \tag{25}$$

$$\lambda = \lambda_0 - i_m \tag{26}$$

Firms' equations

$$I = hY = \dot{K} \tag{12}$$

$$\dot{h} = h\gamma(u - u_n) \tag{13}$$

$$p_e\dot{e} = I - FU \tag{27}$$

$$FU = s_f \pi Y \tag{28}$$

$$FD = (1 - s_f)\pi Y \tag{9}$$

Banks: borrowing decisions

We assume, for simplicity of the analysis, that $i_m = 0$ and that the interest rate on loans, i_l is determined by a mark up (ρ_b) over an exogenous interest rate (i) (as if defined by a monetary authority) such that:

$$i_l = (1 + \rho_b)i \tag{29}$$

Equation 30 is the redundant equation of our stock-flow consistent model:

$$M = L_w \tag{30}$$

$$FB = i_l L_w - i_m M \tag{10}$$

Appendix B - Short-run solution

$$Y = C_w + C_r + I \tag{31}$$

$$Y = \alpha_1(W + i_m M_w - i_l L_w) + Z + \alpha_2[(1 - s_f)\pi Y + i_l L_w - i_m M + i_m M_r] + hY \tag{32}$$

Assuming that $i_m = 0$ and $w = (1 - \pi)$:

$$Y = \alpha_1(wY - i_l L_w) + Z + \alpha_2((1 - s_f)\pi Y + i_l L_w) + hY \tag{33}$$

And further defining $s = 1 - \alpha_1 w - \alpha_2(1 - s_f)\pi$:

$$Y(s - h) = (\alpha_2 - \alpha_1) i_l L_w + Z \tag{34}$$

Since it will be instrumental for deriving the steady state solution of the model, we also present the short run autonomous consumption to output ratio (z) (equation 35). Equation 35 is obtained by dividing both sides of equation 16 by the output level and solving it for the autonomous consumption ratio.

$$z = s - h - (\alpha_2 - \alpha_1) i_l l_w \tag{35}$$

Where $z = \frac{Z}{Y}$ is the autonomous consumption to income ratio.

Appendix C - Steady State Solution

Since under steady state we must have $g_l = g_z$ this then results in:

$$\frac{z^*}{l_w^*} = \varphi_0 - \varphi_1 \frac{i_l}{w} l_w^* \tag{36}$$

If we then assume that under steady state $z = z^*$ and solve the equation above for l_w^*, we find that:

$$\varphi_1 i_l (l_w^*)^2 - \varphi_0 w l_w^* + z^* w = 0 \tag{37}$$

From equation 12 we also have that:

$$h^* = \frac{v g_z}{u_n} \tag{38}$$

Replacing equation 3 into equation 38:

$$h^* = \frac{v}{u_n}\varphi_0 - \frac{v}{u_n}\varphi_1 \frac{i_l}{w} l_w^* \tag{39}$$

Defining $\varphi_0' = \frac{v}{u_n}\varphi_0$ and $\varphi_1' = -\frac{v}{u_n}\frac{i_l}{w}\varphi_1$, we then have that:

$$h^* = \varphi_0' + \varphi_1' l_w^* \tag{40}$$

From equation 35 we must have that:

$$z^* = s - h^* - (\alpha_2 - \alpha_1) i_l l_w^* \tag{41}$$

Replacing equation 40 in the equation above we get that:

$$z^* = s - \varphi_0' - \varphi_1' l_w^* - (\alpha_2 - \alpha_1) i_l l_w^* \tag{42}$$

Defining $\beta_0 = s - \varphi_0'$ and $\beta_1 = \varphi_1' + (\alpha_2 - \alpha_1) i_l$, we get that:

$$z^* = \beta_0 - \beta_1 l_w^* \tag{43}$$

We can also define $\alpha_0' = \dfrac{\varphi_0 w}{\varphi_1 i_l}$ and $\alpha_1' = \dfrac{w}{\varphi_1 i_l}$ and rewrite equation 37 as:

$$(l_w^*)^2 - \alpha_0' l_w^* + \alpha_1' z^* = 0 \tag{44}$$

Consequently, we have the following system of two equations and two variables:

$$\begin{aligned} z^* &= \beta_0 - \beta_1 l_w^* \\ (l_w^*)^2 - \alpha_0' l_w^* + \alpha_1' z^* &= 0 \end{aligned} \tag{45}$$

To solve the system above we replace one equation into the other and we get:

$$(l_w^*)^2 - \alpha_0' l_w^* + \alpha_1' \beta_0 - \alpha_1' \beta_1 l_w^* = 0 \tag{46}$$

Defining $\beta_1' = \alpha_0' + \alpha_1' \beta_1$ and rearranging, we get that:

$$(l_w^*)^2 - \beta_1' l_w^* + \alpha_1' \beta_0 = 0 \tag{47}$$

Where $\beta_1' = \frac{w}{\varphi_1 i_l}(\varphi_0 + (\alpha_2 - \alpha_1) i_l) - \frac{v}{u_n}$ and $\alpha_1' \beta_0 = \frac{w}{\varphi_1 i_l}(s - \frac{v}{u_n}\varphi_0)$, which can also be rewritten as:

$$(l_w^*)^2 + b l_w^* + c = 0 \tag{48}$$

Where $b = -\beta_1' = \frac{v}{u_n} + \frac{w}{\varphi_1 i_l}(i_l(\alpha_1 - \alpha_2) - \varphi_0)$ and $c = \frac{w}{\varphi_1 i_l}(1 - \alpha_1 w - \alpha_2 (1 - s_f)\pi - \frac{v}{u_n}\varphi_0)$

$$l_w^* = -b \pm \sqrt{b^2 - 4c} \tag{49}$$

Substituting (49) into (5), we get the equilibrium growth rate:

$$g^* = \varphi_0 - \varphi_1 \frac{i_l}{w}\left[-b \pm \sqrt{b^2 - 4c}\right] \tag{50}$$

Where the long run growth rate for steady state one is:

$$g_1^* = \varphi_0 - \varphi_1 \frac{i_l}{w}\left[-b - \sqrt{b^2 - 4c}\right] \tag{51}$$

And for steady state two is:

$$g_2^* = \varphi_0 - \varphi_1 \frac{i_l}{w}\left[-b + \sqrt{b^2 - 4c}\right] \tag{52}$$

Appendix D - Steady State Dynamics Derivation

Given that:

$$\dot{h} = h\gamma\left[\frac{v[g_l - (\alpha_1 - \alpha_2)i_l]l_{k_w}}{(s-h)} - u_n\right] \tag{53}$$

First, we look for an equation to describe $\dot{l}_{k_w}$:

$$\dot{l}_{k_w} = \frac{\dot{L}_w}{L_w}\frac{L_w}{K} - \frac{L_w}{K}\frac{\dot{K}}{K} \tag{54}$$

$$\dot{l}_{k_w} = l_{k_w}(g_l - g_K) \tag{55}$$

$$\dot{l}_{k_w} = l_{k_w}(g_l - h\frac{u}{v}) \tag{56}$$

Then we look for an equation to describe $\dot{g}_l$:

$$\dot{g}_l = \frac{\dot{Z}L_w}{L_w^2} - \frac{Z\dot{L}_w}{L_w^2} = \frac{Z}{L_w}\left[\frac{\dot{Z}}{Z} - \frac{\dot{L}_w}{L_w}\right] \tag{57}$$

$$\dot{g}_l = g_l(g_z - g_l) = g_l(\varphi_0 - \varphi_1\frac{i_l}{w}l_w - g_l) \tag{58}$$

Finally, since $l_{k_w} = \frac{L_w}{K} = \frac{L_w}{Y}\frac{Y}{K} = l_w\frac{u}{v}$, we must have:

$$l_w = \frac{l_{k_w}v}{[v(g_l + (\alpha_2 - \alpha_1)i_l)l_{k_w}]/(s-h)} = \frac{(s-h)}{(g_l + (\alpha_2 - \alpha_1)i_l)} \tag{59}$$

Such that:

$$\dot{g}_l = g_l\left[\varphi_0 - \varphi_1\frac{i_l}{w}\frac{(s-h)}{(g_l + (\alpha_2 - \alpha_1)i_l)} - g_l\right] \tag{60}$$

Appendix E - Derivation of the Jacobian matrix and its Routh-Hurwitz determinants

Since our Jacobian matrix is given by:

$$\begin{bmatrix} h_h & h_k & h_g \\ k_h & k_k & k_g \\ g_h & g_k & g_g \end{bmatrix} \tag{61}$$

Where h_h is defined as the partial derivative of $\dot{h}$ with respect to h, h_k with respect to l_{k_w}, h_g with respect to g_l and h_w with respect to l_w. k_x, g_x and w_x represent the partial derivatives of $\dot{l}_{k_w}$, $\dot{g}_l$ and $\dot{l}_w$, respectively. Since it is easy to see that $g_k = 0$, we can write our $[J - \lambda I]$ matrix as:

$$\begin{bmatrix} (h_h - \lambda) & h_k & h_g \\ k_h & (k_k - \lambda) & k_g \\ g_h & 0 & (g_g - \lambda) \end{bmatrix} \tag{62}$$

Therefore, $Det[J - \lambda I]$ will be given by:

$$(g_h) \begin{matrix} h_k & h_g \\ (k_k - \lambda) & k_g \end{matrix} + (g_g - \lambda) \begin{matrix} (h_h - \lambda) & h_k \\ k_h & (k_k - \lambda) \end{matrix} \tag{63}$$

$$Det[J - \lambda I] = g_h[k_g h_k - h_g(k_k - \lambda)] + (g_g - \lambda)[(h_h - \lambda)(k_k - \lambda) - h_k k_h] \tag{64}$$

$$\begin{aligned} = \quad & g_h k_g h_k - g_h h_g k_k + g_h h_g \lambda - g_g h_k k_h + h_k k_h \lambda \\ & +(g_g - \lambda)[h_h k_k - \lambda(h_h + k_k) + \lambda^2] \end{aligned} \tag{65}$$

$$\begin{aligned} = \quad & g_h k_g h_k - g_h h_g k_k + g_h h_g \lambda - g_g h_k k_h + h_k k_h \lambda \\ & +g_g h_h k_k - g_g \lambda(h_h + k_k) + g_g \lambda^2 - \lambda h_h k_k + \lambda^2(h_h + k_k) - \lambda^3 \end{aligned} \tag{66}$$

$$\begin{aligned} = \quad & -\lambda^3 + \lambda^2(h_h + k_k + g_g) + \lambda(h_k k_h + g_h h_g - h_h k_k - g_g h_h - g_g k_k) \\ & +g_g h_h k_k + g_h k_g h_k - g_h h_g k_k - g_g h_k k_h \end{aligned} \tag{67}$$

$$\begin{aligned} = \quad & \lambda^3 - \lambda^2(h_h + k_k + g_g) + \lambda(h_h k_k + g_g h_h + g_g k_k - h_k k_h - g_h h_g) \\ & -g_g h_h k_k - g_h k_g h_k + g_h h_g k_k + g_g h_k k_h \end{aligned} \tag{68}$$

Which then gives us the following characteristic polynomial:

$$a_0 \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 \tag{??}$$

Where: $a_0 = 1$; $a_1 = -(g_g + h_h + k_k)$; $a_2 = (h_h k_k + g_g h_h + g_g k_k - h_k k_h - g_h h_g)$ and $a_3 =$

$g_h h_g k_k + g_g h_k k_h - g_g h_h k_k - g_h k_g h_k$ The Routh-Hurwitz matrix then becomes:

$$\begin{bmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{bmatrix} \tag{69}$$

From Gandolfo (2010, p.239, p.269), the conditions for local stability around the two steady-state equilibria are given by:

$$\begin{cases} \Delta_1 = a_1 > 0 \\ \Delta_2 = a_2 a_1 - a_0 a_3 > 0 \text{ and} \\ \Delta_3 = a_3 \Delta_2 > 0 \end{cases}$$

Appendix F - Derivatives of the Jacobian matrix evaluated at the steady state

$$h_h = \frac{\partial \dot{h}}{\partial h} = \gamma \left[\frac{vs(g_l + (\alpha_2 - \alpha_1) i_l) l_{k_w}}{(s-h)^2} - u_n \right] = \frac{\gamma u_n h^*}{(s - h^*)} \tag{70}$$

$$h_k = \frac{\partial \dot{h}}{\partial l_{k_w}} = \frac{h \gamma v (g_l + (\alpha_2 - \alpha_1) i_l)}{(s-h)} = \frac{h^* \gamma v}{l_w^*} \tag{71}$$

$$h_g = \frac{\partial \dot{h}}{\partial g_l} = \frac{h \gamma v l_{k_w}}{(s-h)} = \frac{h^* \gamma l_w^* u_n}{(s - h^*)} \tag{72}$$

$$k_h = \frac{\partial \dot{l}_{k_w}}{\partial h} = -\frac{s(g_l + (\alpha_2 - \alpha_1) i_l) l_{k_w}^2}{(s-h)^2} = -\frac{u_n^2 l_w^* s}{v^2 (s - h^*)} \tag{73}$$

$$k_k = \frac{\partial \dot{l}_{k_w}}{\partial l_{k_w}} = g_l - \frac{2h(g_l + (\alpha_2 - \alpha_1) i_l) l_{k_w}}{(s-h)} = -g_z^* \tag{74}$$

$$k_g = \frac{\partial \dot{l}_{k_w}}{\partial g_l} = l_{k_w} \left[1 - \frac{l_{k_w} h}{(s-h)} \right] = \frac{l_w^* u_n}{v} \left[1 - \frac{l_w^* u_n h^*}{v(s - h^*)} \right] \tag{75}$$

$$g_h = \frac{\partial \dot{g}_l}{\partial h} = \frac{g_l i_l \varphi_1}{w(g_l + (\alpha_2 - \alpha_1) i_l)} = \frac{g_z^* i_l \varphi_1}{w(g_z^* + (\alpha_2 - \alpha_1) i_l)} \tag{76}$$

$$\begin{aligned} g_g &= \frac{\partial \dot{g}_l}{\partial g_l} = [\varphi_0 - \varphi_1 \frac{i_l}{w} \frac{(s-h)}{(g_l + (\alpha_2 - \alpha_1) i_l)} - g_l] + g_l [-1 - \varphi_1 \frac{i_l}{w} \frac{(s-h)(-1)}{(g_l + (\alpha_2 - \alpha_1) i_l)^2}] \\ &= g_z^* [-1 + \varphi_1 \frac{i_l (s - h^*)}{w (g_z^* + (\alpha_2 - \alpha_1) i_l)^2}] = g_z^* [-1 + \frac{(g_z^* - \varphi_0)}{(g_z^* + (\alpha_2 - \alpha_1) i_l)}] \end{aligned} \tag{77}$$

Appendix G - Trace of the Jacobian matrix

$$T(J^*) = h_h^* + k_k^* + g_g^* \tag{78}$$

We also know from the previous derivation of the jacobian in steady-state that:
i) $h_h^* = \frac{\gamma u_n h^*}{(s-h^*)}$;
ii) $k_k^* = -g_z^*$;
iii) $g_g^* = g_z^*[-1 + \frac{(g_z^* - \varphi_0)}{(g_z^* + (\alpha_2 - \alpha_1)i_l)}]$.

As a result, we must also have that:

$$T(J^*) = \frac{\gamma u_n h^*}{(s - h^*)} - 2g_z^* + g_z^*\Big[\frac{(g_z^* - \varphi_0)}{(g_z^* + (\alpha_2 - \alpha_1)i_l)}\Big] \tag{79}$$

Since:

$$h^* = \frac{v g_z^*}{u_n} \tag{80}$$

Then:

$$T(J^*) = g_z^*\Big[\frac{\gamma v(g_z^* + (\alpha_2 - \alpha_1)i_l) - 2(g_z^* + (\alpha_2 - \alpha_1)i_l)(s - h^*) + (g_z^* - \varphi_0)(s - h^*)}{(s - h^*)(g_z^* + (\alpha_2 - \alpha_1)i_l)}\Big] \tag{81}$$

However, since we are already assuming that $g_z^* > 0$, $(s - h^*) > 0$ and $(g_z^* + (\alpha_2 - \alpha_1)i_l) > 0$, then in order for $T(J^*) < 0$, we must have:

$$\gamma v(g_z^* + (\alpha_2 - \alpha_1)i_l) - 2(g_z^* + (\alpha_2 - \alpha_1)i_l)(s - h^*) + (g_z^* - \varphi_0)(s - h^*) < 0 \tag{82}$$

And since, once again:

$$h^* = \frac{v g_z^*}{u_n} \tag{83}$$

Then:

$$\gamma v(g_z^* + (\alpha_2 - \alpha_1)i_l) - (s - \frac{v g_z^*}{u_n})[g_z^* + 2(\alpha_2 - \alpha_1)i_l + \varphi_0] < 0 \tag{84}$$

And:

$$\begin{aligned} &\gamma v g_z^* + \gamma v(\alpha_2 - \alpha_1)i_l - s g_z^* - s2(\alpha_2 - \alpha_1)i_l - s\varphi_0 \\ &+\tfrac{v g_z^*}{u_n} g_z^* + \tfrac{v g_z^*}{u_n} 2(\alpha_2 - \alpha_1)i_l + \tfrac{v g_z^*}{u_n}\varphi_0 \qquad < 0 \end{aligned} \tag{85}$$

Therefore, in order to have a negative trace we will need:

$$g_z^2 + g_z(\gamma u_n - \frac{s u_n}{v} + 2(\alpha_2 - \alpha_1)i_l + \varphi_0) + \frac{u_n}{v}(\gamma v(\alpha_2 - \alpha_1)i_l - s2(\alpha_2 - \alpha_1)i_l - s\varphi_0) < 0 \tag{86}$$

Which results in the following constraint on g_z^*:

$$-b_1 - \sqrt{b_1^2 - 4c_1} < g_z^* < -b_1 + \sqrt{b_1^2 - 4c_1} \tag{87}$$

Where:

$$b_1 = \gamma u_n - \frac{su_n}{v} + 2(\alpha_2 - \alpha_1)i_l + \varphi_0$$

And:

$$c_1 = \frac{u_n}{v}(\gamma v(\alpha_2 - \alpha_1)i_l - s2(\alpha_2 - \alpha_1)i_l - s\varphi_0)$$

As expected, one of the necessary conditions for the local stability of the steady state results in constraints on the autonomous demand growth rate, which is directly related to limits in the debt-to-income ratio evaluated at the steady state:

$$\frac{w}{i_l\varphi_1}\left[\varphi_0 + b_1 - \sqrt{b_1^2 - 4c_1}\right] < l_w^* < \frac{w}{i_l\varphi_1}\left[\varphi_0 + b_1 + \sqrt{b_1^2 - 4c_1}\right] \quad (88)$$