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# Social common capital accumulation and fiscal sustainability in a wage-led growth economy

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## Abstract

We build a three-dimensional Kaleckian dynamic model, incorporating government-provided social common capital's long-run stock effects and subsequent debt accumulation. We investigate how fiscal stance changes and demand and distributional impacts in a wage-led growth regime affect social common capital accumulation, economic growth, and stability. The Keynesian stability and Domar conditions are necessary for a long-run economically meaningful steady state, while a proactive fiscal stance promotes higher economic growth and a more sustainable economy. A higher wage share stabilises the economy by increasing the likelihood of satisfying the Domar condition, realising an equitable workers/capitalists income distribution, and establishing a resilient economy.

**Keywords:** Social common capital, fiscal sustainability, government debt, wage-led growth, Kaleckian model

**JEL Classification:** E11. E12, E25, H54, O40

## 1 Introduction

One of the most salient aspects of contemporary economies is the government's proactive role. The global financial crisis and COVID-19 pandemic revealed the market economy's fragility; at the same time, the government's role became increasingly significant in building an economy resilient to these shocks. Accordingly, much literature has explored how government interventions can ensure economic stability and act as the driving force for long-run economic growth (Commendatore, Panico, & Pinto, 2011; Dutt, 2013; Hein, 2018; Ko, 2019; Hein & Woodgate, 2021; Obst, Onaran, & Nikolaidi, 2020; Onaran, Oyvat, & Fotopoulou, 2022; Oyvat & Onaran, 2022; Parui, 2021; Ribeiro & Lima, 2019).

Government expenditures support private economic activities as an important component of aggregate demand. Among government expenditures, social common capital promotes private firm investment by expanding social infrastructure such as highways, ports, public transportation, and telecommunication facilities. Moreover, governments impose taxes on the private economy to obtain financial resources but run a fiscal deficit when their expenditures exceed revenue. Thus, government expenditures have three effects on the macroeconomy: they stimulate aggregate demand, supply social common capital accumulation to the private economy, and may lead to fiscal deficits and debt accumulation.

Among government activities, this study focuses on the aspects in economic growth analysis of providing for social common capital and the associated government debt accumulation. Social common capital is a broad category; in particular, Uzawa (2005) highlights the natural environment, social infrastructure, and institutional capital, including health care and education. Social common capital also commonly and broadly benefits a market economy's economic activities. In Uzawa's economic thought, social common capital ensures that citizens have the basic rights to live healthy and culturally minimal lives. Accumulating social infrastructure, a part of social common capital, has been econometrically shown to induce private investment demand and significant productivity gains while supporting an equitable income distribution (Obst et al., 2020; Onaran et al., 2022; Oyvat and Onaran, 2022). Moreover, enhancing social common capital during difficult consecutive crises will be of ever greater importance for establishing a resilient and sustainable economy.

Indeed, the roles of social common capital in macroeconomic dynamics seem to echo the problem of post-Keynesian growth theory, as social common capital has an ideal impact on effective demand and income distribution. Nonetheless, theoretically, the rare comprehensive studies on the relationship between government debt accumulation, social common capital accumulation, income distribution, and economic growth remain quite partial. Specifically, although Commendatore et al. (2011) build a Kaleckian growth model with government expenditures, they do not incorporate its debt accumulation. However, Dutt (2013) is a Keynesian demand-led growth model with debt accumulation. Interestingly, his model explicitly embodies the crowding-in effects of government investment for firm investment but does not consider a functional income distribution's role in economic growth. By contrast, Ko (2019) and Parui (2021) compare how a change in government expenditures impacts growth, employment, and debt accumulation under wage-led and profit-led growth regimes. Ribeiro and Lim (2019) identify the stability conditions required for public debt to converge when a government expenditures ceiling exists. These studies commonly appreciate the positive effect of government expenditures on growth; however, because government expenditures are aggregated, the studies do not differentiate between consumption and investment or their different effects on long-run macroeconomic performance. Hein (2018) and Hein and Woodgate (2021) also employ aggregate treatment of government expenditures, considering them non-capacity creating autonomous expenditures. Focusing on the Harrodian instability issue, the effect of income distribution is not explicitly investigated in these models. Moreover, these studies consider only the flow effect of aggregate government expenditures on economic growth. Consequently, it is not clear how government investment builds social common capital or how the stock of this capital eventually leads to long-run economic growth.

Our model shares the importance of government expenditures with these post-Keynesian growth models but differentiates between consumption and investments in social common capital with debt accumulation. Specifically, we comprehensively examine these relationships in an economy with a wage-led growth regime. Wage-led growth regimes have been empirically confirmed historically in most advanced countries (e.g. Blyth, Pontusson, & Baccaro, 2022; Lavoie & Stockhammer, 2013; Storm & Naastepad, 2012). Therefore, it deserves detailed scrutiny, which motivates our theoretical analyses of a wage-led growth economy. Our model is also inspired by Tavani and Zamparelli's (2016, 2017, 2020) series of works, which incorporate the dual effects of public expenditures for effective demand and labour productivity growth. Compared with these, our model does not incorporate a detailed productivity growth effect. Instead, incorporating the stock effect of social common capital to induce firms' capital accumulation in the long run, our model sheds light more on the wage- and demand-led growth mechanisms and their interactions with a government's debt dynamics. In this sense, our direction is more post-Keynesian than the Marxian or classical models found in Tavani and Zamparelli (2016, 2020). Our model is also an extension of Nishi and Okuma's (2023) model, which considers the flow effect of government expenditures on effective demand. Compared to their study, our model does not allow for both profit-led and wage-led growth regimes; instead, it explicitly considers the long-run stock effect of social common capital and the income distribution effect under a wage-led growth regime in more detail.

A theoretical innovation of our growth model is introducing the long-run stock effects of government-provided social common capital on economic growth and the government's subsequent debt accumulation, which are all endogenously determined. Based on this model, this study investigates whether a proactive fiscal stance, higher economic growth with affluent social common capital, and a government's fiscal sustainability can be simultaneously realised in the long run. Our model supposes the social infrastructure among the variety of social common capital that promotes private investment inter alia, whereas fiscal sustainability is considered based on whether a government's debt-to-GDP ratio converges to a certain level. We also explore the consequences of demand and the distributional impacts on social common capital accumulation, economic growth, and macroeconomic stability.

This study obtains three main results through analytical and numerical approaches. First, the Keynesian stability and Domar conditions are necessary for a long-run economically meaningful steady state and its asymptotical stability. Second, our model explains a proactive fiscal stance reflected by an increase in tax rates, and government expenditure propensity is essential for higher economic growth and a more sustainable economy through affluent social common capital. Third, a higher wage share contributes to stabilising the economy in three ways: by making an economy more likely to satisfy the Domar condition, mitigating distributional conflict between workers and capitalists, and establishing a more resilient economy by providing more affluent social common capital.

The rest of the paper is organised as follows. Section 2 sets up a Kaleckian model and considers its short- and long-run dynamics. Section 3 identifies the existence of the long-run steady state and its stability conditions. Through comparative statics analysis and numerical study, we reveal how the long-run steady state reacts to fiscal, demand, and income distribution shocks while observing the transitional dynamics. Section 4 concludes. The appendices provide mathematical proof for the main results.

## 2 Model

In building a Kaleckian model of social common capital and government debt accumulation, we suppose a closed economy with both private and government sectors. In an oligopolistic market, workers and firms managed by capitalists compose the private sector. Workers supply the labour force to firms, while firms hire workers and pay wages as per their employment. The firms implement productive investment, which we assume is gradually realised with some time lags. The capitalists earn profit by managing the firms and receive interest revenue by holding government bonds. The government sector imposes taxes on both wages and profit at the same rate. Based principally on its tax revenue, the government spends on goods consumption and invests in social common capital. These spendings constitute government expenditures, which are a part of aggregate demand. We call the parameters for government expenditures and tax rates fiscal stances. A proactive fiscal stance is defined as an increase in the propensity to consume and invest and in the tax rate. Our model explicitly incorporates the government investment that eventually generates social common capital stock, boosting private firm investment in the long run. Thus, we suppose that the stock of social common capital provides a foundation for firms to advance their desired investments in the long run. The capital composition changes over time, which reflects the size of an economy's social common capital. Simultaneously, it allows for a government budget deficit and debt financing, as the government's total expenditures always exceed its tax income. Accordingly, the government pays an interest rate on a unit of the issued debt to the capitalists who hold the bonds.

Output $Y$ is produced by firms using the following Leontief-type fixed-coefficient production function:

$$Y = \min(uK, q_L L) \tag{1}$$

where a firm's capital stock $K$ and labour input $L$ are perfect complements. $q_L$ is a constant labour productivity level. The potential output capital ratio is constant and set at unity, while the output capital ratio $u$ represents the capacity utilisation rate.[1] We assume that the output is determined by the operating capital stock under an effective demand constraint, that is, $Y = uK$.

Firms are oligopolistic in the goods market, and they set a mark-up over a unit labour cost to sell their goods at price level $p$. Then, pricing is given by:

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[1] Precisely, $u$ can be decomposed to $\frac{Y}{\bar{Y}} \cdot \frac{\bar{Y}}{K}$, where $\bar{Y}$ is the potential output. Because the potential output capital ratio $\frac{\bar{Y}}{K}$ is assumed to be unity, the output capital ratio $u$ reflects the capacity utilisation rate $\frac{Y}{\bar{Y}}$.

$$p = (1 + \eta) \frac{w}{q_L} \quad (2)$$

where $\eta > 0$ is a mark-up rate. We assume the price level and mark-up rate are constant over time. Thus, the price inflation rate is zero, which implies a change in labour productivity is entirely absorbed by the nominal wage.

As total income $pY$ is divided into wage $wL$ and profit $rpK$, we have:

$$pY = wL + rpK \quad (3)$$

Regarding the income distribution and pricing, we denote the profit share $m$ as

$$m = 1 - \frac{w}{pq_L}. \quad (4)$$

Accordingly, the wage share is expressed by $1 - m$, which we assume is constant over time. In the present setting, there is a one-to-one relationship of $m = \frac{\eta}{1+\eta}$ between the profit share and mark-up rate.

The government levies the same tax rate $\tau \in (0,1)$ on each income category. Thus, the government's tax revenue $T$ is given by:

$$T = \tau(wL + rpK) = \tau pY \quad (5)$$

The workers receive wage income and spend all their disposable income, whereas the capitalists earn profit and interest income from government bonds. We suppose, for simplicity, that the capitalists spend a fraction of their disposable profit but save all the interest income. Then, the private economy's consumption expenditures $C$ are:

$$C = (1 - \tau)(1 - m)uK + (1 - s)(1 - \tau)muK \quad (6)$$

where $s$ denotes the capitalists' savings rate from profit.[2]

We consider short- and long-run dynamics of firm investment decisions and the crowding-in effect of social common capital. Generally, the demand and productivity effects of social common capital

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[2] Originally, the consumption function for capitalists is defined as $(1 - s)(1 - \tau)muK + (1 - s_\delta)i\delta$, where $s_\delta$ is the propensity to save from interest income. By assuming $s_\delta$ is unity, we exclude the debt ratio's feedback to an economy's real side. Naturally, including this feedback is more realistic, but it simply complicates the analytical approach for our purposes, also making an economic interpretation of the results more difficult.

accumulation are not immediate but take time. In the short run, the investment demand and associated capital accumulation rate (i.e. the ratio of investment to capital stock) is supposed to be given by history. Therefore, short run investment demand $I$ is given by:

$$I = gK, \tag{7}$$

where $g$ is a constant. In the long run, however, we suppose the actual accumulation rate $g$ changes to achieve a firm's desired capital accumulation rate $g_d$ when there is a substantial demand effect of social common capital stock.

Government expenditures are principally financed by tax revenue, but the government can depend on debt financing by issuing government bonds. Let $\theta_C > 0$ denote the propensity for government consumption based on the tax income $\tau pY$; then, government consumption $G_C$ is given by:

$$G_C = \theta_C \tau u K \tag{8}$$

Similarly, by letting $\theta_S > 0$ represent the propensity for government investment in social common capital $G_S$, government investment is given by

$$G_S = \theta_S \tau u K \tag{9}$$

Thus, government expenditures are:

$$G_C + G_S = (\theta_C + \theta_S)\tau u K \tag{10}$$

Equations (6), (7), and (10) comprise aggregate demand in the short run. It should be highlighted that our model allows for government debt; consequently government expenditures can exceed tax income. Hence, as we will see below, $\theta_C + \theta_S > 1$ always holds, which dynamically determines the degree of fiscal deficit in the flow term and the debt ratio in the stock term.

### 2.1 Short-run steady state

The short run is a period in which the firm capital accumulation rate remains constant, and accumulations of government debt and social common capital have not yet been realised. The demand for and supply of goods are exclusively and instantaneously adjusted by a change in the capacity utilisation rate. Then, the equilibrium in the goods market can be represented by:

$$Y = C + I + G_C + G_S \tag{11}$$

By substituting Equations (6), (7), and (10) into (11) and dividing both sides by capital stock, the short-run steady state for the capacity utilisation rate is

$$u = \frac{g}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}, \tag{12}$$

where the denominator is assumed to be positive by imposing $sm(1-\tau) > \tau(\theta_C + \theta_S - 1)$.

In Equation (12), the steady state capacity utilisation rate rises following a fall in the savings rate and profit share. Thus, the paradoxes of thrift and cost are true in the short run. Moreover, an increase in the tax rate also increases the capacity utilisation rate, as the government spends more than its tax revenue. It is also true that increases in the two types of government expenditures stimulate the capacity utilisation rate.

### 2.2 Long-run dynamics

The long run is defined as a period where a firm's desired capital accumulation rate is gradually realised. Simultaneously, the economy's capital composition and the government's debt evolve as its social capital accumulation. Importantly, in the long run, the stock effects of social common capital are thoroughly effective. To elaborate, tax income varies according to changes in output, which affects government expenditures. As per its accumulation, an increase in social common capital stock supports firms' desired capital accumulations.[3] During these processes, the government depends on debt financing based on the gap between total government expenditures plus interest payments and tax revenue. This section sets up these dynamic interactions.

First, we suppose a firm's actual accumulation rate $g$ changes gradually to realise its desired capital accumulation rate $g_d$ in an adaptive manner:

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[3] In addition, accumulating social common capital may enhance labour productivity growth (Tavani & Zamparelli, 2020; Oyvat & Onaran, 2022; Nishi & Okuma 2023), subsequently inducing changes in both the income distribution and employment. However, we assume the income distribution and employment rate are constant. This assumption implicitly requires that the nominal wage change rate adjusts to accommodate the labour productivity growth rate (i.e. $\hat{w} = \hat{q}_L$ behind Eq. 4), and that the natural growth rate accommodates the long run economic growth rate (i.e. $\hat{q}_L + n = g^*$ at a steady state). Nishi and Okuma (2023) carefully investigate these dynamics under alternative growth and distribution regimes. We do not explicitly consider these effects, but instead exclusively focus on the causes and consequences of government debt and social common capital accumulation.

$$\dot{g} = \phi(g_d - g) \tag{13}$$

where $\phi$ is a positive parameter representing the speed at which the actual capital accumulation rate adjusts towards its desired rate. We introduce a firm's desired capital accumulation rate $g_d$ as follows:

$$g_d = \alpha + \beta(1-\tau)mu + \gamma\chi \tag{14}$$

where $\alpha > 0$ is a constant term driven by the firm's so-called animal spirits, and $\beta > 0$ represents the sensitivity of the desired capital accumulation to a change in the after-tax profit rate.[4] We exclude an interest rate crowding-out effect on private investment because this effect is empirically small (Oyvat & Onaran, 2022). Importantly, a firm's desired capital accumulation rate complements the stock of social common capital. Its sensitivity $\gamma > 0$ represents how much an increase in the social common capital expenditure eventually induces firm investment and the associated production in the long run. Thus, we define the size of $\gamma$ as the degree of the crowding-in effect of social common capital.[5] Size is exogenously given in our model, but in reality, it naturally concerns the quality and complementarity of social common capital with private capital investment. The better the quality or the more complementary these capitals are, the higher the crowding-in effects are, and vice versa.

By substituting Equation (14) into Equation (13), the dynamics of a firm's capital accumulation rate follows

$$\dot{g} = \phi(\alpha + \beta(1-\tau)mu + \gamma\chi - g). \tag{15}$$

The profit rate is $r = mu$, and substituting Equation (12) into it yields:

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[4] Thus, we employ a traditional Kaleckian investment function (Dutt, 1984; Rowthorn, 1981; Taylor, 1985), where the profit rate (i.e. the product of the capacity utilisation rate and profit share) principally matters for investment decisions. Indeed, choosing the profit rate as an independent variable makes the growth regime more wage-led, but doing allows us to carefully consider whether this regime is compatible with fiscal viability while providing social common capital. In Bhaduri and Marglin type functions, which generate both wage-led and profit-led growth regimes, see also Nishi and Okuma (2023).

[5] The demand effect of government expenditures or social common capital to expand private investment is empirically supported. For instance, Obst et al. (2020) found that for most of the EU's 15 countries, government spending has boosted economic activities while also inducing private investment demand. In a growth regime analysis for the UK, Onaran et al. (2022) confirm that social common capital spending has a positive effect on output and productivity growth.

$$\dot{g} = \phi\left(\alpha + \frac{\beta(1-\tau)m}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)} g + \gamma\chi - g\right), \tag{16}$$

which is the first state variable in our dynamic model. When the capacity utilisation rate becomes constant, $g$ also represents the economic growth rate (i.e. the output growth rate). Thus, we use the capital accumulation and economic growth rates alternatively in the long-run analysis.

Second, government investment generates an accumulation of social common capital (i.e. $\dot{S} = G_S$) in the long run. Therefore, social common capital grows at the following rate $g_S$

$$g_S = \frac{\dot{S}}{S} = \frac{\theta_S \tau u}{\chi} \tag{17}$$

where the capacity utilisation rate $u$ always follows equation (12). For simplicity, we do not consider the depreciation rates of capital stock or social common capital. Hence, the dynamics of capital composition $\chi$ are given by:

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{S}}{S} - \frac{\dot{K}}{K} \tag{18}$$

By substituting Equation (17) with a few arrangements, we obtain

$$\dot{\chi} = g\left(\frac{\theta_S \tau}{\chi\big(sm(1-\tau) - \tau(\theta_C + \theta_S - 1)\big)} - 1\right), \tag{19}$$

which is the second state variable in our dynamic model.

Finally, our model incorporates the government's debt accumulation $D$ over time. As we have seen, government expenditures are $G_C + G_S = (\theta_C + \theta_S)\tau u K$, whereas the tax revenue is $T = \tau u p K$. Additionally, the government must pay interest on the existing debt at a nominal interest rate $i$. As the inflation rate is zero, the nominal interest rate $i$ equals the real rate. Then, total government expenditures are:

$$p(G_C + G_S) + iD = (\theta_C + \theta_S)\tau u p K + i\delta p K. \tag{20}$$

The government depends on debt financing to fill the gap between total government expenditures plus interest payments and tax revenue. Hence, the government budget constraint or the dynamics of debt are defined as

$$\begin{aligned} \dot{D} &= (\theta_C + \theta_S)\tau u p K + i\delta p K - \tau u p K \\ &= (\theta_C + \theta_S - 1)\tau u p K + i\delta p K \end{aligned} \tag{21}$$

where $\theta_C$ and $\theta_S$ discretionarily move over unity according to a government's fiscal stance (i.e.

$\theta_C, \theta_S$, and $\tau$). Thus, government expenditures always exceed revenue, depending on the debt financing. Then, we consider the dynamics of the government's debt-capital ratio $\delta = \frac{D}{pK}$, particularly focusing on whether it will converge to a certain ratio.[6] The rate of change in the government's debt-capital ratio is given by

$$\frac{\dot{\delta}}{\delta} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K} \tag{22}$$

Hence, we have

$$\dot{\delta} = \frac{\dot{D}}{pK} - g\delta \tag{23}$$

where the price inflation rate is zero. By substituting Equations (12) and (21) into (23) with a few arrangements, we get

$$\dot{\delta} = \frac{sm(1-\tau)g}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)} - g - (g - i)\delta \tag{24}$$

which is the third state variable in our dynamic model.

## 3 Analysis

### 3.1 Existence and stability of the long-run steady state

This section confirms the existence of the long-run steady state and derives its stability condition. We examine the long-run effects of social common capital accumulation and its relationship with the fiscal sustainability of the government and economic growth.

Our differential equation system consists of the dynamics of capital composition $\chi$, actual economic growth rate $g$, and the government's debt ratio $\delta$. Additionally, as the quantity adjustment is instantaneous, Equation (12) is always true. The long-run steady state is a solution of simultaneous equations for $\dot{\chi} = 0$, $\dot{g} = 0$, and $\dot{\delta} = 0$ to be realised. The non-trivial steady state values for capital composition, the actual economic growth rate, and the government's debt ratio satisfy the following relationship:

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[6] Because we assume that the potential output capital ratio is unity, we have $K = \bar{Y}$. Thus, the government debt capital ratio is equal to $\delta = \frac{D}{p\bar{Y}}$, which economically reflects the ratio of government debt to the potential GDP.

$$\chi^* = \frac{\tau\theta_S}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)} \tag{25}$$

$$g^* = \frac{\alpha\big(sm(1-\tau) - \tau(\theta_C + \theta_S - 1)\big) + \gamma\tau\theta_S}{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)} \tag{26}$$

$$\delta^* = \frac{g^*\tau}{g^* - i}\left(\frac{\theta_C + \theta_S - 1}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}\right) \tag{27}$$

where the asterisk denotes the long-run steady state value. Because the actual economic growth rate $g$ becomes constant, the long-run capacity utilisation rate is also constant at the following level

$$u^* = \frac{\alpha + \gamma\chi^*}{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)} \tag{28}$$

Importantly, we need the Keynesian stability condition

$$(1-\tau)(s-\beta)m > \tau(\theta_C + \theta_S - 1) \tag{29}$$

so that positive and economically meaningful values for both short-run and long-run economic growth and capacity utilisation rates are obtained.[7] An important implication here is that an excessive increase in the tax rate and government consumption and the investment propensity may violate the stability condition, although it increases capital composition and the economic growth rate.

As far as the Keynesian stability condition and the following Domar condition are satisfied, there exists a unique steady state value for $\chi$, $g$, $\delta$, and $u$. A detailed proof for the asymptotic stability of the long-run steady state is given in Appendix 1. By introducing the main proposition obtained from our model, we briefly discuss its economic implications.

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[7] The Keynesian stability condition means that aggregate supply increases more than aggregate demand in response to an increase in output (Bhaduri and Marglin, 1990). Standard Kaleckian models suppose no time-lag in investment, and the actual accumulation rate is realised simultaneously with the capacity utilisation rate. If so, $g = g_d$ always holds in our framework and

$$(1-\tau)(s-\beta)m > \tau(\theta_C + \theta_S - 1)$$

is necessary to stabilise the capacity utilisation rate adjustment and its positive value. This is exactly the same as the Keynesian stability condition. It also ensures an economically meaningful steady state for the short-run capacity utilisation rate in Equation (12) as $(1-\tau)sm > (1-\tau)(s-\beta)m$.

**Proposition 1.** The long-run steady state is locally and asymptotically stable if the economic growth rate is higher than the real interest rate.

The condition that the economic growth rate is higher than the real interest rate is well known as the Domar condition (Domar, 1944). Although the absolute amount of government debt increases over time, this condition ensures that the government's debt-to-GDP ratio converges to a certain level. When the Domar and Keynesian stability conditions are satisfied, we obtain a long-run steady state with the following characteristics. First, as the capacity utilisation rate is constant, the economic growth rate is the same as the capital accumulation rate. Second, the government's debt in the economy always has a positive value as $\theta_C + \theta_S > 1$ holds. Third, as the debt ratio and capital composition remain constant, social common capital $S$, output $X$, and the government's debt $D$ grow at the same rate. We consider more detailed properties of the steady state and transitional dynamics through comparative statics and numerical analyses in sections 3.2 and 3.3, respectively.

### 3.2 Comparative statics analysis

We focus on the fiscal, demand, and distribution parameters in conducting a comparative statics analysis to obtain their long-run effects on capital composition, the economic growth rate, debt ratio, and capacity utilisation rate. Below, we impose Keynesian and Domar conditions to ensure economically meaningful values and obtain the results. Appendix 2 provides the analysis details. Because how these parameters impact the government's debt ratio $\delta^*$ is not analytically complicated, the results depend on the numerical studies.

**Table 1: Results for comparative statics analysis**

| | (A) Fiscal stance | | | (B) Demand and distribution | | |
|---|---|---|---|---|---|---|
| | $\tau$ | $\theta_C$ | $\theta_S$ | $s$ | $\gamma$ | $m$ |
| $\chi^*$ | + | + | + | − | 0 | − |
| $g^*$ | + | + | + | − | + | − |
| $\delta^*$ | + | + | + | − | − | − |
| $u^*$ | + | + | + | − | + | − |

(Note) $+$ and $-$ mean increases in the parameters increase and decrease the steady state values, respectively.

Table 1 summarises the main results. Overall, our model derives Kaleckian implications and the expected effects of social common capital. Part (A) shows the impact of a proactive fiscal stance, which is defined as an increase in the tax rate and government expenditure propensity. An increase in the tax rate $\tau$, the government's consumption propensity $\theta_C$, and investment propensity $\theta_S$ incudes a higher capital composition, economic growth rate, and capacity utilisation rate. It also leads to an increase in the debt ratio. Thus, a proactive fiscal stance can simultaneously realise affluent social common capital and a higher economic growth rate. Paradoxically, an increase in the tax rate $\tau$ does not reduce the debt ratio in our model. This is because an increase in $\tau$ always urges the government to spend more than before, stimulating effective demand. Consequently, the capital accumulation continues due to the demand expansion on one hand, while it induces the government to borrow more than to increase output on the other hand.[8] Thus, increases in the fiscal stance parameters generally expand the government's debt ratio; however, the ratio does not diverge under the Domar condition. These results can be summarised in Proposition 2.

**Proposition 2.** A proactive fiscal stance contributes to increases in social common capital and the economic growth rate. It also increases the debt ratio.

Meanwhile, part (B) in Table 1 shows a Kaleckian outcome where increases in the profit share $m$ (a decrease in the wage share) and saving rate decrease the capital composition and lower the

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[8] Government expenditures are always linked to the tax rate $\tau$, as Equation (10) shows. Accordingly, an increase in $\tau$ does not improve the fiscal balance but leads to an increase in government expenditures. Thus, we assume the government always runs a deficit because our question is about identifying the condition under which the fiscal deficit is sustainable (i.e. a constant ratio to GDP). Therefore, in our model, raising the tax rate increases the fiscal deficit. In the post-Keynesian literature, the phenomenon that firms' debt reduction behaviour and governments unintentionally increase debt is known as the 'paradox of debt' (Ryoo, 2013). Following the implication of this paradox, the result obtained here may be described as the 'paradox of a tax hike.'

economic growth rate. Thus, wage-led growth and the paradox of thrift are established in the long run. A higher profit share or higher saving rate lowers private consumption and thus, the capacity utilisation rate; moreover, it decreases the tax revenue to be spent on government expenditures. Consequently, accumulating social common capital decelerates, also negatively impacting the economic growth rate; that is, a crowding-in effect negatively works on economic growth. An increase in the profit share and saving rate generally decreases the government's debt ratio by the opposite mechanism mentioned above. Changes in the income distribution, particularly an increase in the wage share (i.e. a decrease in the profit share) has an important implication for a wage-led growth economy, which we explain in detail in the numerical study in Section 3.3. Finally, an increase in the crowding-in effect $\gamma$ increases the economic growth rate. Although it does not impact the capital composition in the steady state, an increase in this effect decreases the government debt ratio. Thus, our model derives an important implication that if the government aims to reduce its debt ratio while increasing the economic growth rate, it should improve the quality and complementarity of social common capital with private capital investment. We can summarise this in Proposition 3.

**Proposition 3.** An increase in the profit share and saving rate simultaneously decreases social common capital and economic growth, whereas an increase in the crowding-in effect increases the economic growth rate without affecting capital composition. In both cases, the government debt ratio converges to a lower value.

### 3.3 Numerical study

The comparative statics analysis shows how the endogenous variables shift from one steady state to another but does not demonstrate how they behave during transitional dynamics. Therefore, we conduct a numerical study to visually trace the dynamic path of capital composition, the economic growth rate, debt ratio, and profit rate due to changes in the relevant parameters. The parameter values to simulate our theoretical model are given in Table 2.[9]

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[9] These parameters satisfy the stability conditions examined in the previous section, but they are not selected based on a calibration for the real economy, which may be seen as a limitation of our study. However, the study's purpose is to visually consider how transitional dynamics change, and they suffice

**Table 2: Parameter values for baseline and shock scenarios**

| | $\alpha$ | $\beta$ | $\gamma$ | $ss\tau$ | $s$ | $m$ | $\theta_C$ | $\theta_S$ | $i$ | $\phi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Baseline | 0.01 | 0.02 | 0.03 | 0.1 | 0.3 | 0.35 | 0.6 | 0.6 | 0.02 | 0.05 |

To examine the consequences of fiscal, demand, and distribution shocks, first, we calculate the initial steady state values by applying these values—approximately $\chi^* = 0.805369$, $g^* = 0.0373167$, and $\delta^* = 0.578511$—to our theoretical model. Then, starting with these initial positions, we apply a 1% positive shock to each fiscal parameter (i.e. $\tau$, $\theta_C$, and $\theta_S$) at $t = 10$ and plot the change in paths in Figure 1. While applying these shocks, other parameters are set to remain constant (i.e. when the tax rate is increased by 1%, the values for $\theta_C$, and $\theta_S$ do not change). Similarly, using the same initial positions, we apply a 1% positive shock to each demand and distribution parameter (i.e. $s$, $\gamma$, and $m$) and plot the change in paths in Figure 2.

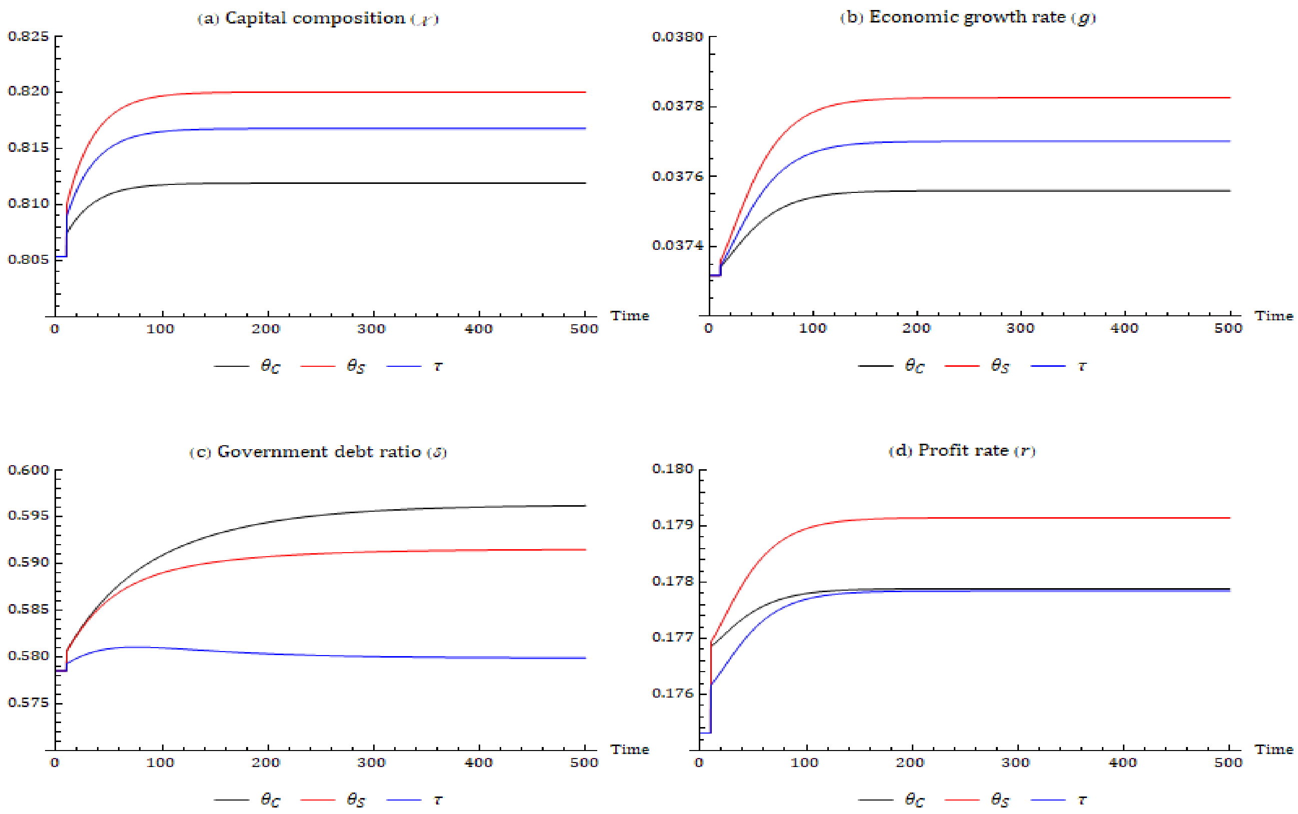


**Figure 1:** Fiscal stance shock and transitional path to the steady states

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for that purpose. The Mathematica codes for the numerical study are available upon request.

Figure 1 shows the transitional path for (a) capital composition, (b) the economic growth rate, (c) the debt ratio, and (d) the profit rate after a change in the fiscal stance. Our numerical study demonstrates that a proactive fiscal stance ultimately increases all these variables, and the transition to the new steady state is monotonic from the initial position. Social common capital accumulation is positively associated with the economic growth rate after these shocks. Thus, we can confirm that output and social common capital primarily grow hand-in-hand. Moreover, this stance also leads to a higher profit rate, which is driven by an increase in the capacity utilisation rate. However, as observed, the magnitude of the fiscal parameters' impact on transitional dynamics differs. Even if the shocks are applied to the same degree (1%), an increase in investment propensity $\theta_S$ increases capital composition more than any of the other shocks; accordingly, the economic growth rate reaches its highest level due to this impact. This occurs because an increase in $\theta_S$ increases effective demand while leading to a social common capital accumulation that eventually stimulates economic growth through the crowding-in effect on the firm's desired accumulation rate.

A policy implication derived from Figure 1 is that each fiscal policy is effective for increasing social common capital and the associated economic growth rate. Thus, our model reveals that social common capital accumulation is a driving force for economic growth. It is important for the government to control the tax rate ($\tau$) and expenditure propensity ($\theta_C$ and $\theta_S$) to realise high and sustainable economic growth. However, a more proactive fiscal stance is necessarily accompanied by a higher debt ratio. Hence, if the government aims to achieve the most affluent social common capital and highest economic growth, it should increase investment propensity. However, if the goal is to restrain the debt ratio increase to the minimum possible while maintaining a certain economic growth rate, it should raise the tax rate.

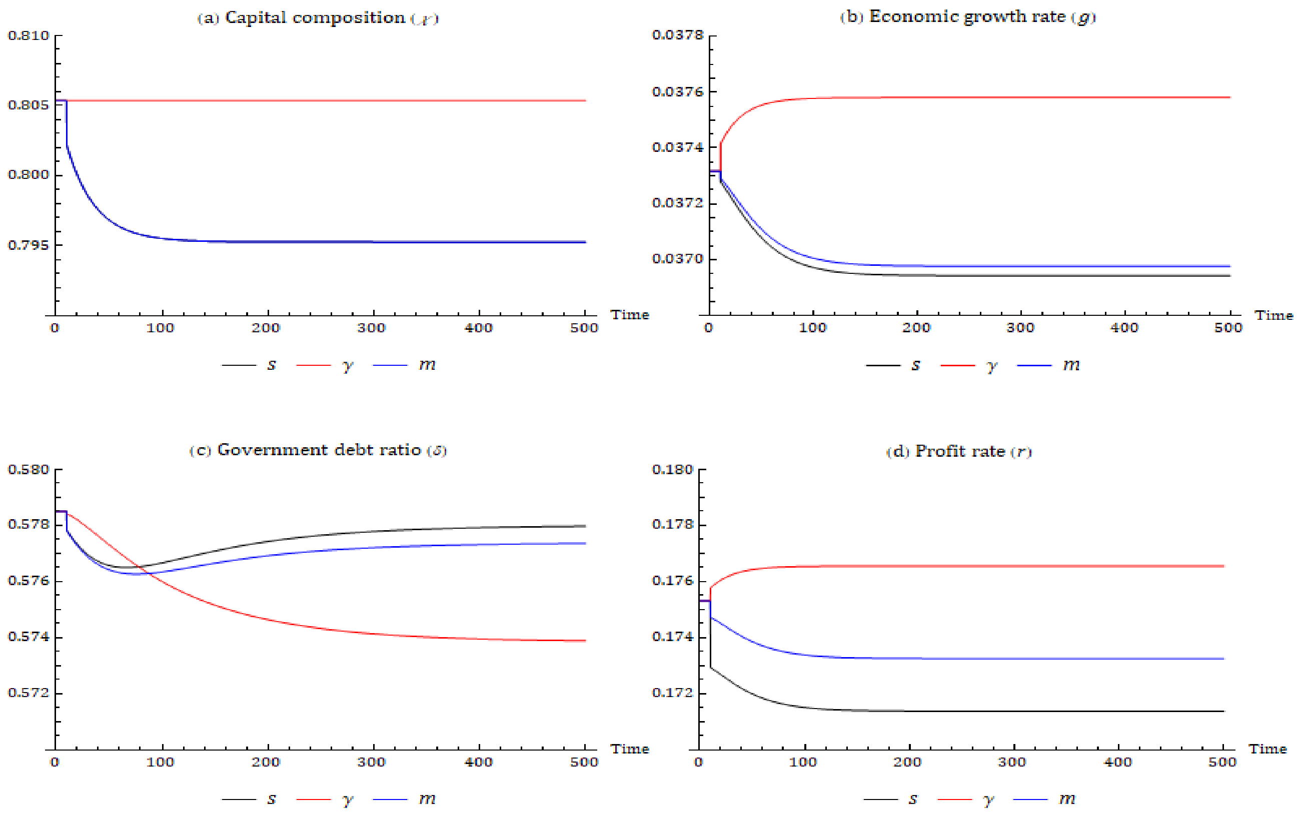


**Figure 2:** Demand and distribution shocks and transitional paths to the steady states

For these same variables, Figure 2 shows the transitional paths due to a shock in the demand and distribution parameters. An increase in the profit share (a decrease in the wage share) and saving rate lowers both the economic growth rate and capital composition by almost the same degree. These shocks also hurt the profit rate. Thus, we note that even if the profit share increases, it does not lead to a higher profit rate because a subsequent decrease in the capacity utilisation rate is more than the initial rise in the profit share. These transitions are monotonic, but the debt ratio moves lower after an increase in these parameters while undergoing a slight overshoot in the short run. This happens because these impacts immediately decrease the capacity utilisation rate, which gradually lowers private capital accumulation under wage-led growth. Meanwhile, an increase in the crowding-in effect $\gamma$ generates different dynamics than those of profit share and saving. Indeed, it does not impact the capital composition. However, an increase in this effect increases the capacity utilisation rate and the subsequent increase in government expenditures. Because the economic growth rate largely rises through the crowding-in effect, it eventually leads to a lower government debt ratio. Thus, the numerical study also highlights the importance of improving the quality and complementarity between social common capital and private capital investment to simultaneously stimulate the economic growth rate and reduce the debt ratio.

Our analytical and numerical studies imply that a change in the income distribution plays an important role in stability and wage-led economic growth. We have demonstrated that an increase in the wage share leads to a higher economic growth rate while promoting social common capital accumulation in an economy. We have also shown that the economic growth rate must be higher than the interest rate to satisfy the stability condition. Moreover, its impact eventually raises the profit rate, although the profit share is squeezed due to a higher wage share. Thus, a higher wage share promotes not only social capital accumulation but also cooperation between the workers and capitalists in an economy, realising an equitable income distribution and higher economic growth. To summarise, a higher wage share contributes to stabilising the economy in three ways First, by raising the economic growth rate, the Domar condition is more likely to be satisfied. Second, by accruing higher wages for workers and a higher profit rate for capitalists, it can mitigate the distributional conflict between workers and capitalists.[10] Third, by providing more social common capital, it contributes to establishing a more resilient economy. Although our approach is based on a formal model, we should not ignore that, in reality, social common capital broadly includes education, healthcare, and the environment as well as the social infrastructure for goods production, as indicated by Uzawa (2005). Not only increasing its availability, but also improving its quality contributes to enhancing the foundation for economic development with an attractive culture, well-being, and basic needs for the current and future generations.

## 4 Conclusion

The roles of the government and social common capital have become ever greater in importance for establishing a resilient and sustainable economy. Social common capital provides society with an attractive culture, well-being, and lifestyle; it also supports private economic activities. Thus, it is essential

[10] We follow Bhaduri and Marglin (1991) in referring to the cooperation and conflict regime for income distribution. Conflict occurs when the wage share and profit rate move in opposite directions, while cooperation occurs if they move in the same direction. Specifically, as a higher wage share stimulates the capacity utilisation rate in our model, it elucidates the existence of stagnation and cooperation regimes in their terminology.

for the market economy and our lives. Simultaneously, the government is asked to be fiscally sustainable in the long run, which has become a major concern of policymakers.

Although Kaleckian models have explored the relationship between economic growth, government expenditures, and the debt ratio, previous studies have performed only a partial analysis focusing on just some of them. Our model is more comprehensive, as it has two types of government expenditures and endogenous accumulation of social common capital and government debt. Our model explicitly incorporates the stock effect of social common capital, which induces the firm's capital accumulation rate in the long run. It also reveals the effects of changes in demand and the income distribution on overall economic performance in a wage-led economy. Based on this model, we derive the following three main conclusions.

First, our analyses show that government can provide affluent social common capital stock, increase the economic growth rate, and control its debt ratio to avoid exploding in the long run; however, two important conditions must be satisfied—the Keynesian stability condition and the Domar condition. The former states that aggregate supply increases more than aggregate demand in response to an increase in output, whereas the latter requires the economic growth rate to be higher than the real interest rate. Although these conditions are well-established in previous studies, our analyses confirm they remain essential for a Kaleckian growth model with explicit government activities. Furthermore, when these conditions are satisfied, we obtain the second and third conclusions.

Second, accumulating social common capital stock through a proactive fiscal stance promotes a higher long-run economic growth rate. An increase in the tax rate and the propensity of government expenditures increase the capital composition and economic growth rate. Among these, an increase in the government's propensity to invest is the most important driving force for social common capital accumulation and subsequent economic growth. Government investment increases not only effective demand in the short run but eventually stimulates long-run economic growth by inducing the firm's desired accumulation rate.

Finally, the model also elucidates the positive impacts of the demand and distribution parameters on growth and stability. The so-called paradox of cost and thrift holds in the long run, and wage-led growth is established. Importantly, we demonstrated that a higher wage share contributes to stabilising an economy in three ways. An increase in the wage share increases both the capital composition and economic growth, making an economy more likely to satisfy the Domar condition. A higher wage share can also mitigate the distributional conflict between workers and capitalists and realise an equitable income distribution and economic growth, as the former receive a higher wage share whereas the latter benefit from a higher profit rate. Furthermore, a wage share increase contributes to establishing a more resilient economy by providing affluent social common capital. Our model supposes the social infrastructure, but in reality, social common capital broadly includes education, healthcare, and the environment. Hence, enhancing its quantity and quality is the foundation of economic development with an attractive culture, well-being, and basic needs for current and future generations.

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## Appendix 1: Proof of proposition 1

The dynamic system consists of Equations (16), (19), and (24), of which the element of Jacobian matrix $\mathcal{J}^*$ evaluated at the long run steady state is given as follows:

$$j_{11} = \frac{\partial \dot{\chi}}{\partial \chi} = -g^*$$

$$j_{12} = \frac{\partial \dot{\chi}}{\partial g} = 0$$

$$j_{13} = \frac{\partial \dot{\chi}}{\partial \delta} = 0$$

$$j_{21} = \frac{\partial \dot{g}}{\partial \chi} = \gamma\phi$$

$$j_{22} = \frac{\partial \dot{g}}{\partial g} = \phi\left(\frac{\beta m(1-\tau)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)} - 1\right)$$

$$j_{23} = \frac{\partial \dot{g}}{\partial \delta} = 0$$

$$j_{31} = \frac{\partial \dot{\delta}}{\partial \chi} = 0$$

$$j_{32} = \frac{\partial \dot{\delta}}{\partial g} = -1 - \delta^* + \frac{sm(1-\tau)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}$$

$$j_{33} = \frac{\partial \dot{\delta}}{\partial \delta} = -g^* + i$$

The characteristic equation associated with the Jacobian matrix can be defined by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \qquad (A1)$$

where $\lambda$ denotes the characteristic root. Coefficients $a_1$, $a_2$, and $a_3$ are respectively given as follows:

$$a_1 = 2g^* - i + \phi\left(\frac{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}\right) \qquad (A2)$$

$$a_2 = g(g^* - i) + \phi(2g^* - i)\left(\frac{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}\right) \qquad (A3)$$

and

$$a_3 = g^*(g^* - i)\phi\left(\frac{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)}\right) \quad (A4)$$

for which we apply the following notation below

$$\sigma \equiv \frac{(1-\tau)(s-\beta)m - \tau(\theta_C + \theta_S - 1)}{sm(1-\tau) - \tau(\theta_C + \theta_S - 1)} \quad (A5)$$

Because we have the Keynesian stability condition by Equation (29), $\sigma$ always takes a positive value.

The steady state values are independent of $\phi$. According to the Routh–Hurwitz criterion, the necessary and sufficient condition for the local asymptotic stability of the long-run steady state is:

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0, \text{ and } a_1 a_2 - a_3 > 0$$

Considering $a_3$ first, it is positive as far as $g^* > i$ is ensured. Suppose this inequality is valid, then both $a_1$ and $a_2$ are positive as can be observed in Equations (A2) and (A3). Moreover, with a few arrangements, we obtain:

$$\begin{aligned} a_1 a_2 - a_3 &= (2g^* - i)(g^* + \phi\sigma)(g^* - i + \phi\sigma) \\ &= (2g^* - i)\sigma^2\phi^2 + (2g^* - i)^2\sigma\phi + (2g^* - i)(g^* - i)g^* \end{aligned} \quad (A6)$$

which is a convex quadratic function of the adjustment speed of actual capital accumulation rate $\phi$. It has a positive y-axis and negative x-axis when the Keynesian stability condition and $g^* > i$ are satisfied. It thus means that $a_1 a_2 - a_3 > 0$ is always true for positive value of $\phi$. Hence, the stability condition can ultimately be summarised as

$$g^* > i \quad (A7)$$

where $g^*$ is given by Equation (26). This is exactly the same as the Domar condition.

## Appendix 2: Comparative statics analysis

(1) The impact of a rise in $\theta_C$:

$$\frac{\partial \chi^*}{\partial \theta_C} = \frac{\tau^2 \theta_S}{\left(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\right)^2} > 0$$

$$\frac{\partial g^*}{\partial \theta_C} = \frac{\tau(\alpha\beta(1-\tau)m + \gamma\tau\theta_S)}{((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1))^2} > 0$$

$$\frac{\partial u^*}{\partial \theta_C} = \frac{\tau}{\left((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\right)^2}\left(\alpha + \frac{\gamma\tau\theta_S\left(m(2s-\beta)(1-\tau) - 2\tau(\theta_C + \theta_S - 1)\right)}{\left(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\right)^2}\right) > 0$$

where $m(2s-\beta)(1-\tau) - 2\tau(\theta_C + \theta_S - 1)) > (s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1) > 0$.

(2) The impact of a rise in $\theta_S$:

$$\frac{\partial \chi^*}{\partial \theta_S} = \frac{\tau(sm(1-\tau) + (1-\theta_C)\tau)}{\big(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} > 0$$

$$\frac{\partial g^*}{\partial \theta_S} = \frac{\tau(m(\alpha\beta + \gamma(s-\beta))(1-\tau) + \gamma\tau(1-\theta_C))}{((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1))^2} > 0$$

$$\frac{\partial u^*}{\partial \theta_S} = \frac{\tau}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2}$$

$$\left( \frac{\alpha + \dfrac{\gamma\tau\theta_S}{s(1-\tau)m - \tau(\theta_C + \theta_S - 1)} + }{\dfrac{\gamma(s(1-\tau)m + \tau(1-\theta_C))((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1))}{(s(1-\tau)m - \tau(\theta_C + \theta_S - 1))^2}} \right) > 0$$

(3) The impact of a rise in $\tau$:

$$\frac{\partial \chi^*}{\partial \tau} = \frac{sm\theta_S}{\big(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} > 0$$

$$\frac{\partial g^*}{\partial \tau} = \frac{\alpha\beta(\theta_C + \theta_S - 1) + (s-\beta)\gamma}{((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1))^2} m > 0$$

$$\frac{\partial u^*}{\partial \tau} = \frac{1}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} \left( \frac{s\gamma\theta_S\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)m}{\big((1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} \right.$$

$$\left. + \big((s-\beta)m + \theta_C + \theta_S - 1\big)\left(\alpha + \frac{\gamma\tau\theta_S}{s(1-\tau)m - \tau(\theta_C + \theta_S - 1)}\right) \right) > 0$$

(4) The impact of a rise in $m$:

$$\frac{\partial \chi^*}{\partial m} = \frac{-s(1-\tau)\tau\theta_S}{\big(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} < 0$$

$$\frac{\partial g^*}{\partial m} = \frac{-(1-\tau)\tau(\alpha\beta(\theta_C + \theta_S - 1) + (s-\beta)\gamma)}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} < 0$$

$$\frac{\partial u^*}{\partial m} = \frac{-(1-\tau)}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2}$$

$$\frac{\partial u^*}{\partial m} = \frac{(s-\beta)(1-\tau)(\alpha + \gamma\chi^*)}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} \gamma\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)\frac{\partial \chi^*}{\partial m} < 0$$

(5) The impact of a rise in $s$:

$$\frac{\partial \chi^*}{\partial s} = \frac{-m(1-\tau)\tau\theta_S}{\big(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} < 0$$

$$\frac{\partial g^*}{\partial s} = \frac{-m(1-\tau)(\alpha\beta(1-\tau)m + \gamma\tau\theta_S)}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2} < 0$$

$$\frac{\partial u^*}{\partial s} = \frac{-(1-\tau)m}{\big((s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)^2}\left(\alpha + \frac{\gamma\tau\theta_S((2s-\beta)(1-\tau)m - 2\tau(\theta_C + \theta_S - 1))}{(s(1-\tau)m - \tau(\theta_C + \theta_S - 1))^2}\right) < 0$$

where $(2s-\beta)(1-\tau)m - 2\tau(\theta_C + \theta_S - 1) > s(1-\tau)m - \tau(\theta_C + \theta_S - 1) > 0$.

(6) The impact of a rise in $\gamma$:

$$\frac{\partial \chi^*}{\partial \gamma} = 0$$

$$\frac{\partial g^*}{\partial \gamma} = \frac{\tau\theta_S}{(s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)} > 0$$

$$\frac{\partial u^*}{\partial \gamma} = \frac{\tau\theta_S}{(s-\beta)(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big(s(1-\tau)m - \tau(\theta_C + \theta_S - 1)\big)} > 0$$